C H A P T E R - I

INTRODUCTION

Preliminary Remarks :

A function is an important notion in Mathematics. Some manipulation like Dirac delta functions in technical literature have motivated Mathematicians to re-examine the concept of a function. The idea of specifying a function not by its value but by its behaviour as a functional is a new concept. This new mode of thinking gave birth to the wheels of research in several branches of Mathematics in rapid motion. The impact of generalized functions on the integral transforms has recently revolutionised the theory of generalized integral transformations.

1.1 Integral Transforms :

The theory of integral transformations provides technique for the resolution of certain types of classical boundary and initial value problems. The integral transforms are used in the solution of problems in applied Mathematics.

The functional equation

$$\bar{f}(s) = \int_a^b K(s,t)\, f(t)\,dt \tag{1.1.1}$$

defines the transform $\bar{f}(s)$ of the function $f(t)$ with respect to the kernel K(s,t) over an internal (a,b). Other important aspects of an integral transform are its inverse theorem, its Abelian theorems, its convolution theorem, and the topological and algebrical structures of its associated object and image spaces.

The transform theory provides a powerful technique to solve an ordinary and a partial differential equation in a direct and systematic manner. A particular differential equation associated with particular initial and boundary conditions requires a suitable integral transform to convert it into an algebrical equation whose solutions are inverted by the inverse transform to obtain the solution of original one.

With the prescription of the different forms of K(s,t) and the range of integration, various theories of integral transforms have been developed. The Laplace, Fourier, Mellin and Hankel transforms are extensively dealt in standard books of Sneddon [63], Titchmarsh [67] and the research paper of Mac Robert [46]. The Laplace transform is separately discussed by Widder [71] and is extended by Boas [2] . Cholewinski [8] and Haime [29] studied the convolution of Hankel transform while Macanley Owen [45] gives its Perseval theorem. The convolution transform in most general form is studied by Hirshmann and Widder [71], Danns and Widdar [13], Tanno [66], Ditzian [17], and Fox [23]. Debnath has discussed Laguerre and Hermite transforms. Deetsch [18] put forth the theory of finite integral transforms for Fourier Transforms. Other transforms extended are Hankel [62], Legendre [68] and [10], Gegenbauer [11], [40] and [65], and Jacobi [61], [14]. Gamkrilidge has compiled an exhaustive survey of integral transforms.

The simplest form of the Laplace transform of a function f(x) is defined by the integral

$$(1.1.2) \qquad F(u) = \int_0^\infty e^{-ux} f(x)dx$$

provided that the function f(x) is of exponential order at infinity [62] .

The other important integral transform, the Hankel transform, arises as a result of separation of variable in the problems we posed in the cylindrical co-ordinates, involving Bessel functions. The Hankel transform has been systematically studied by Waston, Tricomi, Titchmarsh, Sneddon etc. The Hankel transform of a suitably restricted function f(x) is defined by the integral,

$$(1.1.3) \qquad H_\lambda(y) = \int_0^\infty \sqrt{xy}\; J_\lambda(xy)\, f(x)\, dx$$

where $J_\lambda(xy)$ is the Bessel function of the first kind of order λ [62] .

Other well known integral transforms that is Fourier, Mellin, Convolution etc. are extensively described in a number of standard books [62], [63], etc.

The theory of conventional integral transforms was developed by many mathematicians. The names Widder, Sneddon, Tranter, Bosanquet, Boas, Brijmohan, Bhonsle, Saxena K.M., Debnath and some other can hardly be forgotten, who have gone deep in the field and made their contributions.

The problems involving several variables can be solved by applying integral transforms successively with regard to several variables. In physical problems Lapdace transform is generally used first to remove, the time variable and then m other integral transforms on space variables are successively applied. Some examples of the repeated application of transforms are given by Sneddon [62], [63].

It is quite well known that there are several problems which can be solved by the repeated application of Laplace and Hankel transforms. If we construct an integral transform for which the kernel is the product of Laplace and Hankel kernels, we may term this integral transform as the Laplace-Hankel transform (Hladik,1969). If $f(x,y)$ is suitably restricted function on $0 < x < \infty$, $0 < y < \infty$, then its Laplace-Hankel transform ($F(u,v)$ is defined by the integral

$$(1.1.4) \quad \mathcal{LH}_{\lambda}(f)(u,v) = \int_0^{\infty}\int_0^{\infty} e^{-ux}\sqrt{vy}\; J_{\lambda}(vy)\, f(x,y)\, dx\,dy$$

where $J_\lambda(vy)$ is the Bessel function of first kind with order λ real [15], [32] .

1.2 Generalized functions :

British physicist P.A.M.Dirac introduced the Dirac Delta function. The concept of generalized function was first introduced into science in 1927 as a result of Dirac's research into quantum mechanics. The Delta function $\delta(x)$, equals zero everywhere except at the origin, where it is infinite and its integral over the infinite range is one. It is obvious that $\delta(x)$ is not a function in the sense of classical analysis. It is not applicable to theorems of operational calculus. Hence to provide a rigorous Mathematical justification for a correct formulation of the definition and properties of the delta function, one has to generalize the whale concept of classical functions. The generalized function is a generalization of the classical concept of a mathematical function.

For providing a sound mathematical foundation for symbolic functions like Dirac Delta function, different theories were proposed for generalizing the concept of a function by Bochner [3], Soboleff [64], Schwartz [60], Miku-sin-ski [48], Halperin [30] and Weston [73]. Schwartz [60] was led to discover the distributions in an attempt to solve a problem posed by Chequet and Deny [9], with the knowledge of "Radon Measure " of Bourbaki and Carton [4] and duality of normed vector spaces of Dieudonne [14]. The diverse approaches in prevelent theories are unified in elegant theory of Schwartz. We summarise this theory here for future ready reference. However, it is remarked that the representation of certain distributions are given by Carleman [6] and Bremerman [5]. Hormander [34] and Lojasiewicz [44] have nicely tackeled the problem of division of distributions. For detailed discussions of generalized functions Gelfand and

Shilov [26], Friedmann [24], Horvath [33] and Vladimirov [70], Zemanian [78] may be consulted. Grothendick [28] have given very deep results on distributions.

By a conventional function we mean a function whose domain is contained in either R^n or C^n. A function of rapid descent is conventional function f(t) on R^n or C^n such that $|f(t)| = O(|t|^{-m})$ as $|t| \to \infty$ for every integer m R^1. A conventional function is said to be smooth if all its derivatives of all orders are continuous at all points of its domain [77].

Let I be an open set in either R^n or C^n. A set V(I) is called testing-function space if the following three conditions are satisfied :

1. V(I) consists entirely of smooth complex-valued functions defined on I.
2. V(I) is either a complete countably mutinormed space or a complete countable union space.
3. If $\{\phi_\nu\}_{\nu=1}^{\infty}$ converges in V(I) to zero, then, for every non-negative integer K R^n, $\{D^k\phi_\nu\}_{\nu=1}^{\infty}$ converges to the zero function uniformly on every compact subset of I [77].

A generalized function on I is any continuous linear functional on any testing function space on I. In other words, f is called a generalized function if it is a member of the dual space of some testing function space on I.

1.3 Generalized integral transformations :

The topic of the generalized integral transformations requires two theories, the theory of integral transformation and the theory of generalized functions.

There are variety of methods for extending the Laplace transform to generalized functions, some of which are restricted to the one-sided Laplace transformation. Benedetto [1], Cooper [12], Dolezal [19], Garnier and Munster [25], Ishihara [35], Jones [36], Korevaar [38], Lavoine [41], Liverman [43], Miller [47], Mayers [51], Rehberg [69], Schwarz [60], Weston [73] and Zemanian [74], [75]. The original method is due to Schwartz [60] and is based upon his definition of the generalized Fourier transformation.

In contrast to this method presented by Zemanian [77] defines the Laplace transform F(s) of a generalized function f directly as application of f(t) » to e^{-st} :

(1.3.1) $$F(S) = \langle f(t) , e^{-st} \rangle.$$

The first one to extend the Hankel transformation to generalized functions was J.L.Lions [39], who extended imit in such way that the inversion formula could be stated for Hankel transformation.

Let I denotes the open interval (o,∞), and x is a real variable restricted to I. For each real number λ we define a countably multinormed space H_λ as follows. A function $\phi(x)$

is in H_λ if and only if it defined on $0<x<\infty$, it is complex-valued and smooth, and, for each pair of non-negative integers m and k ,

$$(1.3.2) \quad \gamma^{\lambda}_{m,k}(\phi) = \sup_{0<x<\infty} \left| x^{m} (x^{-1}D)^{k} [x^{-\lambda-1/2}\phi(x)] \right|$$

is finite. H_λ is a linear space. Also each $\gamma^{\lambda}_{m,k}$ is a seminorm on H_λ, the $\gamma^{\lambda}_{m,o}$ is a norm and the collection $\{\gamma^{\lambda}_{m,k}\}^{\infty}_{m,k=o}$ is a multinorm. The topology of H_λ is that generated by $\{\gamma^{\lambda}_{m,k}\}^{\infty}_{m,k=o}$. We shall see shortly that H_λ is a testing function space. Let H_λ denotes the space of all continuous linear functionals defined on H_λ. The members of H'_λ are generalized functions on which our Hankel transformation will be defined. Let λ is restricted to the interval $-\frac{1}{2} \leq \lambda < \infty$. We define the generalized Hankel transformation h'_λ on H'_λ as the adjoint of h_λ on H_λ. More specifically, for arbitrary $\Phi \in H_\lambda$ and $\Phi = h_\lambda \phi$ and for arbirary $f \in H'_\lambda$, we define $F = h'_\lambda f$ by

$$(1.3.3) \quad \langle F, \Phi \rangle = \langle f, \phi \rangle$$

or using different symbols, we write

$$(1.3.4) \quad \langle h'_\lambda f, \Phi \rangle = \langle f, h_\lambda \phi \rangle$$

The complex Hankel transformation due to koh and Zemanian [34] is defined as the direct application of generalized function $f(x)$ to the kernel $\sqrt{xy}\ J_\lambda (xy)$.

Zemanian has extended K, Weierstrass and convolution transformations to generalized functions [74], [75], [76]. In the book [77], he has also given the systematic treatment of the extension of integral transformations having kernels arising from orthogonal series expansions.

In series of papers [52], [53], [54], J.N.Pandey has extended Hankel convolution, Weierstrass-Hankel convolution, and Stieltjes transformations to certain class of generalized functions.

O.P.Misra has extended the Stieltjes and Meijer-Laplace transformations to generalized functions [49], [50].

R.S.Pathak has extended in series of papers [55], [57], [58], [59], [56], G-transform, Meijer, Kontorovich-Lebedev, Varma, Hardy transformations to generalized functions.

L.S.Dube has obtained the inversion formulae for Generalized, S_2 transform, finite Hankel transformation and Hankel potential transform in series of papers [20], [21], [22].

Consider $\mathcal{E}(I)$ as the space of all complex-valued smooth functions. We define $\mathcal{E}(I)$ as countably multinormed space as follows :

For each compact subset K of I and for each non-negative integer $K \in R^n$, Let $\gamma_{K,k}$ be seminorm on (I) defined by

(1.3.5) $\gamma_{K,k}(\phi) = \sup_{t \in K} | D^k \phi(t) | < \infty, \quad \phi \in \mathcal{E}(I)$

The collection R of all seminorms $\gamma_{K,k}$ is a multinorm. We assign topology to $\mathcal{E}(I)$ that is generated by R. $\mathcal{E}(I)$ is complete. $\mathcal{E}'(I)$ is a dual space of $\mathcal{E}(I)$, which is also complete. Clearly $\mathcal{E}(I)$ is a testing function space. The members of $\mathcal{E}'(I)$ are generalized functions with compact support. h_λ is Hankel transform operator and h'_λ is adjoint of h_λ defined by

$$\langle h'_\lambda f,\ \Phi \rangle = \langle f,\ h_\lambda \phi \rangle$$

When $F \in \mathcal{E}'(I)$ and $f \in H'_\lambda$ we define Hankel transform of generalized function as :

$$(1.3.6) \quad (H_\lambda f)(y) = F(y) = \langle f(x),\ \sqrt{xy}\ J_\lambda(xy) \rangle$$

Taking product of the kernels of the Laplace and the Hankel transformations, the generalized Laplace-Hankel transform $\Phi(u,v)$ of $\phi(x,y)$ is defined as

$$(1.3.7) \quad \Phi(u,v) = \langle \phi(x,y),\ e^{-ux}\ \sqrt{vy}\ J_\lambda(vy) \rangle$$

where $0 < x < \infty$, $0 < y < \infty$ and $\mathrm{Re}\,u > 0$, $0 < v < \infty$, $J_\lambda(vy)$ is the Bessel function of order λ with λ and a real [7].

1.4 Notation and Terminology :

The notations and terminology of this work follows that of [77].

R^n and C^n denote respectively the real and complex n-dimensional euclidian spaces. Thus, an arbitrary point t in R^n (C^n) is an ordered n-tuple of real (complex) numbers,

$t = \{t_1, t_2, \ldots, t_n\}$ whose magnitude is

$$|t| = \left[\sum_{\nu=1}^{n} | t_\nu | \right]^{1/2}$$

A compact set in R^n or C^n is simply a closed bounded set. If I is an open set in R^n, if K is a compact set in R^n, and if K is contained in I, we say that K is compact subset of I .

The function whose domain is contained in R^n (C^n) and whose range is in R^1 (C^1) then that function is called conventional function. Conventional function is said to be smooth if all its derivatives of all orders are continuous at all points of its domain. The support of a continuous function f(t) defined on some open set in R^n is the closure with respect to of the set of points t where $f(t) \neq 0$. It is denoted by suppf.

If k is nonnegative integer in R^1, the partial differential with respect to x is denoted by $D_x^k = \frac{\partial^k}{\partial x^k}$, we shall use the notation $S^{k'}_{\lambda,y}$ for the

$$(y^{-\lambda-1/2} D_\lambda \; y^{2\lambda+1} \; D_y \; y^{-\lambda-1/2})^{k'}$$

If f is a generated function on R^2 , the notation f(x,y), where $(x,y) \in R^2$ is used merely to indicate that the testing functions, on which f is defined, have (x,y) as their independent variable; it does not mean that f is a function of (x,y). If f $V'(I)$, then $\langle f, \phi \rangle$ denotes the number assigned to $\phi \in V(I)$, when $V'(I)$ is dual space of $V(I)$.

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