P R E F A C E

This dissertation entitled "Killing and Harmonic fields in the space-time of Relativistic Ferrofluid" is based on the central theme of ferrofluid material which is defined as the magnetically soft fluid, Mehta (1989). The basic field equations of Relativistic magnetohydrodynamics are designed by Lichnerowicz (1967). This system deals with infinitely conducting charged fluid distribution under the constraint of constant magnetic permeability. In 1978 Cissoko has presented a system of Relativistic equations for the space-time associated with the infinitely conducting charged fluid with the variable magnetic permeability. Thus the variable magnetic permeability is the main role played in the investigations of ferrofluid system.

The aim of this dissertation is to study the local behaviour of congruences in the space time of ferrofluid. We have attempted to solve the ferrofluid system equations under the geometrical restrictions as groups of motions and collineations.

The highlights of the investigations carried out in the dissertation are presented in the following chapters :

Chapter I :

The basic notions and concepts required for the development of dissertation work are given. The various parameters associated with space time congruences are introduced. The formulation of stress-energy tensor characterizing the ferrofluid is given. The energy conditions regarding this tensor of ferrofluid which is physically transperent are included.

Chapter II :

Section 1 : The introduction for the chapter is given

Section 2 : This section deals with the space-time field equations governing the nature of the ferrofluid, ~~are stated~~. The field equations of gravitation and Maxwell equations for electromagnetic field are given. The consequences of Maxwell equations are - (i) for the shear free, non-expanding flow of the ferrofluid the magnitude of the magnetic field is preserved along the flow iff the magnetic permeability is preserved along the flow. (ii) the magnetic permeability is also conserved along the divergence free magnetic lines iff the 4-acceleration is normal to magnetic lines.

Section 3 : The local conservation laws imply that (i) for the expansion free flow, the matter energy density is preserved along the flow iff the magnetic permeability is preserved along it (ii) the isotropic pressure remains invariant along magnetic lines iff the magnetic permeability is preserved along these lines.

Section 4 : The definitions of Killing vector field and harmonic vector field are given. The necessary conditions for the field of magnetic lines in ferrofluid propagate with fundamental velocity is found. We have proved the following theorems :

1) If the flow lines in ferrofluid are killing then

$$\rho_{;b}u^b = 0, \quad (H^2)_{;b}u^b = 0 \iff \mu_{;b}u^b = 0,$$

$$H^b_{;b} = 0, \quad P_{;b}H^b = 0 \iff \mu_{;b}H^b = 0.$$

2) Further if magnetic lines in ferrofluid are Killing then

$$u^b_{;b} = 0, \quad \rho_{;b}u^b = 0 \iff \mu_{;b}u^b = 0,$$

$$P_{;b}H^b = 0, \quad (H^2)_{;b}H^b = 0 \iff \mu_{;b}H^b = 0.$$

3) If the flow lines of ferrofluid are harmonic then

$$\rho_{;b}u^b = 0 \iff \mu_{;b}u^b = 0,$$

$$P_{;b}H^b = 0, \quad H^b_{;b} = 0 \iff \mu_{;b}H^b = 0.$$

4) If the magnetic lines in ferro fluid are harmonic then

$$\dot{u}_b H^b = 0 \iff \mu_{;b}H^b = 0,$$

$$P_{;b}H^b = 0 \iff \mu_{;b}H^b = 0.$$

5) If flow lines and magnetic lines in ferrofluid are both Killing then

$$\theta = 0, \quad \sigma_{ab} = 0, \quad \dot{u}_a = 0, \quad (H^2)^{\cdot} = 0, \quad \dot{\rho} = 0, \quad \dot{\mu} = 0,$$

$$H^b_{;b} = 0, \quad \mathcal{E} \neq 0, \quad \mathcal{J}_{ab} \neq 0, \quad (H^2)_{;b}H^b = 0, \quad \mu_{;b}H^b = 0.$$

6) If flow lines and magnetic lines in ferrofluid are both harmonic then

$$\theta = 0, \quad w_{ab} = 0, \quad \dot{u}_a = 0, \quad \mathcal{E} \neq 0, \quad H^b_{;b} = 0, \quad \bar{R}_{ab} = 0,$$

$$p_{;b}H^b = 0, \quad \mu_{,b}H^b = 0 \text{ and } (H^2)^{\cdot} = 0, \quad \dot{\rho} = 0 \iff \dot{\mu} = 0.$$

7) If flow lines in ferrofluid are Killing and magnetic lines are harmonic then

$$\theta = 0, \quad \sigma_{ab} = 0, \quad \dot{u}_a = 0, \quad (H^2)^{\cdot} = 0, \quad \dot{\rho} = 0, \quad \dot{\mu} = 0,$$

$$H^b_{;b} = 0, \quad \bar{R}_{ab} = 0, \quad \mathcal{E} \neq 0, \quad p_{;b}H^b = 0, \quad \mu_{;b}H^b = 0.$$

8) If flow lines in ferrofluid are harmonic and the magnetic lines in ferrofluid are Killing then

$$\theta = 0, \quad w_{ab} = 0, \quad \dot{u}_a = 0, \quad \dot{\rho} = 0, \quad \dot{\mu} = 0,$$

$$H^b_{;b} = 0, \quad \mathcal{E} \neq 0, \quad \mathcal{J}_{ab} = 0, \quad p_{;b}H^b = 0, \quad \mu_{;b}H^b = 0.$$

Chapter III :

Section 1 : Introduction presents the historical survey to Ricci collineation.

<u>Section 2</u> : The conservation law generatorsemerging due to Ricci collineation are discussed.

<u>Section 3</u> : The properties of the space-time of ferro-fluid admitting the Ricci collineations are examined. The results found are -

<u>Theorem 1</u> : For ferrofluid with $\rho \neq P + \mu H^2$ and $\rho \neq P + 5/3\ \mu H^2$, if $\dot{\mu} = 0$ then

$$\underset{u}{\pounds} R_{ab} = 0 \iff \underset{u}{\pounds} g_{ab} = 0.$$

<u>Theorem 2</u> : For the space time of ferrfluid with $\rho \neq P + 5/3\ \mu H^2$ if $\mu_{;b} H^b = 0$ then

$$\underset{H}{\pounds} R_{ab} = 0 \iff \underset{H}{\pounds} g_{ab} = 0.$$

<u>Section 4</u> : Here the space-time of the ferrofluid governed by two simultaneous conditions as Ricci collineations and Isometrics is studied. The main result of this Section is "If the matter density is preserved along the Killing magnetic lines and the isotropic pressure is conserved along the Killing flow then

$$\underset{H}{\pounds} R_{ab} = 0 \iff \underset{u}{\pounds} R_{ab} = 0.$$

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