

P R E F A C E

The present dissertation entitled "Fixed point theory and its applications in Hilbert space" is the outcome of the research work carried out by me in the field of fixed point theory and its applications.

The dissertation consists of four chapters. Chapter-I is introductory which is divided into two parts. Part-I deals with the basic concepts, fundamental fixed point theorems and some useful fixed point results needed in the development of the work in the subsequent chapters of this dissertation. In Part-II of this chapter, applications of some fixed point theorems are discussed. One would be interested to know the applications of fixed point theorems to an infectious disease model and to establish existence theorem of equilibria in monetary economies.

In Chapter-II, a theorem on Common fixed points of a family of more restrictive strictly pseudo-contractive mappings using generalisation of Ishikawa iteration scheme has been established.

In Chapter-III, the following definition of generalised contraction mapping in Hilbert space has been introduced by us.

Definition : (Generalised contraction) : Let C be a closed convex subset of a Hilbert space H . A mapping $T : C \rightarrow C$ is said to be generalised contraction if for all $x, y \in C$

$$\|Tx - Ty\|^2 \leq a_1 \|x - y\|^2 + a_2 \|x - Ty\|^2 + a_3 \|y - Tx\|^2 + a_4 \|(I - T)x - (I - T)y\|^2,$$

where $a_i \geq 0$, $\sum_{i=1}^4 a_i < 1$.

Based upon this generalised contraction some fixed point theorems using Ishikawa sequence of iterates have been established.

Chapter-IV deals with the fixed point theorems of reasonable wanderer maps in Hilbert space. Following the technique of Ishikawa, it was shown that demicontractive and hemicontractive maps are reasonable wanderer maps in C , where C is a closed convex subset of a Hilbert space H . A theorem on fixed point of a Lipschitzian demicontractive selfmap T of a bounded closed convex subset C of a Hilbert space H has been established by further assuming that T is reasonable wanderer in C and demicompact. Finally it was shown that the generalised contraction self map T of a closed convex subset C of a Hilbert space H defined by the author in Chapter-III is reasonable wanderer in C . A partial text of this chapter has been presented in paper form by

the author in 55th Conference of the Indian Mathematical Society held at Delhi from 27th Dec. to 30th Dec. 1989.

The theorems, definitions, corollaries, remarks and equations are numbered sequentially sectionwise in each chapter.

The references are given at the end of the dissertation and they are arranged in the alphabetical order. In the text, they have been referred to by putting within rectangular brackets. For example [7] means the 7th reference given at the end of this dissertation.