CHAPTER - I

INTRODUCTION

1.1 Preliminary Remarks :

A function is an important notion in Mathematics. Some manipulations like Dirac delta function in technical literature have motivated the votaries of mathematics to re-examine the concept of a function. The idea of specifying a function not by its value but by its behaviour as a functional on some space of testing functions is a new concept. This new mode of thinking gave birth to the theory of generalized functions, which put the wheels of research in several branches of mathematics in rapid motion. The impact of generalized functions on the integral transforms has recently revolutionised the theory of integral transformations.

1.2 Integral Transforms :

The theory of integral transformations provides technique for the solution of certain types of classical boundary and initial value problems. The integral transforms are used in the solution of problems in applied mathematics.

Many functions in analysis can be expressed as Lebesque integrals or improper Riemann integrals of the form

$$F(s) = \int_{-\infty}^{\infty} K(s,x)f(x)dx. \qquad \ldots(1.2.1)$$

A function F defined by an equation of this type (in which s may be real or complex) is called an integral transform of f. The function K which appears in the integrand is referred to as the kernel of the transformation, it being assumed, of course, that the infinite integral in equation (1.2.1) is convergent. When the range of integration $(-\infty, \infty)$ is replaced by a finite range (a,b), F(s) is usually called the finite transform of f(x).

The oldest systematic technique for the solution of the partial differential equations of mathematics and physics is the method of separation of variables, introduced by d'Alembert, Daniel Bernculi and Euler in the middle of the eighteenth century. It remains a method of great value today and lies at the heart of the use of integral transforms in the solution of problems in applied mathematics.

The use of integral transformation will often reduce a partial differential equation in n independent variables to one in (n-1) variables. Thus reducing the difficulty of the problem under discussion. A particular differential equation associated with particular initial and boundary conditions requires a suitable integral transform to convert it into an algebric equation whose solution is inverted by the inverse transform to obtain the solution of the original equation.

The theory of conventional integral transforms was developed by many mathematicians. The conventional Stieltjes transform was introduced for the first time in 1894 by T.J. Stieltjes, in connection with his work on continued fractions, and latter on many eminent mathematicians studied certain properties of Stieltjes transformation.

The integral transform defined by the equation

$$f(x) = \int_0^\infty \frac{\phi(t)}{x + t}\, dt \qquad \dots (1.2.2)$$

(where x is a point of the complex plane cut along the negative real axis) is called the Stieltjes transform. The Stieltjes transform is obtained by the first iteration of the Laplace transform. The theoretical foundations of the Stieltjes transform have been set down by Widder [78] and Titchmarsh [68].

Widder [76,77] studied the successive iterates of the Stieltjes kernel expressed in terms of the elementary functions. N.Pollard [57] also studied Stieltjes transform. Harry Pollard [56] has obtained an inversion formula for the Stieltjes transform. He has also obtained for the first time a necessary and sufficient condition for the representation of a function f(x) in the most general convergent Stieltjes transform. Vuckovich [75] has proved some theorems on the Stieltjes transform. Wintner [79] studied stable distributions and the transform of Stieltjes and Le Roy. Goldberg [29] has derived an inversion formula for the Stieltjes transform, starting with inversion of the integral

$$G(x) = \int_0^\infty \frac{\psi(t)t.dt}{x^2 + t^2}, \quad 0 \leq x < \infty .$$

Greenstein [30] has obtained the precise bounds for Stieltjes transforms and positive real functions. An [1] has obtained a complex Tauberian theorem for the Stieltjes transform. Woolcock [80,81] and Zimering Shimshan [83] have studied some asymptotic behaviour of Stieltjes transform. Jain [31] has obtained a relation between Laplace and Stieltjes transform of two variables. Celidze [16] has studied the double Stieltjes transform. Dahiya and Singh Bhagat [21] have proved a theorem relating the Hankel, Stieltjes and k-transforms. Dahiya [19] has obtained a theorem on the Stieltjes transform that connects the Hankel and the k-transforms of a function and then used that theorem to obtain the Stieltjes transform of functions involving Bessel functions and G-functions. Singh Bhagat [63] has proved some theorems on Stieltjes and k-transform. Kof [38] has obtained the representation of certain classes of functions by means of the Stieltjes transform. Mukherjee and Bhattacharya [48] have obtained the necessary and sufficient conditions for a function f(x) to be a Stieltjes transform of g(t) with

$g(t) \in L_1(0,R)$ for all $R > 0$. Erdelyi [23] has studied Stieltjes transformation on weighted LP-spaces.

The generalized Stieltjes transform has been studied by many mathematicians. Sumner [66] has obtained an inversion formula involving a complex integral for the generalized Stieltjes transforms

$$f(x) = \int_0^\infty \frac{d\alpha(t)}{(x+t)^\rho}, \qquad \ldots (1.2.3)$$

and

$$f(x) = \int_0^\infty \frac{\phi(t)}{(x+t)^\rho} dt \qquad \ldots (1.2.4)$$

Byrne and Love [12] established some complex inversion formula for the generalized Stieltjes transform (1.2.3). Their results are significant improvements over the work of Sumner [66]. Dahiya [20] has proved one theorem connecting the Hankel transform of order ν, k-transform of order ν and generalized Stieltjes transform of order ρ . Carmichael [13] and Carmichael and Hayashi [14] have proved Abelian theorems for the generalized Stieltjes transform

$$F(s) = \int_0^\infty \frac{f(t)}{(s+t)^{\rho+1}} dt \quad . \qquad \ldots (1.2.5)$$

Arya [9] has proved a representation theorem for a generalized Stieltjes transform

$$h(y) = \int_0^\infty (x+y)^{-\sigma} e^{(x+y)/2} W_{k,\mu}(x+y) f(x) dx \ . \qquad \ldots (1.2.6)$$

Chatterje [17] has obtained an inversion formula for the generalized Stieltjes transform (1.2.6). Pandey [52] has introduced the transform

$$\phi(p) = \int_0^\infty (p+x)^{-\sigma-\mu+1/2} \exp(p+x)\Gamma(2\mu,p+x)f(x)dx \quad \ldots (1.2.7)$$

where $\Gamma(\mu,x)$ is the incomplete gamma function, as a generalization of the Stieltjes transform and established an inversion formula for this transform. Simple generalization of the transform (1.2.4) given by the integral

$$f(x) = \int_0^\infty (x^m + t^m)^{-\rho} \phi(t)dt \quad \ldots (1.2.8)$$

has been studied by Arora [2]. He has obtained some theorems analogous to theorems known for the Laplace, Hankel and Mellin transform. Varma [73] has analyzed the generalized Stieltjes transform given by

$$f(s) = \frac{\Gamma(2m+1)}{s\Gamma(m-k+3/2)} \int_0^\infty F(2m+1,1;m-k+3/2;-\frac{t}{s})\phi(t)dt \ldots (1.2.9)$$

Saksena and Sagar [59] have extended the transform (1.2.9) to the transform

$$f(s) = \frac{\Gamma(-2n)\Gamma(2n+1)\Gamma(2m+2n+1)\ s^{-2n-1}}{\Gamma(1/2-n-1)\Gamma(m+2n-k+3/2)}$$

$$\times \int_0^\infty t^{2n}\ {}_2F_1(2n+2m+1,n+1+1/2;m+2n-k+3/2;-t/s)\phi(t)dt$$

...

$$+\frac{\Gamma(2n)\Gamma(2m+1)s^{-1}}{\Gamma(1/2+n-\underline{1})\Gamma(m-k+3/2)}$$

$$\times\int_{\rho}^{\infty} {}_3F_2(2m+1,\ 1,\ \underline{1}-n+1/2;\ m-k+3/2 - 2n+1; -t/s)\phi(t)dt.$$

... (1.2.10)

Arya [3,4] proved an Abelian theorem, convergence theorem and asymptotic properties of a generalized Stieltjes transform

$$f(s) = \frac{\Gamma(2m+1)}{\Gamma(m-k+3/2)}\frac{1}{s}\int_{c}^{\infty} F(2m+1,1,m-k+3/2;\ -t/s)d\alpha(t).$$ ...(1.2.11)

He [5,6,7,8] has also proved real inversion theorems and complex inversion formula for a generalized Stieltjes transform (1.2.11). Saxena [60] has evaluated some examples and formulated some integral identities showing the connection of the generalized Stieltjes transform with other integral transform. Swaroop Rajendra [67] has given several complex inversion formulae and proved the uniqueness theorem and a parseval type property for the generalized Stieltjes transform

$$\phi(p,\lambda,\mu,\nu) = \frac{\Gamma(\lambda)\Gamma(\mu)}{\Gamma(\nu)}\frac{1}{p}\int_{c}^{\infty} F(\lambda,\mu;\nu;-\frac{t}{p})f(t)dt\ .$$ ... (1.2.12)

Verma [74] has stated some convergence theorems and asymptotic properties of the generalized Stieltjes transform

$$f(s) = s^{-1}\ [\Gamma(\underline{1}+m+3/2)\Gamma(-\underline{1}-m+3/2)]\ /\ \Gamma(\underline{1}-k+2)$$

$$\times\int_{0}^{\infty} {}_2F_1(\underline{1}+m+3/2,\ \underline{1}-m+3/2;\ \underline{1}-k+2;\ 1/2-a-t/s)\phi(t)dt.$$

...(1.2.13)

Joshi [34,35] has studied the generalized Stieltjes transform

$$\phi(s) = [\Gamma(\beta+\eta+1)/\Gamma(\alpha+\beta+\eta+1)][\Gamma(\beta+1)/s] \times \int_0^\infty (y/s)^\beta F(\alpha,\beta+1,b;-y/s)f(y)dy. \quad \ldots(1.2.14)$$

Mehra and Saxena [46] have proved two theorems on the transform

$$\phi(x) = \prod_{i=1}^{k}\left\{\Gamma(a_i)/\Gamma(b_i)\right\} \times \int_0^\infty (xu)^\lambda {}_kF_k(a_1,\ldots a_k:b_1,\ldots b_k;-xu)h(u)du \quad \ldots(1.2.15)$$

and derived some known results by taking k=1 and 2. Mukherjee [49] has obtained an inversion formula for the generalized Stieltjes transform

$$g(p) = p^{-1} \int_c^\infty G^{2,2}_{3,3}\left[\frac{x}{p}\,\middle|\,\begin{matrix}c,-m,-\mu,-1/2-\nu+\mu\\ c,\ k-m-1/2\end{matrix}\right] g(x)dx. \quad \ldots(1.2.16)$$

Saxena and Gupta [61] have proved three theorems on a generalized Stieltjes transform involving Meijer's G-function. These theorems are very general and are useful for the evaluation of complicated integrals involving Meijer's G-function, which reduce to well-known functions. Pandey [53] has defined the integral transform

$$f(s) = \int_{c}^{\infty} K(s,t)\phi(t)dt \qquad \ldots (1.2.17)$$

where the kernel K(s,t) is Meijer's G-function.

Kapoor [36,37] has obtained an inversion theorem for the generalized Stieltjes transform

$$\phi(s) = \int_{0}^{\infty} t^{-1}\, G^{m,n+1}_{p+1,q}\left[\frac{s}{t}\middle|\begin{matrix} c,\ a_1,\ a_2,\ \ldots\ a_p \\ b_1,\ b_2,\ \ldots\ b_q \end{matrix}\right] g(t)dt. \qquad \ldots (1.2.18)$$

Jaiswal [32] has proved two theorems and their converses for the generalized Stieltjes transform

$$\phi(s) = \int_{0}^{\infty} e^{-\alpha st}\, G^{m,n}_{p,q}\left[\beta st \middle| \begin{matrix} (\alpha_p) \\ (\beta_q) \end{matrix}\right] f(t)dt\ . \qquad \ldots (1.2.19)$$

Golas [26] has defined an operator for the generalized Stieltjes transform. Golas and Gupta [28] have developed an inversion operators for the generalized Stieltjes transform

$$f(s) = \left(\Gamma(b+c+1)\Gamma(b+1)s^{-1}/\Gamma(a+b+c+1)\right)$$

$$\times \int_{0}^{\infty} \left(\frac{t}{s}\right)^{b} {}_2F_1\,(b+c+1,b-1,a+b+c+1;-t/s)\theta(t)dt \ldots (1.2.20)$$

and

$$f(s) = s^{-1} \int_{0}^{\infty} G^{2,2}_{3,3}\left(\frac{t}{s}\middle| \begin{matrix} c,\ -m-1,\ 1/2-\gamma+1 \\ 2\underline{1},o,k-m-1/2 \end{matrix}\right)\theta(t)dt \qquad \ldots (1.2.21)$$

Mehra [45] considered the generalized Stieltjes transform

$$\phi(s) = \int_0^\infty t^{-1} H^{m,n+1}_{p+1,q} \left[\alpha s^\lambda t^{-\lambda} \,\Big|\, \begin{matrix} (\theta,\lambda), & (a_p,\alpha_p) \\ & (p_q,\beta_q) \end{matrix} \right] g(t)dt \qquad \ldots(1.2.22)$$

and obtained an inversion formula by means of the Mellin transform with convergence in the ordinary sense. Sharma [62] has studied the generalized Stieltjes transform involving Fox's H-function in the kernel. Golas [27] has proved an inversion and the representation theorem for a generalized Stieltjes transform. Srivastava [65] has given some remarks on a generalization of the Stieltjes transform. Joshi [33] has proved an Abelian theorem for a generalized Stieltjes transform and Love and Byrne [41,42] have proved real inversion theorems for a generalized Stieltjes transform.

1.3 Generalized Integral Transforms :

The theory of integral transforms is a classical subject in mathematics whose literature can be traced back through at least one and half century. The theory of generalized functions, on the other hand, is of recent origin, its advent being the publication of Laurent Schwartz's works, which appeared from 1944 onwards, most notably of them is his two volume work, "Theorie des Distributions," published in 1950 and 1951. Some fragments of the theory appeared still earlier in the works of Bochner [11], around 1927, and Soboleff [64], around 1936.

Generalized Stieltjes transformation has been defined by Zemanian [82] and Benedetto [10]. Benedetto has defined some elementary properties of one-sided, one-dimensional distributional Stieltjes transformation. Zemanian [82] has defined convolution and inversion of generalized Stieltjes transformation.

The transform (1.2.2) and (1.2.4) have been studied in the distributional sense by Pandey [50,51] and Pathak [54,55] respectively. The transform (1.2.4) has been also extended to generalized functions both by direct approach and the method of adjoints, and the resulting extensions are correlated and inversion formulae are also developed, as is the application of fractional integration to these transforms by Erdelyi [22]. Recently the generalization of the transform (1.2.8) given by the integral

$$F(s) = \Gamma(\varrho) s^{m\varrho - 1} \int_0^\infty \frac{f(t)\, dt}{(s^m + t^m)^\varrho}, \quad (m, \varrho > 0) \qquad \ldots (1.3.1)$$

has been studied by Malgonde and Saxena [44]. They have extended this transform to a certain class of generalized functions interpreting convergence in the weak distributional sense. Moreover Chaudhari [18] has proved a complex inversion theorem for the generalized Stieltjes transform (1.3.1) for the space of generalized functions developed by Zemanian [82].

Some generalizations of the transform (1.2.2) have been studied in the distributional sense by Ghosh [24,25], Tiwari and others. Tiwari [69] has extended the generalized Stieltjes transform (1.2.6) to a class of generalized functions. He has also obtained an inversion formula for the distributional generalized Stieltjes transform

$$F(s) = \left\langle f(\tau), \frac{\Gamma(A)}{s\Gamma(B)}\, {}_2F_1(A,1;B; -\frac{\tau}{s}) \right\rangle, \quad 0 < s < \infty \qquad \ldots (1.3.2)$$

by putting it into the form of convolution transform [70]. Misra [47], Lavoine and Misra [39,40] and Carmichael and Milteon [15] have proved Abelian theorems for the distributional Stieltjes transformation. Rao has also proved Abelian theorems for a distributional generalized Stieltjes transform [58]. Tiwari and Pandey [72] have shown that the two complex inversion formulas of Byrne and Love [12] for the generalized Stieltjes transformation are also valid for a class of distributions. The Gauss-hypergeometric transform given by

$$F(s) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(c)} \int_0^\infty {}_2F_1\,(a,b;c;-\varrho x) f(x)\,dx \qquad \ldots (1.3.3)$$

and studied by Swaroop [67] has been extended to a class of generalized functions by Mahato [43] and very recently Tiwari and Koranne [71] have extended a generalized Stieltjes transform

$$F(x) = \frac{\Gamma(\beta+\eta+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+\eta+1)} \frac{1}{x} \int_0^\infty (t/x)^\beta$$

$$\times \quad F(\beta+\eta+1,\ \beta+1,\ \alpha+\beta+\eta+1;\ -t/x) f(t)dt \qquad \ldots (1.3.4)$$

to generalized functions (distributions) and some Abelian theorems for the distributional generalized Stieltjes transform are also proved.

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