CHAPTER - 4

STATE VARIABLE ANALYSIS AND SYNTHESIS

STATE VARIABLE ANALYSIS AND SYNTHESIS

4.1) STATE VARIABLES :-

The term "State-Variable" is related to "State variable theory" which provides a systematic means of formulating the differential equations of large system. A state diagram showing interconnection of all the mathematical operations may be constructed, as part of such a formulation. Inreality, state diagram is a mathematical form of an analog computer simulation.[6]

The zero input response in an RLC network is completely determined when the initial inductor currents and capacitor voltages are known. Hence, we call the initial capacitor voltages and inductor currents (initial conditions) as the initial states of the system. The knowledge of capacitor voltages and inductor currents, at a given time, of a given network is sufficient to calulate any of the network variables (current and voltages) at the particular time. Hence, the capacitor voltages and the inductor currents at a specified time, are called "state variables"[1] of the network.

The state of a network is normally defined as a set of real or complex quantities that satisfy the following conditions.

(a) The state at any time t_1 and the input from t_1 to t (t>t1) uniquely determine the state at a time t.

(b) The state at time \`t' and the inputs at time \`t' of any network variable.

For example, consider the network shown in fig. (4.1)

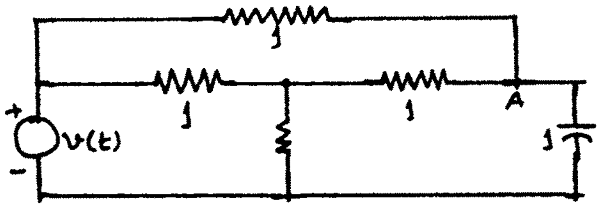

fig.(4.1)

If the capacitor voltage is known at time 't' then we can replace it by a known voltage source Vc(t) as shown in fig.(4.2)

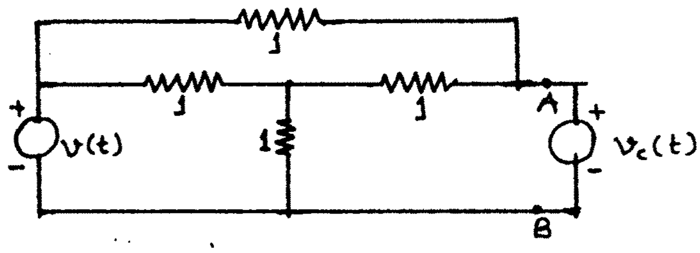

fig.(4.2)

Now, the voltages across and currents through the resistors can be found by solving a set of algebraic equations (say, mesh equation). Hence,Vc(t) is the state of the network.

States can also be considered as variables which carry sufficient information about the history of systems. Obviously,a purely resistive network has no states at all. Only a reastive network or a network with energy elements (capacitor with $1/2cv^2$& inductor, with $1/2LI^2$) has states.[(1)]

The motivation in the study of state variable is that the state variable form is most convinient for computer solutions. [either digital or analog computer]

T.R. Bashkow in 1957, has suggested the strategy for state variable analysis of any network (reactive network) which is accomplished in the following steps.

(1) Select a tree containing all capacitors but not indictors.

(2) The state variables are the branch capacitor voltages in this tree and the inductor currents of the chords.

(3) Write a node equation for each capacitor.

(4) Manipulate each equation, if necessary, until it involves only the variables selected in (2) plus the inputs.

(5) Write a loop equation using each inductor as a chord in the tree of (1).

(6) Repeat step (4).

(7) Manipulate the eqns. as may be necessory (devision by constants, for example) until they appear in the standard form of the eqns. (4.1).

$$dx_1/dt = a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots + a_{1n}x_n + y_1$$

$$dx_2/dt = a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots + a_{2n}x_n + y_2$$

$$\vdots$$

$$dx_n/dt = a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots\ldots\ldots\ldots + a_{nn}x_n + y_n$$

If solution is to be accomplished by computer, then the state space formulation (like eqn.4.6) offers advantagesand writting the eqns. correctly is the only requirement. If the network to be analyzed contains one element or several elements that are non-linear or time-variable, then the state space formulation is recommended and the solution by computer methods is the only practical possibility.

If the solution is to be accomplished using a pencil and pad of paper, then it is ordinarily simpler to use node or loop formulations.

4.2) STATE-VARIABLE REALIZATION [infinite-gain]:-

The method of synthesis using op.amps. may be set up through the use of state variables.

To see this consider the open-circuit voltage transfer function

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \dots\dots\dots + a_n s^n}{b_0 + b_1 s + b_2 s^2 + \dots\dots\dots + b_n s^n} \qquad (4.8)$$

Where the co-efficient a_i and b_i are real but may be positive or negative (or zero). We shall assume that the usual requirements of stability are met.

On multiplying eqn^n. 4.8 , throught by x/s^n, we get

$$T(s) = \frac{a_0 x/s^n + a_1 x/s^{n-1} + a_2 x/s^{n-2} + \dots\dots + a_{n-1} x/s + a_n x}{b_o x/s^n + b_1 x/s^{n-1} + b_2 x/s^{n-2} + \dots + b_{n-2} x/s + b_n x} \qquad (4.9)$$

The numerator and denominator are seperately written as input and output by proper scaling, therefore

$$V_2(s) = \frac{a_0 x}{s^n} + \frac{a_1 x}{s^{n-1}} + \frac{a_2 x}{s^{n-2}} + \dots\dots + \frac{a_{n-1} x}{s} + a_n x \qquad (4.10)$$

$$V_1(s) = \frac{b_0 x}{s^n} + \frac{b_1 x}{s^{n-1}} + \frac{b_2 x}{s^{n-2}} + \dots\dots + \frac{b_{n-1} x}{s} + b_n x \qquad (4.11)$$

In eqns. (4.10) & (4.11) the term x/s^i, where i = 1,2,..... is a " state-variable"

Equation (4.11) can be modified and rewritten as

$$b_n x = V_1(s) - \frac{b_0 x}{s^n} - \frac{b_1 x}{s^{n-1}} - \frac{b_2 x}{s^{n-2}} - \ldots\ldots - \frac{b_{n-1} x}{s} \quad (4.12)$$

Where $V_1(s)$ is usually the input voltage availlable from external source.

So, the equation (4.10) & (4.12) may be realized by means of the interconnections of the n integrator (each realized by an op. amp.,a resistor, and a capacitor) and two summers (each also requiring an op.amp.)

The overall configuration is as shown in fig.(4.3).

Here, first summer (Σ (1)) is used to represent the equation for $b_n x$, and second summer (Σ(2))torepresent the output equation for v_2.

In this fig. it has been assumed that the integration are non-inverting (Modification of the circuit for the case where the conventional inverting integretors are used is easily made). It should be noted that such a realization requires only n capacitors and a miximum of n+2 operationalamplifiers.[11]

4.3) REALIZATION OF A SECOND-ORDER FUNCTION[8]:-

The second order voltage transfer ratio given by

$$T(s) = \frac{as^2 + bs + c}{ds^2 + es + f} \quad (4.13)$$

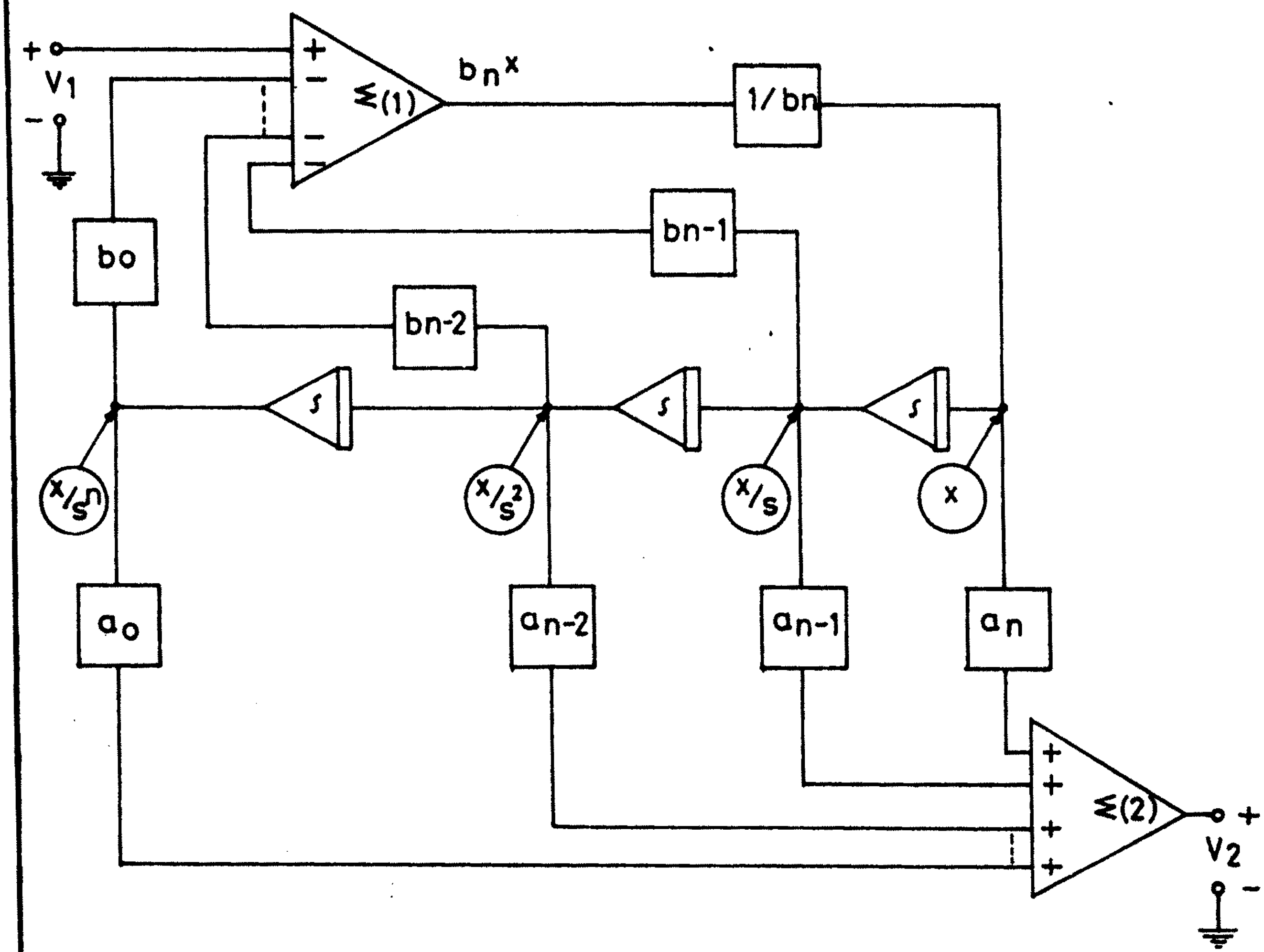

Fig. 4·3 – A state-variable-gain network configuration.

on transformation to

$$T(s') = \frac{a}{d} + \frac{\dfrac{b - \dfrac{ae}{d}}{dGB} s' + \dfrac{c - \dfrac{af}{d}}{dGB^2}}{s'^2 + \dfrac{e}{dGB} s' + \dfrac{f}{dGB^2}} \qquad (4.14)$$

Using $\dfrac{1}{s'} = \dfrac{GB}{s}$

is represented by the following state space eqns.

$$V_\emptyset = \frac{a}{d} V_i + \frac{c - \dfrac{af}{d}}{dGB^2} x_1 + \frac{b - \dfrac{ae}{d}}{dGB} x_2 \qquad (4.15)$$

$$\dot{x}_1 = x_2 \qquad (4.16)$$

and

$$\dot{X}_2 = -\frac{f}{dGB^2} x_1 - \frac{e}{dGB} x_2 + V_i \qquad (4.17)$$

The realization[47] is shown in fig. (4.4).

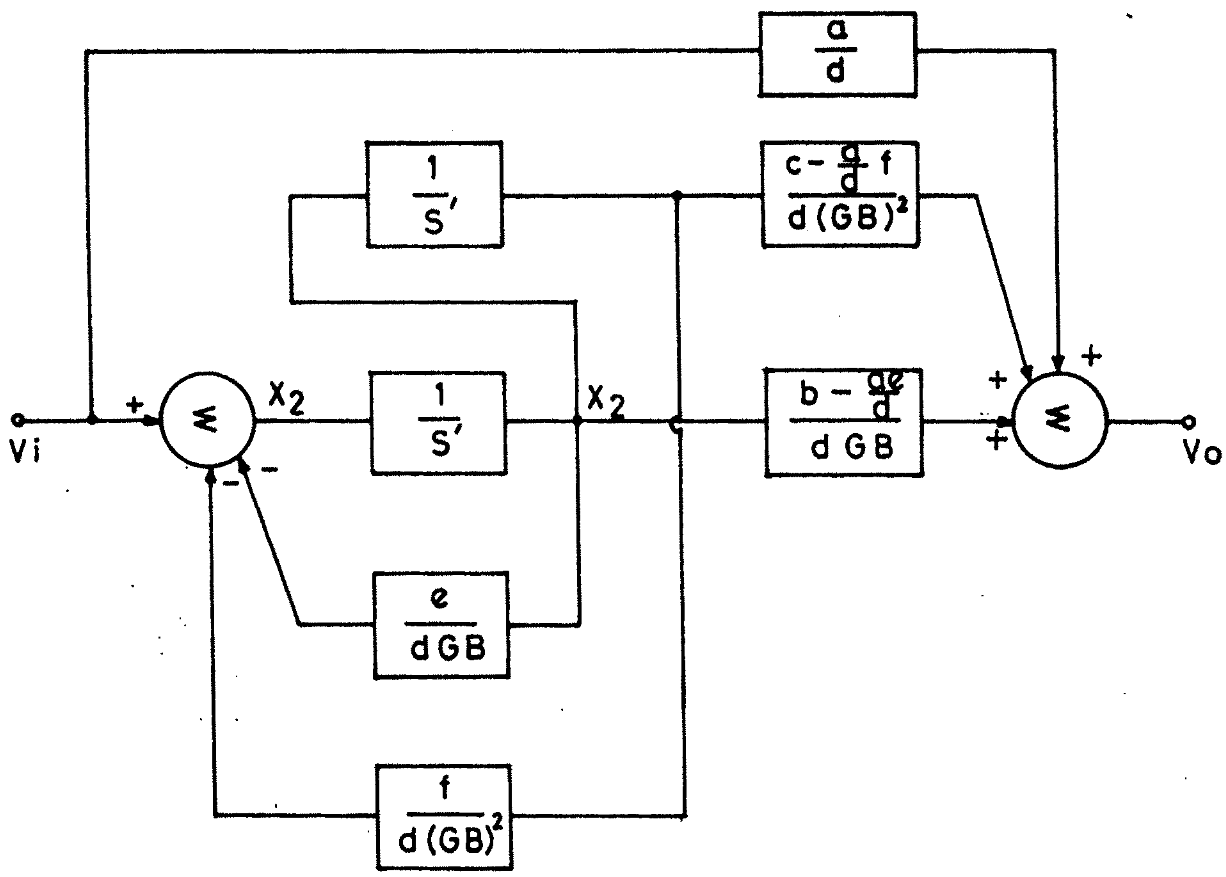

Fig. 4·4 - Realization of a general second-order transfer function using the operational amplifier pole.