CHAPTRE I

Brief Survey of Einstein - Cartan Theory

BRIEF SURVEY OF EINSTEIN - CARTAN THEORY

1. INTRODUCTION

At present, Einstein - Cartan theory of space time has attracted a wide, attention in which the intrinsic spin of matter is incorporated as the source of the torsion of the space-time mainfold. Mass and spin are two basic characters of an elementary particle system according to relativistic quantum mechanies.

In Einstein theory of general relativity mass plays an important role but not the spin. Here the density of energy momentum is the source of curvature, one can obtain an intersting link between theory of gravitation and theory of special relativity by introducing torsion and relating it to spin. The Einstein - Cartan theory, by introducing torsion with the density of angular momentum restores, the analogy between mass and spin.

This analogy between mass and spin leads to the principle of equivalance at least in its weak form.

According to this principle of equivalance the world line of the test particle without spin, moving under the influence of gravitational fields, depends only on its initial position and velocity but not on its mass and also the motion of spin depends on the initial data instead of magnitude of the spin of the particle.

Eddington (1921) while discussing the possible extensions of general relativity mentioned casually the notion of an affine connection. He remarked that the applications in microphysics are conceivable but didnot develop his idea.

Elie Cartan (1922, 1923, 1924) introduced torsion as the axisymmetric part of an symmtric affine connection. He stated a simple generalisation of Einstein's theory of gravitation and proposed to consider a model of space-time. This model is a four dimensional differentiable manifold with a metric tensor and linear connection compatible with the metric but not a symmetric in general. In Cartan theory torsion tensor of the space time is related to the density of intrinsic angular momentum of matter and it would vanish in matter free regions. This Cartan's generalization is only a slight departure form the Einstein's theory.

2. THE NECESSITY OF EINSTEIN - CARTAN THEORY.

Recently there has been a growing interest in the foundation of Einstein's theory of general relativity. A number of relativistic theories on gravitation were putforth. The theories put up by, Brans and Dicke (1961), Bergaman (1968), Wagoner (1970), Nordtvedt (1920), Sen and Dunn (1971) are note worthy.

Their predictions obtained with the available experimental results and the observational data are compared with those of older theories. Also the question of singularity is much worried problem in general relativity. Penrose (1965), Hawking (1966) and Geroch (1966) have established that the occurence of space-time singularties is a general prediction of the theory and not just the consequence of the symmetry of the models.

It was suggested by the Trautman (1973) that the spin and torsion in Einstein-Cartan theory may avert gravitational singularities for a Friedman type of universe with a minimum radius, Ro at t = 0. In Trautman's view Einstein-Cartan theory is the simplest and the most natural modification of the original theory of gravitation.

According to Hehl, Einstein-Cartan theory is an even more beautiful theory than Einstein's general relativity.

3. HISTORICAL BACKGROUND OF EINSTEIN-CARTAN THEORY.

Einstein-Cartan theory (E-C theory) begins with Scima (1961, 1964) and Kibble (1964). Further it was developed by Trautman (1972, a, b, c, 1973a, 1975),Kerlic (1973), Kuchowicz (1975a, 1976), Hehl (1973, 1974),Tafel (1975), Steward and Hajicek (1973), Kupezynsk,(972, 1973), Raychaudhri (1975), Prasanna (1974, 1975, a, b, c) etc. The important field of application is for Einstein-Cartan theory is relativistic astrophysics which deals with the interiors of stellar objects like neutron stars with constituent particles having same aligment of spins and under the conditions when torsion may produce some observable effects. Therefore it is desirable to understand the implications in full for finite distributions, like fluid spheres with non zero presure in Einstein-Cartan theory. Considering this point the problem of static fluid spheres in Einstein-Cartan theory was considered by Prasanna (1975a). Adopting Hehl's approach (1973, 1974), he obsered that the solutions of the static fluid spheres in Einstein-Cartan theory are analogus to the solutions of Tolman (1939) in general relativity. It was found by him that, a space time metric simillar to Schwarzschild interior solutions, would no longer represent a homogeneous fluid sphere in the presence of spin density.

Raychaudhuri and Banerji (1977) constructed a specific solution corresponding to a collapsing sphere and they showed that it bounces at a radius greater than Schwarzschild redius.

Banerji (1978) pointed out that Einstein-Cartan sphere must bounce out side Schwarzschild redius, if it bounces at all. Singh and Yadav (1979) studied the static fluid sphere in Einstein-Cartan theory and obtained the solution by the method of quadrature in an analytic form.

Naduka (1977) by considering static charged fluid sphere in Einstein-Cartan theory generalized the Prasanna's work. Spatially homogeneous cosmological models of Binchi types VI and VII based on Einstein-Cartan theory were studied by Tsoubelis (1979). A class of solutions representing a static in coherent spherical dust distribution in equilibrium under the influence of spin and torsion were obtained by Som and Bedran (1981). A singularity free solutions of a static charged fluid sphere in Einstein-Cartan theory were given by Krori et al (1981). Einstein-Cartan field . equations for a static spherically symmetric fluid spheres were studied by Panday et at (1982), by a suitable assumption in the metric potential g_{11} Component.

Kalyanshetti and Waghmode (1982) considered a static cosmological model in Einstein-Cartan theory based on modified Riemannian geometry. A class of conformally flat solutions for a charged spheres were obtained by Wang Xingxiang (1987).

4. THE STRUCTURE EQATIONS OF EINSTEIN-CARTAN THEORY.:

Let g be a Lorntz metric defined on a four dimensional differential manifold M. The metric g and connection W are expressed with respect to the co-frame θ^i chosen by the metric components g_{ij} by a set of one forms W^i_j.

Here we have

$$ds^2 = g_{ij}\ \theta^i \otimes \theta^j \qquad \ldots \qquad (4.1)$$

where W^i_j are given by

$$W^i_j = \Gamma^i_{jk}\ \theta^k \qquad \ldots \qquad (4.2)$$

The torsion and curvature two-forms are given by

$$\begin{aligned} \Theta^i &= D\theta^i \\ &= d\theta^i + W^i_j \wedge \theta^i \\ &= \frac{1}{2} Q^i_{jk}\ \theta^j \wedge \theta^k \qquad \ldots \quad \ldots \quad \ldots \qquad (4.3) \end{aligned}$$

$$\begin{aligned} \Omega^i_j &= dw^i_j + W^i_k \wedge W^k_j \\ &= \frac{1}{2} R^i_{jkl}\ \theta^k \wedge \theta^l \qquad \ldots \quad \ldots \quad \ldots \qquad (4.4) \end{aligned}$$

Where D denotes the exterior covarient derivative and Q^i_{jk} and R^i_{jkl} are the torsion & the curvature tensors respectively.

Now here we introduce the completely antisymmetric pseudo-tensor η_{ijkl}, where

$$\eta_{1234} = \left| \det g_{ij} \right|^{1/2}$$

The zero-form η_{ijkl} with the forms

$$\eta_{ijk} = \theta^l \eta_{ijkl} \quad , \qquad \eta_{ij} = \frac{1}{2} \quad \theta^k \wedge \eta_{ijk}$$

$$\eta_i = \theta^j \wedge \eta_{ij} \qquad \eta = \frac{1}{4}\ \theta^i \wedge \eta_i \quad \ldots \qquad (4.5)$$

We obtain the Einstein - Cartan field equations from the veritional principle

$$\delta \int (S + KL) = 0$$

Where L is the material Lagrangian four-form and is given by

$$L = L \left[\psi_A, D\psi_A, \theta^i, g_{ij} \right].$$

Lagrangian L is locally depending on the spinor or tensor fields ψ_A, their covarient derivatives $D\psi_A$, and the metric K is the gravitational constant and S is the Ricci four-form defined globally as

$$S = 1/2 \ \eta^l_k \wedge \Omega^k_l = 1/2 \ R \ \eta \qquad \ldots \qquad \ldots \qquad (4.7)$$

where $R = g^{ln} \delta^m_k R^k_{lmn}$ and η is the volume four-form.

Varing the total action with respect to the metric, the connection W^i_j and the fields ψ_A indepedently we obtain the following equations

$$\frac{\delta L}{\delta \psi_A} = 0 \ , \ e_i = kt_i \ . \qquad C^j_i = KS^j_i \qquad \ldots \qquad (4.8)$$

where

$$e_i = \frac{1}{2} \eta_{ijk} \wedge \Omega^{jk} \ , \qquad C^j_i = - \ D\eta^j_i$$

$$t_i = \frac{\delta L}{\delta \theta^i} \qquad S^j_i = \frac{1}{2} \frac{\delta L}{\delta w^i_j} \qquad (4.9)$$

From the equations (4.8) and (4.9), using equations (4.3) and (4.4) we get Einstein - cartan equations.

$$R^j_i - 1/2\, R\, \delta^j_i = -\, K\, t^i_j \qquad \ldots \qquad (4.10)$$

$$Q^k_{ij} - \delta^k_i\, Q^l_{lj} - \delta^k_j\, Q^l_{il} = -\, KS^k_{ij} \qquad \ldots \qquad (4.11)$$

where S^i_j is the spin density and t_i is the energy momentum vector valued 3-form. t^j_i and S^k_{ij} are defined by the relations

$$t_i = \eta_j\, t^j_i \;, \quad S_{ij} = \eta_k\, S^k_{ij} \qquad \ldots \qquad (4.12)$$

For classical description of spin

$$S^i_{jk} = u^i\, S_{jk} \quad \text{with} \quad u^k\, S_{jk} = 0 \qquad \ldots \qquad (4.13)$$

where u^i is the velocity four vector.

5. COMPARISION WITH EINSTEIN'S THEORY.

Torsion is only found inside spinning matter. In some vacuume we have the usual Riemann space- time geometry where the Einstein tensor vanishes. As torson is tied to matter it cannot propadate in vacuum. Accordingly propagation of gravity is the same as in Einstein's theory in the vaccum. The difference springs for the metric dependent part of gravity from redefined sources.

The metric energy momentum tensor of General Relativity in this view is replaced by the combined energy-momentum tensor of Einstein – Cartan theory. The soureces look different but the field is the same.

The long distance behaviour of Einstein – Cartan theory is the same as in General Relativity but the short distance behaviour is distinctly different.

In General Relativity, the test particles which are neutral and and point like fall along geodesics of the Riemannian Space-time V_4 of General Relativity. This behaviour can be derived from the field equations or from the law of Reneral Relativity. In Einstein – Cartan theory a typical massive test mass carries spin and therefore falls neither along a shortest line ("geodesic") nor along a straightest line ("auto parallel").

6. BRIEF SURVEY OF OUR INVESTIGATIOUS.

In our present work, we consider a static charged and uncharged fluid spheres and obtain the non singular solutions by different techniques. Also we study the properties of the solutions.

In chapter I, we have discussed Einstein – Cartan theory with equations of structure and have given the field equations following Trautman (1973).

In chapter II, we consider a static fluid sphere in Einstein - Cartan theory and present the field equations. Follwing Prasanna, by Hehl's (1973, 1974) approach, we solve these field equations with the equation of state $\bar{P} = \bar{\rho}$ (Zeldovich-fluid). For specific values of an arbitrary constants, our model will reduce to static Einstein-Universe.

In chapter III, we deal with a static charged conformally flat perfect fluid distribution in Einstein - Catran theory and obtain the field equations and these field equations are solved with the assumption that the charge $Q = Ar^3$, whare A is an arbitrary constant and spins of particles are all aligned in the radial direction only.

The solutions obtained are regular at all points even at the origin r = 0, leading to the satisfactory model for the point charge. Also the properties of our solutions are discussed.

Chapter IV deals with a static conformally flat perfect fluid distribution in Einstein - Cartan theory. Here we obtain the field equations and they are solved with the assumption that the metric coeficient,

$$e^{-\mu} = (A + Br^2 + cr^3)$$

Also the properties of solution are discussed. It is interesting to note that in the absence of charge our solution will reduce to solutions of S.B. Kylanshetti and

Waghamode (1982) for conformally flat perfect fluid distribution.

It is worth to note that if the charge is absent then density $\rho = 12AB$ as seen by Som and Bedran (1981). The arbitrary constants appearing in the solutions can be evaluated by comparing the metric with the Reissner-Nordstrom metric for a mass m_o, redius r_o and charge Q_o. The constant A_1 appearing in the solution can be evaluated in terms of central density ρ_o.

Here we see that our solutions are free from singularities.