CHAPTER 1

STATEMENT AND DEFINITION OF KNOWN RESULTS

ABSTRACT

In this chapter we give a list of some definitions and statements of the results required for us during the course of investigation. The relevant references are cited at the end of this chapter.

STATEMENT AND DEFINITION OF KNOWN RESULTS

Definition : Let $U = \{z : z \text{ is a complex Number } |z| < 1\}$

Definition : A complex valued function f (z) is said to be holomorphic in a domain D in the complex plane if it has uniquely determined derivative at each point of D

Definition : A complex valued function f(z) is said to be meromorphic in a domain D if it is analytic except at finite number of poles which are the singularities.

Definition : A function f (z) holomorphic in domain D is said to be p-valent in D if f (z) has p roots. The Taylor series for f(z) for p-valent functions is given by

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n$$

Definition : A domain containing the origin is said to be starlike with respect to the origin if it is intersected by any straight line through the origin in a linear segment. Starlike with respect to the origin we will refer to as simply starlike.

Definition : Let f(z) be holomorphic at z = 0, let α be a real number satisfying $o < \alpha < p$. Then f (z) is said to be p-valent starlike of order α denoted by $S^*p(\alpha)$

$$\text{if} \quad \operatorname{Re}\left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha, \quad o \le \alpha < p$$

Definition : A convex function is one which maps the unit disc conformally on to a convex domain.

Definition: Let K be the subclass of S, S being the family of univalent function whose members map every disc $|z| < \xi$, $0 < \xi < 1$ on to a convex domain.

Definition: Let α be a real number such that $o \le \alpha < p$. Then the analytic representation for p-valent convex function of order α is given by

$$\operatorname{Re}\left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > \alpha \quad o \le \alpha < p$$

Definition : A function f holomorphic in the unit disc is said to be close-to-convex if there is a convex function g such that

$$\operatorname{Re}\left\{\frac{f'(z)}{g'(z)}\right\} > o \text{ for all } z \text{ in } U$$

We denote by C the class of close - to - convex function, we note that every convex function is obviously close - to - convex. More generally every starlike function is close-to-convex. The relationship between the family univalent / multivalent functions can be presented in the following chain proper inclusions

$$K \subset S^* \subset C \subset S$$

S-denoting the family of univalent / p-valent function.

Definition : Let $\rho(n, R)$ denote the family of polynomials

$$p(z) = a \prod_{k=1}^{n} (z-z_k)$$

whose all the n zeros lie outside or on the circle with centre at origin and radius $R \geq 1$, Here 'a' will be always be a constant to be approximately selected.

We need the following several lemmas for our research,

Lemma (Basgöze) [1.]

If $z = re^{i\theta}$, $z_1 = Re^{i\phi}$ where $o < r < R$ Then

$$-\frac{r}{R-r} \leq \operatorname{Re}\frac{z}{z-z_1} \leq \frac{r}{R+r}$$

Equality holds in the first of inequality if and only if $z = (r/R)\, z1$ and the second inequality if and only if $z = (r/R)\, z1$. Consequently allowing $r \to 1$ we need the following corollary which we require in section (I)

Corollary : 2 If $z = re^{i\theta}$, $z_1 = Re^{i\phi}$ where $o \leq r < 1$ and $R > 1$ then

$$-\frac{1}{R-1} \leq \operatorname{Re}\frac{z}{z-z_1} \leq \frac{1}{R+1}$$

Lemma : 3

Let f be holomorphic / meromorphic function in

$|z| < r$ ($r < 1$) $f'(z) \neq 0$. If $f \in C(\delta)$

where $o \leq \delta < p$ then

$$-\delta\pi < \int_{\theta 1}^{\theta 2} \mathrm{Re}\left(1 + \frac{z f''}{f'}\right) d\theta < \delta\pi + 2\pi$$

where $o \leq \theta 1 < \theta 2 < 2\pi$

We have also a mention of the fact that each of the above inequality separately implies that $f \in C(\delta)$ $o \leq \delta < p$

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