CHAPTRE IV

Conformally Flat Charged Perfect Fluid Distribution In Einstein - Cartan Theory

CONFORMALLY FLAT CHARGED PERFECT FLUID DISTRIBUTION IN EINSTEIN-CARTAN THEORY.

1. INTRODUCTION :

The possibilities of exsistence of electromagnatic fields which are conformal to some empty space times were investigated by Singh and Roy (1966). Stephani after studing the conformally flat electrovc solutions showed that these fields with non-null electromagnatic fields would form product manifolds. Collinson (1968) proved that the Wey-l tensor and electromagnatic fields both must be non - null type and non zero for the electrovac fields of class -I. While Baros (1974) showed, class -I perfect fluid was to possess at least one of the following properties : (i) conformal flatness (ii) the flow is geodesic (iii) it admits a three dimensional group of isometries with two dimensional space like trajectories. Bayen and Flato (1976) gave original comments on a conformally covarient formulation of Maxwell's equations with a linear group condition on a compactified Minkowski space.

Tariq and Mchenaghan (1978) expected for the Bertotti Robinson solution as the most general conformally flat solution of the source-free Einstein - Maxwell equations of non null electromagnatic fields. Raychaudhuri and Maiti (1979) presented a proof of the theorm that the only static conformally flat metric for a perfect fluid distribution is the Schwarzchild interior metric. In Brans-Dicke (1961) and Sen-Dunn (1971) theories of gravitation Reddy (1979) obtained the exact solutions for the static spherically symmetric conformally flat metric. Panday and Tiwari (1981) obtained the solutions of Einstein's field equations for static charged conformally flat perfect fluid distribution with spherical symmetry.

Prasanna (1975) obtained the analogue of some static spherically symmetric solutions of Tolman (1939) in Einstein - Cartan theory. By considering a static charged fluid sphere in Einstein - Cartan theory, Naduka (1977) generalised the Prasanna's work. Also he solved the resulting field equations by an entirely different technique. While Kuchowicz (1975) has discussed various methods of deriving exact solutions of spherical symmetry in Einstein - Cartan theory. Singh and Yadav (1979) obtained an analytic solution by quadrature method. Krori et al. (1981) obtained a singularity free solution for a static charged fluid sphere with spin. Panday and Tiwari (1982) solved the Einstein - Cartan equations for a static spherically symmetric fluid sphere.

In this chapter we consider a static conformally flat perfect fluid distribution in Einnstein - Cartan theory and obtain the field equations.

These field equations are solved with the assumption of the metric coefficient

$$e^{-\mu} = (A + Br^2 + Cr^3)$$

Here we also disscuss the properties of the solutions.

It is interesting to note that in the absence of the charge our solutions will reduce to the solutions of S.B. Kalyanshetti and B.B. Waghamode (1982) for conformally flat perfect fluid distribution.

It is worth to note that if the charge is absent then the density ρ = 12AB, as seen by Som and Bedran (1981).

The aribitrary constants appearing in the solutions can be evaluated by comparing the metric with the Reissner-Nordstrom metric for a mass m_o, redius r_o and charge Q_o.

2. METRIC AND FIELD EQUATIONS :

The Einstein-Cartan Maxwell equations for the perfect fluid are

$$R_{ij} - 1/2\, Rg_{ij} = - 8\pi T_{ij} \quad \ldots \quad (2.1)$$

$$\left[(-g)^{1/2} F^{ij}\right]_{,j} = 4\pi J^i (-g)^{1/2} \quad \ldots \quad (2.2)$$

$$F_{[ij;k]} = 0 \quad \ldots \quad (2.3)$$

where R_{ij} is the Ricci tensor, T_{ij} is the energy momentum tensor. F^{ij} is the electromagnatic field tensor and J^i is the current four vector.

Here we conseder a static conformally flat spherical symmetric metric in the form

$$ds^2 = e^{2\mu}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 - dt^2) \quad \ldots \quad (2.4)$$

Where μ is a function of r only. Here in the system under the study the energy momentum tensor T^i_j splits into two parts t^i_j and E^i_j for the matter part and for the charge part respectively.

$$T^i_j = t^i_j + E^i_j \quad \ldots \quad (2.5)$$

The non vanishing components of t^i_j are

$$t^1_1 = t^2_2 = t^3_3 = -P \quad \text{and} \quad t^4_4 = \rho \quad \ldots \quad \ldots$$

while the non vanishing component of F^{ij} is

$$F^{14} = -F^{41}$$

Therefore the non vanishing components of E^i_j are

$$E^4_4 = E^1_1 = -E^2_2 = -E^3_3 = -\frac{1}{8\pi} g_{44} g_{11} (F^{41})^2$$

Equation (2.3) is satisfied by the choice of F^{ij} while equation (2.2) reduces to

$$F^{41} = \frac{Q(r)}{r^2} e^{-\mu} \quad \ldots \quad (2.6)$$

where Q(r) is the charge upto redius r,

$$Q(r) = 4\pi \int_0^r J^4 r^2 e^{-\mu} dr \quad \ldots \quad \ldots \quad (2.7)$$

It is clear from the equation (2.7) that Q(r) is a constant Qo (say) outside the fluid sphere. Qo is the total charge. From (2.6) we get the asymptotic form of the electric field as Q_o/r^2.

The Einstein-Cartan-Maxwell equations for the metric (2.4), from the equation (2.1), are

$$e^{-2\mu}\left[3\mu'^2 + 4\frac{\mu'}{r}\right] + \frac{1}{4}k^2K^2 = 8\pi P - 8\pi E^1_1 \quad \ldots \quad (2.8)$$

$$e^{-2\mu}\left[2\mu'' + \mu'^2 + 2\frac{\mu'}{r}\right] + \frac{1}{4}k^2K^2$$
$$= 8\pi P - 8\pi E^2_2 \quad \ldots \quad (2.9)$$

$$-e^{-2\mu}\left[2\mu'' + \mu'^2 + 4\frac{\mu'}{r}\right] + \frac{1}{4}k^2K^2$$
$$= 8\pi\rho + 8\pi E^4_4 \quad \ldots \quad (2.10)$$

where dashes denote differentiation with respect to r.

These field equations take the form

$$e^{-2\mu}\left[3\mu'^2 + \frac{4\mu'}{r}\right] = 8\pi\bar{P} - 8\pi E^1_1 \quad \ldots \quad (2.11)$$

$$e^{-2\mu}\left[2\mu'' + \mu'^2 + 2\frac{\mu'}{r}\right] = 8\pi\bar{P} - 8\pi E^2_2 \quad \ldots \quad (2.12)$$

and $$-e^{-2\mu}\left[2\mu'' + 2\mu'^2 + 4\frac{\mu'}{r}\right] = 8\pi\bar{\rho} + 8\pi E^4_4 \quad (2.13)$$

By following Hehl's approach we redefine the pressure and density as,

$$\bar{\bar{P}} = (P - 2\pi K^2)$$

and $$\bar{\rho} = (\rho - 2\pi K^2) .$$

3. SOLUTIONS OF THE FIELD EQUATIONS.

By eliminating $\bar{P}$ between the equations (2.11) and (2.12) we have

$$\mu'' - \mu'^{2} - \frac{\mu'}{r} + 8\pi E_2^2 e^{2\mu} = 0 \qquad \ldots \qquad (3.1)$$

Here P, Q, μ, and ρ are four unknowns and we have only three equations. In this view we assume that μ is known.

Let $\quad e^{-\mu} = (A + Br^2 + Cr^3) \ \ldots \ (3.2)$

where A, B, C are arbitrary constants.

From equation (3.2) we get,

$$\mu' = - e^{-\mu} (2Br + 3Cr^2) \qquad \ldots \qquad (3.3)$$

and $$\mu'' = e^{2\mu} (2Br + 3Cr^2) - e^{\mu} (2B + 6Cr) \qquad (3.4)$$

Hence from equation (3.1)

$$8\pi E_2^2 = (3Cr)(A + Br^2 + Cr^3) \qquad \ldots \qquad (3.5)$$

Also from equation (2.11)

$$8\pi\bar{P} = 8\pi \left[p - 2\pi K^2 \right]$$

$$= 8\pi P - 16\pi^2 K^2$$

$$= e^{-2\mu} \left[3\mu'^{2} + \frac{4\mu'}{r} \right] - 8\pi E_2^2$$

$$\therefore 8\pi\bar{P} = e^{-2\mu} \left[3e^{2\mu}(2Br + 3Cr^2)^2 - 4e^{\mu}(2B + 3Cr) \right] - 3Cre^{-\mu}.$$

$$= 4B^2r^2 - 8AB + 15C^2r^4 + 16BCr^3 - 12ACr$$

$$\therefore\ 8\pi P = 16\pi^2K^2 + 4B^2r^2 - 8AB + 15C^2r^4 + 16BCr^3 - 12ACr$$

$$= 16\pi^2A_1^2(\ A + Br^2 + Cr^3)^2 + 4B^2r^2 - 8AB + 15C^2r^4 + 16BCr^3 - 12ACr \qquad \ldots \qquad (3.6)$$

where spin density K is given by

$$K = A_1e^{-\mu}$$

Also density ρ is given by from equation (2.13) as

$$8\pi\bar{\rho} = 8\pi(\ \rho - 2\pi K^2\ )$$

$$= 12AB + 3BCr^3 + 27ACr$$

$$8\pi\rho = 12AB + 3BCr^3 + 27ACr + 16\pi^2A_1^2(\ A + Br^2 + Cr^3)^2 \qquad \ldots \qquad \ldots \qquad (3.7)$$

We have the charge

$$Q = Cr^3$$

In the absence of charge i.e. C = 0, then the equations (3.6) and (3.7) will reduce to,

$$8\pi P = 4(\ B^2r^2 - 2AB\ ) + 16\pi^2A_1^2(\ A + Br^2)^2 \ \ldots \qquad (3.8)$$

and

$$8\pi\rho = 12AB + 16\pi^2A_1^2(\ A + Br^2)^2 \qquad \ldots \qquad (3.9)$$

Our metric takes the form

$$ds^2 = \frac{-1}{(A + Br^2 + Cr^3)^2} \times$$

$$\times\left\{\ dr^2 + r^2d\theta^2 + r^2\sin^2\theta\ d\phi^2 - dt^2\right\} \qquad \ldots\ldots \qquad (3.10)\ \cancel{(4.10)}$$

The solutions are non singular. We see that our solutions are free from singularities. The arbitrary constants A, B, C appearing in the solutions can be evaluated by comparing the metric with Reissner-Nordstorm metric for a mass m_o, redius r_o and charge Q_o. The constant A_1 appearing in the solution can be obtained in terms of central density ρ_o.

4. DISCUSSIONS AND PROPERTIES OF THE SOLUTIONS.

Equqtions (3.6) and (3.7) are

$$8\pi P = 16\pi^2 A_1^2 (A + Br^2 + Cr^3)^2 + 4B^2 r^2 - 8AB + 15C^2 r^4 + 16BCr^3 - 12ACr$$

and $$8\pi \rho = 12AB + 3BCr^3 + 27ACr + 16\pi^2 A_1^2 (A + Br^2 + Cr^3)^2$$

In the absence of charge the equations (3.8) and (3.9) are

$$8\pi P = 4(B^2 r^2 - 2AB) + 16\pi^2 A_1^2 (A + Br^2)^2$$

and $$8\pi \rho = 12AB + 16\pi^2 A_1^2 (A + Br^2)^2$$

These solutions are same as shown by S. B. Kalyanshetti (1982).

If the spin density K = 0, then

$$8\pi\rho = 12AB \qquad \dots \qquad \dots \qquad (4.1)$$

as obtained by Som and Bedran (1981). It is worth to ~~have~~ note that non singular solutions can be obtained by assuming

$$e^{-\mu} = (A + Br^2 + Cr^3)^n \qquad \ldots \qquad \ldots \qquad (4.2)$$

Here particularly we have solved the field equations for n = 1 , in the above assumption.

In chapter 2 we have obtained the non singular solutions with the assumption of the charge $Q = Ar^3$ and we have seen that if $Q = 0$ then there exists no sphere. Here every thing would be of electromagnatic in origin.But in this chapter we have obtained non singular solutions by assuming the metric coefficient,

$$e^{-\mu} = (A + Br^2 + Cr^3)$$

In the absence of charge our solutions will reduce to solutions obtained by S. B. Kalyanshetti and B. B. Waghamode for conformally flat perfect fluid distribution in Einstein - Cartan theory and our solutions in this case will lead to a satisfatctory static uncharged fluid sphere.