P R E F A C E

The present dissertation entitled "On Relativistic Ferrofluid" is the subject matter of relativistic continuum mechanics. The relativistic rheology dealing with deformation and flow of relativistic matter moving with high speed has emerged very recently. Hence the survey of literature suggest very few names like Radhakrishna (1978), Walwadkar (1983) and Shah (1987) who are working in this field. The ferrofluid system consist of an infinitely conducting relativistic charged fluid possesing variable magnetic permeability. The deformation properties of this ferrofluid System are investigated in this dissertation. Some special types of transport equation are used to examine these properties. The salient features of the dissertation work are given below (chapterwise).

Chapter I: BASIC CONCEPTS AND PREREQUISITES :

Section : 1 : A brief introduction of the chapter is presented.

Section : 2 : The kinematical parameters associated with the time like flowvector and space like magnetic field vector are introduced.

Section : 3 : The evolution of the stress energy tensor for infinitly conducting perfect fluid with variable magnetic permeability is given. So also the Ricci Tensor expression for ferrofluid is obtained.

Section : 4 : The local conservation laws of energy and momentum tensor emerging out of twice contracted Bianchi identities are studied with special reference to the ferrofluid distribution.

Section : 5 : Different types of transportation equations for tensor fields along the time like congruences are described.

Chapter II : THE DEFORMATION TENSOR FIELD :

Section : 1 : It includes the brief sketch of the historical remarks of classical and relativistic rheology. A list of contributors working in the relativistic rheology is given.

Section : 2 : The deformation tensor, Shear tensor and rotation tensor are defined through the 3-space projection of the flow gradient tensor. The derivation of kinematical equation of strain tensor along the time like vector is performed by choosing the signature (-, -, -, +). The same is obtained already by Carter and Quintana (1977) for the metric of signature (+,+,+,-).

Section : 3 : This section is related to the dynamical form the strain variation equation in context of ferrofluid system. This reveals the effects of matter density, pressure, magnetic field and the magnetic permeability on the deformation properties of ferrofluid space-time.

Section : 4 : We have derived the strain variation equation along the space like congruences which included the parameters associated with time like congruence $\bar{u}$ and space like congruence $\bar{h}$. This equation provides the deformation rate along the space like congruence.

Section : 5 : The above strain variation equation along the space like congruence is reviewed with special reference to ferrofluid system. The effects of dynamical variables like density and pressure on the deformation rate are examined. This section deals with the kinematical form of strain variation equation under the restriction like inertial reference frame and uniform magnetic field.

Chapter III : TRANSPORTS EQUATIONS AND FERROFLUID SYSTEM :

Section : 1 : This provides the introduction of different types of transports that are used in the study of kinematical and dynamical rheology of ferrofluid distribution.

Section : 2 : Ferrofluid distribution is studied with reference to some special types of space-times namely the self similar space and Einsten space. The prominant results regarding the ferrofluid system are as follows

1) The self similar space time is incompatible for unit time like flow.
2) The ferrofluid space time cannot be equivalent to Einsten space.

Section : 3 : This section deals with the Jaumann Transport. If the Jaumann stress rate of ferrofluid system vanishes then the matter density, pressure, magnetic permeability and the magnitude of magnetic field are preserved with respect to the geodesic flow. This section also relates to the constitutive equations of hypoelasticity with the usage of Jaumann stress rate giving the rheology of relativistic hypoelastic materials. It deals with the four types of materials corresponding to the tensor λ_{abcd} with expressions

a) $\lambda_{abcd} = h_{ab} h_{cd}$,

b) $\lambda_{abcd} = h_{ab} T_{cd}$,

c) $\lambda_{abcd} = T_{ab} h_{cd}$,

d) $\lambda_{abcd} = h_{ac} h_{bd} + h_{ad} h_{bc}$.

Accordingly we have

I) The Jaumann stress rate of ferrofluid system with $\lambda_{abcd} = h_{ab} h_{cd}$ the following relations are equivalent

A) $\underset{u}{\underline{J}}\, T_{ab} - \lambda_{abcd}\, \theta^{cd} = 0,$

B) $\dot{\rho} = \dot{p} = \dot{u}_a = \dot{\mu} = H^2 = \theta = 0,$

$\dot{H}_a - H_c\, W^c{}_a = 0.$

II) For the case $\lambda_{abcd} = h_{ab} T_{cd}$ the following results hold

$$\frac{J}{u} T_{ab} = \lambda_{abcd} \; \theta^{cd} \Leftrightarrow \dot{\rho} = \dot{p} = \dot{\mu} = \dot{H}^2 = 0,$$

$$\theta = 0, \quad \sigma_{ab} = \theta_{ab}.$$

III) If for the ferrofluid distribution with the uniform magnitude of the magnetic field the response tensor is governed by $\lambda_{abcd} = T_{ab} h_{cd}$, then

$$\frac{J}{u} T_{ab} - \lambda_{abcd} \; \theta^{cd} = 0 \Leftrightarrow \dot{\rho} = \dot{p} = \dot{\mu} = \dot{u}_a = 0,$$

$$\theta = 0 \text{ and } \theta_{ab} = \sigma_{ab}.$$

IV) For the constitutive relation $\lambda_{abcd} = h_{ac} h_{bd} + h_{ad} h_{bc}$ the following result hold for the expansion free flow.

$$\frac{J}{u} T_{ab} = \lambda_{abcd} \; \theta^{cd} \Leftrightarrow \dot{\rho} = \dot{p} = \dot{\mu} = \dot{H}^2 = 0$$

$$\text{and} \quad \sigma_{ab} = \theta_{ab}$$

Section : 4 : The aim of this section is to study the rheological properties of ferrofluid under Truesdell transport. It is provied that if the magnetic field vector is Truesdell transported then

a) The magnetic permeability is invariant along expansion free flow.

b) The magnetic permeability is invariant along the divergence free magnetic lines.

The necessary and sufficient conditions for the ferrofluid to be of expansion free flow are derived.

Section : 5 : The features of Fermi-Walker and Convective transports of stress energy Tensor for ferrofluid system along the space like vector are examined.

The Tensors $\overset{*}{\sigma}_{ab}$ and $\overset{*}{W}_{ab}$ resides in a 2-space since

$$\overset{*}{\sigma}_{ab}u^{b} = \overset{*}{\sigma}_{ab}h^{b} = \overset{*}{W}_{ab}u^{b} = \overset{*}{W}_{ab}h^{b} = 0.$$

Further we denote

$$2\overset{*}{\sigma}^{2} = \overset{*}{\sigma}_{ab}\overset{*}{\sigma}^{ab},\ 2\overset{*}{W}^{2} = \overset{*}{W}_{ab}\overset{*}{W}^{ab}.$$

In order to obtain the decomposition of $h_{a;b}$ we write

$$h_{a;b} = \overset{*}{\sigma}_{ab} + \overset{*}{W}_{ab} + 1/2\,\overset{*}{\theta}\,p_{ab} + \dot{h}_{a}u_{b} -$$

$$- h'_{a}h_{b} - u_{a}h_{c}(\sigma^{c}{}_{d}h^{d}h_{b} + \sigma^{c}{}_{b} + W^{c}{}_{b}), \quad .....(2.15)$$

Wheere $h'_{a} = h_{a;b}h^{b}$.

3. THE STRESS-ENERGY TENSOR FOR FERROFLUID :

The only part of Einstien's field equations that provides a complete description of the dynamical features of the space-time is the stress-energy tensor.

The relativistic perfect charged fluid with infinite electric conductivity is decomposed into two parts : (a) with constant magnetic permeability (designates as magneto-fluid) (b) with variable magnetic permeability (is called as relativistic ferrofluid). The magnetofluid system is introduced by Lichnerowicz (1967) and developed by Taub (1970), Yodzis (1971), Date (1972), Maugin (1972), Greco (1974). The propogation equations for the kinematical parameters in the space-time filled with magnetofluid are computed by