CHAPTER - 3

SENSITIVITY

SENSITIVITY

In the design of active R filters,the design parameters are assumed to be constants. The parameters may be passive, like resistors or active like gain bandwidth product. Parameters depend upon many factors like temperature,humidity,aging etc. As such , these parameters cannot be considered as constants. This deviation of active and passive parameters from their nominal values result in a change in the response of the network.

In fact,for a highly selective network this may induce instability and hence oscillations. The variation of the network response due to an increamental change in the value of the parameter is expressed in terms of the " sensitivity" of the particular network function with respect to that parameter.

The sensitivity of a network function `F′ with respect to a parameter `X′ is defined as

$$S_X^F = \frac{X}{F} \frac{dF}{dX} \tag{3.1}$$

various network functions give rise to the various sensitivity functions. Sensitivity of a transfer function T(S) with respect to a parameter `X′ is called transfer function sensitivity. In general, it is a complex quantity . The real part is called the `gain sensitivity′ and imaginary part is called the `phase sensitivity ′. Definitions of different kinds of sensitivities and ways of evaluating the sensitivity of a circuit are discussed below.

3.1) ω and Q Sensitivity :-

In a qualitative sense ,the sensitivity of a network is a measure of the degree of variation of its performance from nominal, due to changes in the elements constituting the network. A biquadratic filter function can be expressed in terms of the parameters ω_p, ω_z, Q_p ,Q_z and K, as

$$T(S) = K \frac{S^2 + \frac{\omega_z}{Q_z} S + \omega_z^2}{S^2 + \frac{\omega_p}{Q_p} S + \omega_p^2} \qquad (3.2)$$

Sensitivities of these biquadratic parameters to the elements can be studied and evaluated as given below.

The sensitivity of the pole frequency ω_p to a change in a resistor R i.e. pole sensitivity is defined as the per unit change in the pole frequency , $\Delta\omega_p/\omega_p$, caused by a per unit change in the resistor , $\Delta R/R$. Mathematically

$$S_R^{\omega_p} = \lim_{\Delta R \to 0} \frac{\frac{\Delta\omega_p}{\omega_p}}{\frac{\Delta R}{R}} \qquad (3.3)$$

$$= \frac{R}{\omega_p} \frac{\partial\omega_p}{\omega_R} \qquad (3.4)$$

This is equivalent to

$$S_R^{\omega_p} = \frac{\partial(\ln \omega_p)}{\partial(\ln R)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.5)$$

It should be noted that the cost of manufacturing a component is a function of the percentage change ($100 \times \Delta R/R$) rather than the absolute change(ΔR) of the component . For this reason it is desirable to measure sensitivity in terms of the relative changes in components , as is done in equation (3.3).

The sensitivities of the parameters ω_z, Q_p, Q_z and K to any element of the network are defined in a similar way.

$$S_R^{Q_p} = \frac{R}{Q_p}\frac{\partial Q_p}{\partial R}, \quad S_R^{Q_z} = \frac{R}{Q_z}\frac{\partial Q_z}{\partial R},$$

$$S_R^{K} = \frac{R}{K}\frac{\partial K}{\partial R}, \quad S_R^{\omega_z} = \frac{R}{\omega_z}\frac{\partial \omega_z}{\partial R}$$

Equation 3.4 can be used to develop some useful rules that simplify sensitivity calculations. The sensitivity of parameter p to an element x is

$$S_x^P = \frac{x}{p}\frac{dp}{dx} = - \frac{\partial(\ln p)}{\partial(\ln x)} \tag{3.6}$$

If p is not a function of x (e.g., P = a constant) , then

$$S_x^P = 0 \tag{3.7}$$

If P = cx, where c is a constant

$$S_x^{cx} = \frac{\partial(\ln cx)}{\partial(\ln x)} = \frac{\partial(\ln c)}{\partial(\ln x)} + \frac{\partial(\ln x)}{\partial(\ln x)} = 1 \tag{3.8}$$

Another useful relationship is

$$S_x^P = - S_x^{1/P} \tag{3.9}$$

This follows from equetion 3.4, since

$$- S_x^{1/P} = - \frac{\partial(\ln {}^1/P)}{\partial(\ln x)} = - \frac{\partial[-(\ln {}^1/P)]}{\partial(\ln x)} = S_x^P$$

In a similar way

$$S_x^P = - S_{1/x}^P \qquad (3.10)$$

Other useful relationships that can be easily be proved are :

$$S_x^{P_1 P_2} = S_x^{P_1} + S_x^{P_2} \qquad (3.11a)$$

$$S_x^{P_1/P_2} = S_x^{P_1} - S_x^{P_2} \qquad (3.11b)$$

$$S_{x^n}^P = {}^1/_n \, S_x^P \qquad (3.11c)$$

$$S_x^{P^n} = n \, S_x^P \qquad (3.11d)$$

$$S_x^{P_1+P_2} = \frac{P_1 S_x^{P_1} + P_2 S_x^{P_2}}{P_1 + P_2} \qquad (3.11e)$$

$$S_x^{cf(x)} = S_x^{f(x)} \qquad (3.11f)$$

3.2) <u>MULTI-ELEMENT DEVIATIONS</u> :-

An expresion for the change in biquadratic parameter due to change in a particular circuit element is obtained in the last section. For instance, the change in resistance causes the frequency to change by (equation 3.2)

$$\Delta\omega_p = \lim_{\Delta R \to 0} S_R^{\omega_p} \frac{\Delta R}{R} \omega_p \qquad (3.12)$$

For small deviation in R

$$\Delta\omega_p \simeq S_R^{\omega_p} \frac{\Delta R}{R} \omega_p \qquad \text{..................} \qquad (3.13)$$

This is the change due to one element.In general, the change due to simultaneous variation of all the elements in the circuit can be considered as follows.

For example, the change in ω_p due to deviations of all the circuit elements x_j(where the elements can be resistors or the parameters describing the active device). The change $\Delta\omega_p$ may be obtained by expanding it in a Taylor series, as

$$\Delta\omega_p = \frac{\partial\omega_p}{\partial x_1} \Delta x_1 + \frac{\partial\omega_p}{\partial x_2} \Delta x_2 + \ldots\ldots\ldots + \frac{\partial\omega_p}{\partial x_m} \Delta x_m$$

+ second and higher-order terms

where `m´is the total number of elements in the circuit. Since the changes in the components Δx_j are assumed to be small, the second-and higher-order terms can be ignored. Thus

$$\Delta\omega_p \approx \sum_{j=1}^{m} \frac{\partial\omega_p}{\partial x_j} \Delta x_j \qquad \text{..............} \qquad (3.14)$$

To bring the sensitivily term into evidence, (3.14) may be written as

$$\Delta_p \approx \sum_{j=1}^{m} \left[\frac{\partial\omega_p}{\partial x_j} \Delta x_j \right] (\Delta x_j / x_j)\omega_p$$

$$= \sum_{j=1}^{m} S_{x_j}^{\omega_p} V_{x_j} \omega_p \qquad \text{......} \qquad (3.15)$$

where $V_{x_j} = \Delta x_j / x_j$ is the per unit change in the element x_j and is known as the variability of x .From (3.15) the per-unit change in ω_p is

$$\frac{\Delta\omega_p}{\omega_p} = \sum_{j=1}^{m} S_{x_j}^{\omega_p} V_{x_j} \qquad \ldots\ldots\ldots \qquad (3.16)$$

similarly the per-unit change in pole Q, ω_z, Q_z and K, due to the simultaneous deviations of all the components, are given by

$$\frac{\Delta Q_p}{Q_p} = \sum_{j=1}^{m} S_{x_j}^{Q_p} V_{x_j} , \qquad \frac{\Delta K}{K} = \sum_{j=1}^{m} S_{x_j}^{K} V_{x_j}$$

$$\frac{\Delta\omega_z}{\omega_z} = \sum_{j=1}^{m} S_{x_j}^{\omega_z} V_{x_j} , \qquad \frac{\Delta Q_z}{Q_z} = \sum_{j=1}^{m} S_{x_j}^{Q_z} V_{x_j} \qquad \ldots\ldots \qquad (3.17)$$

3.3) GAIN SENSITIVITY :-

Filter requirements are usually stated in terms of the maximum allowable deviation in gain over specified bands of frequencies. In this section it is shown that how this gain deviation is related to the biquadraticparameter sensitivities. Furthermore, the ways of adapting the design process to minimize the gain deviation are also suggested.

Assume that the filter function has been factored into biquadratics,as

$$T(S) = \prod_{i=1}^{N} Ki \frac{S^2 + \frac{\omega_{zi}}{Q_{zi}} S + \omega_{zi}^2}{S^2 + \frac{\omega_{pi}}{Q_{pi}} S + \omega_{pi}^2} \qquad \ldots\ldots\ldots\ldots \qquad (3.18)$$

The gain in dB is given by

$$G(\omega) = 20 \log_{10} |T(j\omega)|$$

$$= \sum_{i=1}^{N} 20 \log_{10} |S^2 + \frac{\omega_{zi}}{Q_{zi}} S + \omega_{zi}^2 |_{s=j\omega}$$

$$- \sum_{i=1} 20 \log_{10} |S^2 + \frac{\omega_{pi}}{Q_{pi}} S + \omega_{pi}^2|_{s=j\omega} + 20\log_{10}|ki|$$

.... (3.19)

"Gain sensitivity" is defined as the change in gain in dB due to a per unit change in an element (or parameter) x :

$$G^{G(\omega)} = \frac{\partial G(\omega)}{\partial x/x}$$

$$= x \frac{\partial G(\omega)}{\partial x} \text{ dB} \qquad \text{........} \qquad (3.20)$$

From this equation

$$\Delta G(\omega) = \lim_{\Delta x \to 0} G_x^{G(\omega)} \frac{\Delta x}{x}$$

and small changes in x

$$\Delta G(\omega) \approx G_x^{G(\omega)} \frac{\Delta x}{x} \qquad \text{..........} \qquad (3.21)$$

change in gain $\Delta G(\omega)$ (hereafter abbreviated as ΔG), due to the element variabilities V_{x_i} is of interest. Since the gain function is the sum of similar second-order functions, the results of which can easily be extended to the summed expression of equation (3.13). Let us consider the contribution to the gain deviation of the second-order numerator term

$$T(S) = S^2 + \frac{\omega_z}{Q_z} S + \omega_z^2 \quad \ldots\ldots\ldots\ldots \quad (3.22)$$

The corresponding gain is

$$G(\omega) = 20 \log_{10} \left| S^2 + \frac{\omega_z}{Q_z} S + \omega_z^2 \right|_{s=j\omega} \ldots \quad (3.23)$$

Since G(w) is a function of the variables ω_z and Q_z, a Taylor series expansion of ΔG will have the form :

$$\Delta G = \frac{\partial G}{\partial Q_z} \Delta Q_z + \frac{\partial G}{\partial \omega_z} \Delta \omega_z + \frac{\partial^2 G}{\partial Q_z^2} (\Delta Q_z)^2 + \frac{\partial^2 G}{\partial \omega_z^2} (\Delta \omega_z)$$

$$+ \frac{2\partial^2 G}{\partial \omega_z \partial Q_z} (\Delta Q_z \Delta_z) + \ldots.$$

for small changes in the components, the corresponding changes in Q_z and ω_z will be small, so that the second and higher-order can be ignored Then

$$\Delta G \approx \frac{\partial G}{\partial Q_z} \Delta Q_z + \frac{\partial G}{\partial \omega_z} \Delta \omega_z \quad \ldots\ldots\ldots\ldots \quad (3.24)$$

substituting for $\Delta\omega_z$ and ΔQ_z from (3.16) and (3.17), respectively, we get

$$\Delta G \approx \sum_{j=1}^{m} \left[Q_z \frac{\partial G}{\partial Q_z} S_{x_j}^{Q_z} V_{x_j} + \omega_z \frac{\partial G}{\partial \omega_z} S_{x_j}^{\omega_z} V_{x_j} \right] \ldots\ldots\ldots \quad (3.25)$$

From the defination of gain sensitivity this expression reduces to

$$\Delta G \approx \sum_{j=1}^{m} \left(G_{Q_z}^{G} S_{x_j}^{Q_z} V_{x_j} + G_{\omega_z}^{G} S_{x_j}^{\omega_z} V_{x_j} \right) \quad (3.26)$$

This equation gives the change in the gain in dB due to the simultaneous variation in all the elements realizing the second-order function :

$$S_z^2 + \frac{\omega_z}{Q_z} S + \omega_z^2$$

The gain change for the complete transfer function of equation 3.19 is obtained by adding the gain contributions to each of the second order functions.

3.4) Factors Affecting Gain Sensitivity :-

The approximation function, the circuit and the components are the three major factors affecting the gain deviations.

3.4.1) Contribution of the approximation function :-

Suppose the approximation function has been expressed as a product of biquadratics, each term being of the form :

$$T(S) = K \frac{S^2 + \frac{\omega_z}{Q_z} S + \omega_z^2}{S^2 + \frac{\omega_p}{Q_p} S + \omega_p^2} \quad \ldots\ldots\ldots \quad (3.27)$$

The corresponding gain in dB is

$$G(\omega) = 20 \log_{10} \left| S^2 + \frac{\omega_z}{Q_z} S + \omega_z^2 \right|_{s=j\omega}$$

$$= 20 \log_{10} \left| S^2 + \frac{\omega_p}{Q_p} S + \omega_p^2 \right|_{s=j\omega} + 20\log|K| \quad . \quad (3.28)$$

The biquadratic parameters describing the approximation function contribute to the gain deviation expression (eqation 3.28) via the biquadratic parameter sensitivity terms.

$$G^{G}_{\omega_z} \quad , \quad G^{G}_{Q_z} \quad , \quad G^{G}_{\omega_p} \quad ,$$

$$G^{G}_{Q_p} \quad \text{and} \quad G^{G}_{k} \; .$$

These sensitivities can be evaluated from the definition of gain sensitivity[128] (equation 3.20).

(i) The biquadratic parameter sensitivities depend only on the approximation function, at a given frequency

(ii) The sensitivity in the pass band increases with the pole Q.

3.4.2) Choice of the circuit :-

The circuits with lower component sensitivities will be desirable for better performance.Also the gain deviation depends on the number of elements used in the circuit realization of the approximation function.

The gain deviation increases with:

(i) The component sensitivities

(ii) The number of components used to synthesize the given function.

3.4.3) Choice of component types :-

After the filter has been designed,the next step is to select the components (like resistors, operational amplifiers) to be used in the manufacture of the circuit.Practical elements deviate from their nominal values due to manufacturing tolerances, temperature and humidity changes and due to chemical changes that occur with the aging of the elements. A Practical solution is to select the components with low sensitivities.

The sensitivity to component types can be reduced by choosing components that have low spread in their inital manufacturing tolerence, in their temperature aging and humidity coifficients.