CHAPTER - I

SOME DEFINITIONS

ABSTRACT

In the present Chapter we give in detail the definitions and statements of known results which we are making use in the course of our research.

Notation -

Definition :- Let $E = \{ Z : Z$ is a complex number and $|z| < 1 \}$

Definition :- A complex valued function $f(z)$ is said to be holomorphic in a domain D in the complex plane if it is differentiable at every point of the domain D.

Definition :- A single valued function f is said to be univalent (or schlicht) in a domain $D \subset \mathbb{C}$, $\mathbb{C}$ denoting the complex plane, if it never takes the same value twice, that is, if $f(z_1) \neq f(z_2)$ for all points z_1 and z_2 in D, with $z_1 \neq z_2$

Definition :- Let S be the class of all functions $f(z)$ holomorphic and univalent in E and normalised by $f(o)=0$, $f'(o)=1$.

This family of functions is designated as the normalised univalent functions. We will in general confine our attention to this above family of functions.

Definition :- A domain containing the origin is starlike with respect to the origin if it is intersected by any straight line through the origin in a linear segment. Starlike with respect to the origin will be referred to as simply starlike.

Definition :- Let S^* be the subclass of S whose members map every disc $|z| < \rho$, $(0 < \rho < 1)$ onto a starlike domain .

Definition :- Let $f(z)$ be holomorphic at $z=0$ and satisfy $f(o)=0$, $f'(o)\neq 0$ there. Then the radius of univalence, denoted by Uo is defined to be the largest value of 'r' such that $f(z)$ is holomorphic and univalent for $|z|<r$.

Definition :- let $f(z)$ be holomorphic at $z = 0$ and satisfy $f(o)=0$ and $f'(o)\neq 0$, there. Let α be a real number satisfying $0 \leqslant \alpha < 1$. The radius of starlikeness of order α denoted by S_α, is defined to be the largest value of 'r' such that $f(z)$ is holomorphic and $Re\left\{ z\frac{f'(z)}{f(z)}\right\} > \alpha$ for $|z| < r$.

Definition :- Let K be the subclass of S, whose members map every disc $|z| \leqslant \rho$, $0 \leqslant \rho < 1$ onto a convex domain.

Definition :- Let $f(z)$ be holomorphic at $z=0$ and satisfy $f(o)=0$ and $f'(o)\neq 0$ there, let α be a real number satisfying $0 \leqslant \alpha < 1$. The radius of convexity of order α, C_α defined to be the largest value of 'r' such that $f(z)$ is holomorphic and $Re\left\{ 1 + z\frac{f''(z)}{f'(z)}\right\} > \alpha$ for $|z| < r$

Theorem - [6, page 221] Suppose $f(z)$ is holomorphic in E and $f(o)=0$, $f'(o)=1$, then $f(z) \in S^*$ if and only if

$$Re\left\{ z f'(z)/ f(z)\right\} > 0 \quad \text{for } z \in E$$

Theorem - [6 page 223] Suppose $f(z)$ is holomorphic in E and $f(o)=0$ and $f'(o)=1$, then $f(z) \in K$, the class of convex functions if and only if $Re\left\{1 + z\frac{f''(z)}{f'(z)}\right\} > 0 \quad z \in E$

Theorem - (5) Suppose $f(z) \in K$, then for $Z \in E$

$$Re\left\{ z f'(z)/f(z) \right\} > \tfrac{1}{2}$$ 10

<u>Definition</u> :- (Kaplan (24)). A function $f(z) \in S$ is close-to-convex with respect to the convex function $e^{i\alpha} g(z)$ # where $g(z) \in K$ and $0 \leq \alpha < 2\pi$ if

$$Re\left\{ f'(z) / e^{i\alpha} g'(z) \right\} > 0$$ 10

For $z \in E$ let C be the subclass of S of close-to-convex functions ie. $f(z) \in C$ if $f(z)$ is close-to-convex with respect to some $e^{i\alpha} g(z)$, $g(z)$ in K.

<u>Definition</u> :- Let $f(z)$ be holomorphic at $z=0$ and satisfy $f(o)=0$, $f'(o)\neq 0$ there. Then the radius of close-to-convexity is defined to be the largest value of r such that $f(z)$ is holomorphic and close-to-convex for $|z| < r$.

<u>Remark</u> :- A function satisfying either the close-to-convex condition, the starlike condition, or the convex condition is univalent.

Theorem - (4) page 173

Suppose $f(z)$ is holomorphic in E, $f'(z)\neq 0$, $(Z \in E)$, and $f(o)=0$, $f'(o)=1$, Then $f(z) \in C$ if and only if

$$\int_{\theta_1}^{\theta_2} Re\left(1 + z f''(z)/f'(z) \right) d\theta > -\pi \quad , \quad Z = re^{i\theta}$$

for any r, θ_1 and θ_2, $0 < r < 1$, $0 \leq \theta_1 < \theta_2 \leq 2\pi$

REMARKS :

The following inclusion relationships hold for normalised classes of functions, K, S^*, C, S: $K \subset S^* \subset C \subset S$.

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