CHAPTER : 3

SENSITIVITY

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3.1 INTRODUCTION :-

In the design active R filters, the design parameters are assumed to be constants. These parameters are like gain bandwidth product, values of components. These parameters depend on many factors like temp. humidity, aging etc. These parameters are not considered as a constants. This deviation of active and passive parameters from their nominal values result in a change in the response of the network.

Actually for highly selective network they induce instability and hence oscillations. This variation of the network response due to incremental change in the values of the parameters is expressed in terms of the 'Sensitivity' of the particular network function with respect to that parameter.

The sensitivity of any network function 'F' with respect to a parameter 'X' is defined as

$$S_X^F = \frac{X}{F} \cdot \frac{d_F}{d_X} \qquad \text{....(3.1)}$$

Various network functions give rise to the various sensitivity functions. Sensitivity of a transfer function T (S) with respect to a parameter 'X' is called transfer function sensitivity. Actual it is complex quantity. The real part is called 'gain sensitivity' and its imaginary part is called 'phase sensitivity'. The various definitions of sensitivities and different ways of evaluating the sensitivity of a circuit are given below

3.2 ω & Q SENSITIVITY :-

In a qualitative sense the sensitivity of a network is a measure of the degree of variation of its performance from nominal, due to changes in the elements constituting the network.

The bi-quadratic filter function can be expressed in terms of the paramters ω_p, Q_p, Q_z and K as

$$T(S) = K \frac{S^2 + \frac{\omega_z}{Q_z} \cdot S + \omega_z^2}{S^2 + \frac{\omega_p}{Q_p} S + \omega_p^2} \qquad \text{.....(3.2)}$$

Sensitivies of these bi-quadratic parameters to the elements can be studied and evaluated as below.

The sensitivity of the pole frequency ω_p to a a change in a resistor R i.e. pole sensitivity is defined as the per unit change in the pole frequency. $\Delta\omega_p/\omega_p$ caused by a per unit change in the resistor, $\Delta R/R$, Mathematically,

$$S_R^{\omega_p} = \lim_{\Delta R \to 0} \frac{\frac{\Delta\omega_p}{\omega_p}}{\Delta R/R} \qquad (3.4)$$

$$= \frac{R}{\omega_p} \cdot \frac{\partial\omega_p}{\partial R} \qquad (3.5)$$

This can also written as

$$S_R^{\omega_P} = \frac{\partial(\ln\omega_p)}{\partial(\ln R)} \qquad (3.6)$$

It is noted that the cost of the manufacturing component is a function of the percentage change ($100 \times \Delta R/R$) rather than the absolute change (ΔR) of the component. For this reason it is desirable to measure sensitivity in terms of the relative changes in components.

The sensitivities of the parameters ω_2, Q_p, Q_z and K to any element of the network are defined in similar way,

$$S_R^{Q_p} = \frac{R}{Q_p} \cdot \frac{\partial Q_p}{\partial R}, \qquad S_R^{Q_z} = \frac{R}{Q_z} \cdot \frac{\partial Q_z}{\partial R}$$

$$S_R^{K} = \frac{R}{K} \cdot \frac{\partial K}{\partial R} \qquad S_R^{\omega_z} = \frac{R}{\omega_z} \cdot \frac{\partial \omega_z}{\partial R}$$

The sensitivity of parameters 'P' to an element X is

$$S_X^P = \frac{X}{P} \cdot \frac{d p}{d x} = \frac{- \partial (\ln p)}{\partial (\ln x)} \quad (3.7)$$

If P is not a functioin of X (P = const), then

$$S_X^P = 0$$

If P = Cx, where C is constant

$$S_x^{cx} = \frac{\partial (\ln cx)}{\partial (\ln x)} = \frac{\partial (\ln c)}{\partial (\ln x)} + \frac{\partial (\ln x)}{\partial (\ln x)} = 1 \quad (3.8)$$

Another useful equation is

$$S_X^P = -S_X^{1/0} \quad (3.9)$$

In a similar way,

$$S_X^P = -S_{1/X}^{P_1}$$

Another useful relation-ships that can easily proved as

$$S_X^{P_1 P_2} = S_X^{P_1} + S_X^{P_2} \qquad (3.10)$$

$$S_X^{P_1 / P_2} = S_X^{P_1} - S_X^{P_2} \qquad (3.11)$$

$$S_x^{P^{1/n}} = 1/n \, S_x^{P} \qquad (3.12)$$

$$S_X^{P^n} = n \, . \, S_X^{P} \qquad (3.13)$$

$$S_X^{P_1 + P_2} = \frac{P_1 S_X^{P_1} + P_2 S_X^{P_2}}{P_1 + P_2} \qquad (3.14)$$

$$S_X^{F(X)} = S_X^{F(X)} \qquad (3.15)$$

3.3 **MULTI-ELEMENT DEVIATIONS :-**

The change in bi-quadratic parameter due to change in a particular circuit element is obtained.

Also, the change in resistance causes the frequency to change by

$$\Delta \omega_p = \lim_{\Delta R \to 0} S_R^{\omega_p} \frac{\Delta R}{R} \omega_p \qquad (3.16)$$

for small change in resistor,

$$\Delta \omega_p = S_R^{\omega_p} \frac{\Delta R}{R} \omega_p \qquad (3.17)$$

This change occurs due to element. In general the change due to simultaneous variation of all the elements in the circuit can be considered as follows.

For example the change in "ω_p' due to deviations of all the circuit elements.

The change $\Delta \omega_p$ may be obtained by expanding it in a Taylor Series as ,

$$\Delta \omega_p = \frac{\partial \omega_p}{\partial x_1} \cdot \Delta_1 x + \frac{\partial \omega_p}{\partial x_2} \Delta_2 x + \ldots +$$

$$\frac{\partial \omega_p}{\partial x_n} \cdot \Delta x_n \quad + \text{Second \& Higher order terms}$$

Where n is total number of elements in the circuit. Since changes in component assumed to be small the second and higher order terms are neglected, Thus

$$\Delta \omega_p = \sum_{j=1}^{n} \frac{\partial \omega_p}{\partial x_j} . \Delta x_j \qquad (3.18)$$

To bring the sensitivity in actual practice

$$\Delta_p = \sum_{j=1}^{n} \left[\frac{\partial \omega_p}{\partial x_J} . \Delta x_j \right] \qquad (\Delta x_j / x_j) \omega_p$$

$$= \sum_{j=1}^{n} S_{xj}^{\omega p} \cdot V_{xj} \, \omega_p \qquad (3.19)$$

Where $X_{xj} = \Delta xj/xj$ is per unit change in the element. Xj is called variability of X.

The per unit change in '$_p$' is

$$\frac{\Delta \omega_p}{\omega_p} = \sum_{j=1}^{n} S_{xj}^{\omega p} \, V_{xj} \qquad (3.20)$$

Similarly the per unit change in pole Q, ω_z, Q_z and K due to simultaneous deviations of all the components is given by

$$\frac{\Delta Q_p}{Q_p} = \sum_{j=1}^{n} S_{xj}^{Q_p} \cdot V_{xj} \qquad \frac{\Delta K}{K} v = \sum_{j=1}^{n} S_{xj}^{K} \cdot V_{xj}$$

$$\frac{\Delta \omega_z}{\omega_z} = \sum_{j=1}^{n} S_{xj}^{\omega z} \, V_{xj} \qquad \frac{\Delta Q_z}{Q_z} = \sum_{j=1}^{n} S_{xj}^{Q_2} \, V_{xj} \qquad (3.21)$$

3.4 **Gain Sensitivity :-**

The requirements of the filter are usually stated in terms of the maximum allowable deviation in gain over specified bands of frequencies. In this part we have to show how this bi-quadratic parameter is related to gain deviation.

If we assume the filter function which is in bi-quadratics as

$$T(S) = \sum_{j=1}^{N} Ki \frac{S^2 + \frac{\omega_{zi}}{Q_{zi}} S + \omega_{zi}^2}{S^2 + \frac{\omega_{Pi}}{Q_{po}} S + \omega_p^2 i} \quad (3.22)$$

This gain in dB is given by

$$G(\omega) = 20 \log_{10} |T(J\omega)|$$

$$= \sum_{i=1}^{N} 20 \log \left| S^2 + \frac{\omega_{zi}}{Q_{zi}} s + \omega^2_{zi} \right|_{s=j\omega}$$

$$= \sum_{i=1} 20 \log_{10} \left| S^2 + \frac{\omega_{pi}}{Q_{pi}} S + \omega^2_{pi} \right|_S$$

$$= jw + 20 \log_{10} |Ki| \quad (3.23)$$

Gain sensitivity is defined as the change in gain in dB due to a per unit change in an element

$$G(\omega) = \frac{\partial G(\omega)}{\partial x / x}$$

$$= x \cdot \frac{\partial G(\omega)}{\partial x} \cdot dB \quad (3.24)$$

From this equation,

We can write the equation

$$\Delta G(\omega) = \lim_{\Delta x \to 0} G_x^{G(\omega)} \frac{\Delta x}{x}$$

and small changes in X

$$\Delta G(\omega) = G_X^{G(\omega)} \frac{\Delta X}{X} \qquad (3.25)$$

Since the gain function is the sum of similar second order functions, the results of which can easily be extended to summed operation.

If we consider the contribution to gain deviation of the second order numerator term

$$T(S) = S^2 + \frac{\omega_z}{Q} - S + \omega^2_{z_z} \qquad (3.26)$$

The corresponding gain is

$$G(\omega) = 20 \log_{10} \left| S^2 + \frac{\omega_z}{Q_z} - S + \omega^2_z \right|_{S} = jw \qquad (3.27)$$

Since $G(\omega)$ is a function of the variables ω_z and Q_z

A Taylor series expansion of ΔG will have the form

$$\Delta G = \frac{\partial G}{\partial Q_z} \Delta Q_z + \frac{\partial G}{\partial \omega_z} \Delta \omega_z + \frac{\partial^2 G}{\partial Q^2_z} (\Delta Q_z)^2 +$$

$$+ \frac{\partial^2 G}{\partial \omega^2_z} (\Delta \omega_z) + \frac{z \partial^2 G}{z \omega_z Q_z} (\Delta Q_z \Delta_z) + \ldots\ldots$$

For small change in component the corresponding changes in Q_z and ω_z will be small, so that the second and higher order can be ignored, then

$$\Delta G = \frac{\partial G}{\partial Q_z} \cdot \Delta Q_z + \frac{\partial G}{\partial \omega_z} \Delta \omega_z \qquad (3.28)$$

Substituting for $\Delta \omega_z$ and ΔQ_z from 1 & n respectively, we get

$$\Delta G = \sum_{j=1}^{n} \left[Q_z \frac{\partial G}{\partial Q_z} \cdot S^{Q_z}_{xj} V_{xj} + \omega_z \frac{\partial G}{\partial \omega_z} S^{\omega_z}_{xj} V_{xj} \right] \qquad (3.29)$$

From the definition of gain sensitivity, this expression reduces to

$$\Delta G = \sum_{j=1}^{n} \left[G^G_{G_z} S^{Q_z}_{xj} V_{xj} + G^G_{\omega_z} S^{\omega_z}_{xj} V_{xj} \right] \qquad (3.30)$$

From the definition of gain sensitivity, this expression becomes,

$$\Delta G = \sum_{j=1}^{N} \left[G^G_{Q_z} S^{Q_z}_{xj} V_{xj} + G^G_{\omega_z} S^{\omega_z}_{xj} V_{xj} \right] \qquad (3.31)$$

This equation gives the change in the gain in dB due to the simultaneous variation in all the elements realizing the second-order function.

$$S_z^z + \frac{\omega_z}{Q_z} S + \omega_z^z$$

The gain change for complete transfer functiion of equation(3.31)is obtained by adding the gain contributions to each of the second order functions.

3.5 FACTORS AFFECTING GAIN SENSITIVITY :-

The approximation function, the circuit and the components are major three factors affecting the gain derivatives.

Contribution of the approximation function :-

If we suppose the approximation function has been bi-quadratic, each term becomes

$$T(S) = K \frac{S^z + \frac{\omega_z}{Q_z} - S + \omega_z^z}{S^2 + \frac{\omega_p}{Q_p} - S + \omega^2_p} \qquad (3.32)$$

The corresponding gain in dB is

$$G(\omega) = 20 \log_{10} \left| S^z + \frac{\omega_z}{Q_z} - S + \omega_z^z \right|_{z = J\omega}$$

$$- 20 \log_{10} \left| S^z + \frac{\omega p}{Q_p} - S + \omega^z_p \right|_{s = jw} + 20 \log |K| \qquad (3.33)$$

The bi-quadratics parameters describing the approximation function contribute to the gain deviatioin expression - via bi-quadratic parameter sensitivity term -

$$G^G_{\omega_z} \quad , \quad G^G_{Q_z} \quad , \quad G^G_{\omega p}$$

$$G^G_{Q_p} \quad \text{and} \quad G^G_K$$

These sensitivities can be evaluated from the definition of gain sensitivity -

i) The bi-quadratic parameter sensitivity depend only on the approximatioin function, at a given frequency

ii) The sensitivity in the pass band increases with the pole Q.

* **Choice of the Circuit :-**

The lower component sensitivity circuits will be desirable for better performance. Also the gain deviation depends on the number of elements used in circuit realization of the approximation function.

The gain deviation increases with

i) Component sensitivities.

ii) The number of components used to synthesize the given function.

* **Choice of Component types :-**

After the filter designing the next step is to select the components (like resistors, operational amplifiers) to be used in the assembly of the circuit. Practical elements deviate from their normal values due to manufacturing tolerances, temperature and humidity changes and due to chemical changes that occur with the aging of the elements.

A practical solution is to select the components with low sensitivities.

The sensitivity is component types can be reduced by choosing components that have low spread in their initial manufacturing, tolerance, in their temperature aging and humidity coefficients.