CHAPTER III

DYNAMICAL INHERITANCE PROPERTY AND MAGNETO-DUST DISTRIBUTION

1. INTRODUCTION :

The critical study of the work due to Maarteen and Maharaj (1986) suggests that there do exist proper Conformal Killing vectors in FRW models and perfect fluid distribution. This suggested clue to Duggal (1992) to modify the concept of curvature collineation suitable for proper Conformal Killing vector and other related proper symmetries. Accordingly he introduced a new symmetry called as Curvature Inheritance defined as,

$$\underset{\xi}{\pounds} R^{a}_{bcd} = 2\,\alpha\, R^{a}_{bcd}\,.$$

This implies Ricci Inheritance property,

$$\underset{\xi}{\pounds} R_{ab} = 2\,\alpha\, R_{ab}\,.$$

The prime goal of this work is to explore the geometrical and dynamical properties of Dynamical Inheritance associated with Magneto-Dust Distribution described by energy momentum tensor I (3.4) through the definition given below :

2. DEFINITION : DYNAMICAL INHERITANCE :

The space-time is said to admit Dynamical Inheritance along vector field ξ if,

$$\underset{\xi}{L}\, T_{ab} = 2\,\alpha\, T_{ab}\,. \qquad 1$$

where α is scalar function of co-ordinates and T_{ab} is the stress energy tensor.

This stress energy tensor I (3.4) involves two eigen vectors one of which is the time-like eigen vector u^a with eigen value $e_1 = \rho + \frac{1}{2}\mu h^2$ and the other is the space-like eigen vector h^a with eigen value

$$e_2 = \frac{1}{2}\mu h^2$$

Hence the study of Dynamical Inheritance described by (1) is governed by the choice of the Inheritance vector ξ with two separate cases as

$$\xi^a = u^a \quad \text{and} \quad \xi^a = h^a$$

Equation (1) can be expressed as

$$(T_{ab\,;\,c})\, \xi^c + (T_{db})\, (\xi^d{}_{;\,a}) + (T_{ad})\, (\xi^d{}_{;\,b}) = 2\,\alpha\, T_{ab}. \qquad 2$$

We study this expression with the following two specific types of vector ξ

CASE I : If $\xi^a = u^a$

For this choice equation (2) can be written as

$$(T_{ab\,;\,c})\, u^c + (T_{db})\, (u^d{}_{;\,a}) + (T_{ad})\, (u^d{}_{;\,b}) = 2\,\alpha\, T_{ab}\,. \qquad 3$$

Theorem 1 : If the Magneto-Dust Distribution admits Dynamical Inheritance property along flow lines then the active gravitational mass density is inherited iff $(\mu h^2)^{\bullet} + \dot{\mu} h^2 + \mu h^2\, \theta = 0$.

Proof : If equation (3) is transvected with g^{ab}, we get

$$(g^{ab}\, T_{ab;\,c})\, u^c + (T_{db}\, g^{ab})\, (u^d{}_{;\,a}) + (g^{ab}\, T_{ad})\, (u^d{}_{;\,b}) =$$
$$= 2\,\alpha\, g^{ab}\, T_{ab} \qquad \because g^{ab}{}_{;\,c} = 0$$

i.e. $(T_{;\,c})\, u^c + T^a{}_d\, (u^d{}_{;\,a}) + T^b{}_d\, (u^d{}_{;\,b}) = 2\,\alpha\, T$,

i.e. $(T_{;\,c})\, u^c + 2\, T^a{}_d\, (u^d{}_{;\,a}) = 2\,\alpha\, T$,

i.e. $(T_{;\,c})\, u^c + 2\, T^{ad}\, (u_{d\,;\,a}) = 2\,\alpha\, T$. $\qquad$ 4

Putting $T = \rho$ (vide I - 4.7) and using expression of T^{ad} (vide I – 3.4) in equation (4), we have

$$(\rho_{\,;\,c})\, u^c + 2\, [\, (\rho + \mu h^2)\, u^a u^d - \tfrac{1}{2}\, \mu\, h^2\, g^{ad} - \mu\, h^a h^d \,]$$
$$[\, u_{d\,;\,a} \,] = 2\,\alpha\, \rho\,. \qquad 5$$

We have $\quad u^a{}_{;a} = \theta, \rho_{;c}u^c = \dot{\rho}, u^d{}_{;a}u_d = 0.$

Hence equation (5) can be written as

$$\dot{\rho} + 2\,[\,(\rho + \mu h^2)\,u^a u^d u_{d;a} - \tfrac{1}{2}\,\mu\, h^2 g^{ad} u_{d;a} - \mu\, h^a h^d u_{d;a}\,] = 2\,\alpha\,\rho,$$

i.e. $$\dot{\rho} - \mu h^2\,\theta - 2\,\mu\, u_{d;a}h^d h^a = 2\,\alpha\,\rho\,. \tag{6}$$

Maxwell equation I (8.7) directs

$$\mu\, u_{d;a}h^d h^a = -\dot{\mu}h^2 - \mu h^2\,\theta - (\mu/2)\,(h^2)^{\cdot} \tag{7}$$

Hence from (7), equation (6) can be written as

$$\dot{\rho} - \mu h^2\,\theta - 2\,[\,-\dot{\mu}h^2 - \mu h^2\,\theta - (\mu/2)\,(h^2)^{\cdot}\,] = 2\,\alpha\,\rho\,, \tag{8}$$

i.e. $$\dot{\rho} + \mu h^2\,\theta + 2\,\dot{\mu}h^2 + \mu\,(h^2)^{\cdot}\,] = 2\,\alpha\,\rho\,, \tag{9}$$

i.e. $$[\,\rho + \mu h^2\,]^{\cdot} + \dot{\mu}h^2 + \mu h^2\,\theta = 2\,\alpha\,\rho\,, \tag{10}$$

i.e. $$\underset{u}{L}\,(\rho + \mu h^2) = 2\,\alpha\,\rho - \dot{\mu}h^2 - \mu h^2\,\theta\,, \tag{11}$$

This implies that

$$\underset{u}{L}\,\rho = 2\,\alpha\,\rho \quad \text{iff} \quad (\mu h^2)^{\cdot} + \dot{\mu}h^2 + \mu h^2\,\theta = 0\,, \tag{12}$$

But $\quad \rho = T \quad$ vide I (4.7),

Hence from (12), we get

$$\underset{u}{L}\,T = 2\,\alpha\,T \quad \text{iff} \quad (\mu h^2)^{\cdot} + \dot{\mu}h^2 + \mu h^2\,\theta = 0\,.$$

Hence the theorem.

Theorem 2 : If the Magneto-Dust Distribution admits Dynamical Inheritance property along flow lines then time-like eigen value is inherited along flow lines.

Proof : On multiplying equation (3) by $u^a u^b$, we get

$$u^a u^b (T_{ab\,;\,c}) u^c + u^a u^b (u^d{}_{;a}) T_{ab} + u^a u^b (u^d{}_{;\,b}) T_{ab} = 2\,\alpha\, u^a u^b T_{ab} \qquad 13$$

The three terms on left hand side of equation (13) are simplified in following manner.

Consider, $u^a u^b (T_{ab\,;\,c}) u^c =$

$$= u^a u^b [(\rho + \mu h^2) u_a u_b - \tfrac{1}{2} \mu h^2 g_{ab} - \mu\, h_a h_b]_{;\,c} u^c$$

vide I (3.4)

$$= u^a u^b [(\rho + \mu h^2) u_a u_b]_{;\,c} u^c - (\tfrac{1}{2} \mu h^2)_{;\,c}\, u^a u^b g_{ab} u^c - (\mu\, h_a h_b)_{;\,c} u^c u^a u^b u^c , \qquad 14$$

Using $u_{a\,;\,c} u^a = 0$, $u^a h_a = 0$, $g^{ab}{}_{;\,c} = 0$ and $u^a u_a = 1$ in equation (14), we get

$u^a u^b (T_{ab\,;\,c}) u^c =$

$$= (\rho + \mu h^2)_{;\,c} u^c + (\rho + \mu h^2) u_a u_b u_{a\,;\,c} u_b u^c + + (\rho + \mu h^2) u^a u^b u_{b\,;\,c} u_a u^c - - \tfrac{1}{2} [g_{ab} (\mu_{;\,c} u^a u^b h^2 + h^2{}_{;\,c} u^a u^b \mu)] u^c - [\mu_{;\,c} u^a u^b h_a h_b + \mu\, h_{a\,;\,c} u^a u^b h_b + \mu\, h_{b\,;\,c} h_a u^a u^b)] u^c ,$$

$$= (\rho + \mu h^2)_{;\,c} u^c - \tfrac{1}{2} (\mu_{;\,c} h^2 + h^2{}_{;\,c} \mu) u^c$$

$$= (\rho + \mu h^2)_{;\,c} u^c - \tfrac{1}{2} (\mu h^2)_{;\,c} u^c .$$

Finally we get

$$u^a u^b (T_{ab\,;\,c}) u^c = (\rho + \tfrac{1}{2} \mu h^2)_{;\,c} u^c . \qquad 15$$

Now consider

$u^a u^b (u^d{}_{;\,a}) T_{ab} =$

$$= u^a u^b [(\rho + \mu h^2) u_d u_b - \tfrac{1}{2} \mu h^2 g_{ab} - \mu\, h_d h_b]\; u^d{}_{;\,a}$$

vide I (3.4)

$$= (\rho + \mu h^2) u^a u^b u_d u_b u^d{}_{;\,a} - \tfrac{1}{2} \mu h^2 g_{db} u^a u^b u^d{}_{;\,a} - - \mu\, h_d h_b u^a u^b u^d{}_{;\,a} , \qquad 16$$

We have $u^d{}_{;a}u_d = 0$, $u^a u_a = 1$, $u^b h_b = 0$,

Hence from equation (16), we get

$$u^a u^b (u^d{}_{;a}) T_{db} = 0 . \quad 17$$

Further consider

$$u^a u^b (u^d{}_{;b}) T_{ad} = u^a u^b [(\rho + \mu h^2) u_a u_d - \tfrac{1}{2} \mu h^2 g_{ad} - \mu h_a h_d] u^d{}_{;b}$$

vide I (3.4)

$$= (\rho + \mu h^2) u^a u^b u_a u_d u^d{}_{;b} - \tfrac{1}{2} \mu h^2 u^a u^b g_{ad} u^d{}_{;b} - - \mu h_a h_d u^a u^b u^d{}_{;b} ,$$

$$u^a u^b (u^d{}_{;a}) T_{ad} = 0 . \quad 18$$

$\because u^d{}_{;b}u_d = 0$, $u^a u_a = 1$, $u^a h_a = 0$

Thus from (15), (17) and (18) equation (13) reduces as

$$(\rho + \tfrac{1}{2} \mu h^2)_{;c} u^c = 2 \alpha u^a u^b T_{ab} , \quad 19$$

This implies that

$$(\rho + \tfrac{1}{2} \mu h^2)_{;c} u^c = 2 \alpha (\rho + \tfrac{1}{2} \mu h^2) \qquad \text{vide I (4.2)}$$

i.e. $$\underset{u}{L} (\rho + \tfrac{1}{2} \mu h^2) = 2 \alpha (\rho + \tfrac{1}{2} \mu h^2) , \quad 20$$

i.e. $$\underset{u}{L} e_1 = 2 \alpha e_1 , \qquad \text{vide I (4.2)} \quad 21$$

Here the proof of theorem is completed.

Note 1 : Equation (20) can be rearranged as

$$\dot{\rho} + \tfrac{1}{2} \dot{\mu} h^2 + \tfrac{1}{2} \mu (h^2)^{\bullet} = 2 \alpha (\rho + \tfrac{1}{2} \mu h^2) . \quad 22$$

Making use of continuity equation I (9.10) in (22), we get

$$(\tfrac{1}{2}\, \dot{\mu} h^2 - \rho\, \theta) + \tfrac{1}{2}\, \dot{\mu} h^2 + \tfrac{1}{2}\, \mu\, (h^2)^{\bullet} =$$

$$= 2\, \alpha\, (\rho + \tfrac{1}{2}\, \mu h^2) .$$

i.e. $$\dot{\mu} h^2 - \rho\, \theta + \tfrac{1}{2}\, \mu\, (h^2)^{\bullet} = 2\, \alpha\, (\rho + \tfrac{1}{2}\, \mu h^2) . \qquad 23$$

This equation (23) exhibits the effect of dynamical entities on Inheritance scalar function α .

Theorem 3 : If the Magneto-Dust Distribution admits Dynamical Inheritance property along flow lines then

$$\underset{u}{L}\, \mu h^2 = \alpha\, \mu h^2 \quad \text{iff} \quad \mu h^2 + 2\, \mu h^2\, \theta = 0 .$$

Proof : On transvecting equation (3) with $h^a h^b$, we get

$$h^a h^b\, (T_{ab\,;\,c})\, u^c + h^a h^b\, T_{db}\, (u^d{}_{;a}) + h^a h^b\, T_{ad}\, (u^d{}_{;\,b})$$

$$= 2\, \alpha\, h^a h^b\, T_{ab} \qquad 24$$

Each term on left hand side of equation (24) is simplified in following manner.

Consider $h^a h^b\, (T_{ab\,;\,c})\, u^c =$

$$= h^a h^b\, [\, (\rho + \mu h^2)\, u_a u_b - \tfrac{1}{2}\, \mu h^2\, g_{ab} - \mu\, h_a h_b\,]\, _{;\,c} u^c$$

vide I (3.4)

$$= [\, (\rho + \mu h^2)_{;\,c} u^c\, u_a u_b h^a h^b + (\rho + \mu h^2)\, u_{a\,;\,c} u_b h^a h^b$$
$$+ (\rho + \mu h^2)\, u_{b;\,c} u_a h^a h^b\,]\, u^c - \tfrac{1}{2}\, [\, \mu h^2{}_{;\,c} h^a h^b + \mu_{;\,c}\, h^2\, h^a h^b]$$
$$g_{ab}\, u^c - [\, \mu_{;\,c} h_a h_b h^a h^b + \mu\, h_{a\,;\,c} h_b h^a h^b + \mu\, h_{b\,;\,c} h_a h^a h^b\,]\, u^c . \qquad 25$$

We have $u_{a\,;\,c} u^a = 0$, $h_{a\,;\,c} u^c = h_a$, $h_a h^a = -\tfrac{1}{2}\, (h^2)$, $u^a h_a = 0$, $h^a h_a = -h^2$, $\rho_{\,;\,c} u^c = \dot{\rho}$ Hence from equation (25)

$$h^a h^b\, (T_{ab\,;\,c})\, u^c =$$

$$= -\ \tfrac{1}{2}\, [\, \mu_{\,;\,c} u^c\, h^2 h^a h_a + \mu\, h^2{}_{;\,c} u^c h^a h_a\,]$$

$$- [\, \mu_{;c} u^c h^4 + \mu\, h_{a;c} h_{a;c} h^a (-h^2)\, u^c + \mu\, h_{b;c} u^c (-h^2)\, h^b \,], \qquad 26$$

i.e. $h^a h^b (T_{ab;c}) u^c =$

$$= -\tfrac{1}{2} [\, \dot{\mu}\, h^2 (-h^2) + \mu\, (h^2)^{\cdot} (-h^2) \,] - [\, \dot{\mu}\, h^4 + \mu\, (\dot{h}_a)\, h^a (-h^2) + \mu\, (\dot{h}_b)\, h^b (-h^2) \,] \qquad 27$$

$$= \tfrac{1}{2}\, \dot{\mu}\, h^4 + \tfrac{1}{2}\, \mu h^2 (h^2) - \mu\, h^4 - \tfrac{1}{2}\, \mu h^2 (h^2)^{\cdot} - \tfrac{1}{2}\, \mu h^2 (h^2)^{\cdot}$$

$$= -\tfrac{1}{2}\, \dot{\mu}\, h^4 - \tfrac{1}{2}\, \mu h^2 (h^2)^{\cdot}$$

$$= -\tfrac{1}{2} h^2 [\, \dot{\mu} h^2 + \mu\, (h^2)^{\cdot} \,]$$

$$h^a h^b (T_{ab;c}) u^c = -\tfrac{1}{2} h^2 [\, \mu h^2 \,]^{\cdot}. \qquad 28$$

Now consider

$$h^a h^b T_{db} (u^d{}_{;a}) =$$

$$= h^a h^b [\, (\rho + \mu h^2)\, u_d u_b - \tfrac{1}{2}\, \mu h^2 g_{db} - \mu\, h_d h_b \,]\, u^d{}_{;a}$$

vide I (3.4)

$$= (\rho + \mu h^2)\, u_d u_b h^a h^b - \tfrac{1}{2}\, \mu h^2 g_{db} h^a h^b - \mu\, h^a h^b h_d h_b \,]\, u^d{}_{;a},$$

$$= (\rho + \mu h^2)\, u^d{}_{;a} u_d u_b h^a h^b - \tfrac{1}{2}\, \mu h^2 u^d{}_{;a} g_{db} h^a h^b - \mu\, u^d{}_{;a} h^a h^b h_d h_b. \qquad 29$$

We have $h_{a;c} u^c = \dot{h}_a$, $\dot{h}_a h^a = -\tfrac{1}{2} (h^2)$, $u^a h_a = 0$, $h^a h_a = -h^2$, $\rho_{;c} u^c = \dot{\rho}$

Hence from equation (29), we get

$$h^a h^b T_{db} (u^d{}_{;a}) = -\tfrac{1}{2}\, \mu h^2 u^d{}_{;a} h^a h_d + \mu h^2 u^d{}_{;a} h^a h_d$$

$$= \tfrac{1}{2}\, \mu h^2 [\, u_{a;d} h^a h^d \,]$$

$$= \tfrac{1}{2} h^2 [\, -\dot{\mu} h^2 - \mu h^2 \theta - (\mu/2)\, (h^2)^{\cdot} \,] \qquad 30$$

vide I (8.7)

$$h^a h^b (T_{db}) u^d{}_{;a} = -h^2/4 [2 \dot{\mu} h^2 + 2 \mu h^2 \theta + \mu (h^2)] \quad 31$$

Further consider,

$$h^a h^b T_{ad} (u^d{}_{;b}) = h^a h^b [(\rho + \mu h^2) u_a u_d - ½ \mu h^2 g_{ad} - \mu h_a h_d] u^d{}_{;b} \quad 32$$

$$= [(\rho + \mu h^2) u^d{}_{;b} h^a h^b u_a u_d - ½ \mu h^2 g_{ad} h^a h^b u^d{}_{;b} - \mu u^d{}_{;b} h^a h^b h_a h_d] . \quad 33$$

If we put $u^a h_a = 0$, $h^a h_a = -h^2$, $\dot{h}_a h^a = -½ (h^2)^{\dot{}}$, $u^d{}_{;b} u_d = 0$ in equation (33), we get

$$h^a h^b T_{ad} (u^d{}_{;b}) = -½ \mu h^2 u^d{}_{;b} h_d h^b + h^2 u^d{}_{;b} h^b h_d ,$$

$$h^a h^b T_{ad} (u^d{}_{;b}) = ½ \mu h^2 u_{d;b} h^d h^b ,$$

$$= ½ h^2 [-\mu h^2 - \mu h^2 \theta - (\mu/2)(h^2)^{\dot{}}],$$

vide I (8.7)

$$= -h^2/4 [2 \dot{\mu} h^2 + 2 \mu h^2 \theta + \mu (h^2)^{\dot{}}] . \quad 34$$

Thus from (28), (31) and (34), equation (24) reduces to

$$-½ h^2 (\mu h^2)^{\dot{}} - ½ h^2 [2 \dot{\mu} h^2 + 2 \mu h^2 \theta + \mu (h^2)^{\dot{}}] = 2 \alpha h^a h^b T_{ab} , \quad 35$$

i.e. $$-½ h^2 (\mu h^2)^{\dot{}} - ½ h^2 [2 \dot{\mu} h^2 + 2 h^2 \theta + \mu (h^2)^{\dot{}}] = -2 \alpha h^4, \text{ vide I (4.4)}$$

$$2 (\mu h^2)^{\dot{}} + \dot{\mu} h^2 + 2 \mu h^2 \theta = 2 \alpha \mu h^2 \quad 36$$

This implies that

$$2 \underset{u}{L} (\mu h^2) = 2 \alpha \mu h^2 - \dot{\mu} h^2 - 2 \mu h^2 \theta . \quad 37$$

Hence the proof of theorem is completed.

Theorem 4 : If Magneto-Dust Distribution admits Dynamical inheritance property along flow lines then acceleration is orthogonal to magnetic lines.

Proof : On contracting equation (2) with $u^a h^b$, we get

$$u^a h^b (T_{ab\,;\,c}) u^c + u^a h^b T_{db} (u^d{}_{;\,a}) + T_{ad} u^a h^b (u^d{}_{;\,b}) = 2\,\alpha\, u^a h^b T_{ab}, \quad 38$$

i.e. $$u^a h^b (T_{ab\,;\,c}) u^c + T_{db} u^a h^b (u^d{}_{;\,a}) + T_{ad} u^a h^b (u^d{}_{;\,b}) = 0 . \quad \text{vide I (4.5)} \quad 39$$

The three terms on left hand side of (39) are simplified in following manner :

Consider,

$$\begin{aligned} u^a h^b (T_{ab\,;\,c}) u^c &= u^a h^b [(\rho + \mu h^2) u_a u_b \\ &\quad - ½\, \mu h^2 g_{ab} - \mu h_a h_b]_{;\,c} u^c, \\ &= [(\rho + \mu h^2)_{;\,c} u^a h^b u_a u_b + (\rho + \mu h^2) u_{a\,;\,c} \\ &\quad u_b u^a h^b + (\rho + \mu h^2) u_{b\,;\,c} u_a u^a h^b] u^c \\ &\quad - ½ [g_{ab}\, \mu_{;\,c} h^2 u^a h^b + g_{ab}\, \mu h^2{}_{;\,c} u^a h^b] u^c \\ &\quad - [\mu_{;\,c} u_a h^b h_a h_b + \mu h_{a\,;c} h_b u^a h^b + \\ &\quad + \mu\, h_{b\,;\,c} h_a u^a h^b] u^c \end{aligned} \quad 40$$

Using $u_a h^a = 0$, $\dot{u}_a = u_{a\,;\,c} u^c$, $\dot{u}_a u^a = 0$ in (40), we get

$$\begin{aligned} u^a h^b (T_{ab\,;\,c}) u^c &= (\rho + \mu h^2) \dot{u}_b h^b + \mu h^2 \dot{h}_a u^a , \\ &= (\rho + \mu h^2) \dot{u}_b h^b - \mu\, h^2\, \dot{u}^a h_a , \\ &\qquad \because\ \dot{u}_a h^a = - \dot{h}_a u^a \\ &= (\rho)\, \dot{u}_b h^b. \end{aligned} \quad 41$$

Now consider,

$$\begin{aligned} u^a h^b T_{db} u^d{}_{;\,a} &= u^a h^b [(\rho + \mu h^2) u_d u_b \\ &\quad - ½\, \mu h^2 g_{db} - \mu h_d h_b] u^d{}_{;\,a}, \end{aligned}$$

$$= (\rho + \mu h^2)\, u^d{}_{;a}\, u^a h^b u_d u_b$$
$$- \tfrac{1}{2}\mu h^2\, u^d{}_{;a}\, u^a h^b g_{db} - \mu\, u^d{}_{;a} u^a h^b h_d h_b \qquad 42$$

Using $u^a u_a = 1$, $u^a h_a = 0$, $u^d{}_{;a} u^a = \dot{u}^d$, $u^d{}_{;a} u_d = 0$ in (42), we get

$$u^a h^b\, T_{db}\, u^d{}_{;a} = -\tfrac{1}{2}\mu h^2\, u^d{}_{;a}\, u^a h_d - \mu\, \dot{u}^d (-h^2)\, h_b,$$
$$= -\tfrac{1}{2}\mu h^2\, \dot{u}^d h_d + \mu h^2\, \dot{u}^d h_b,$$
$$= +\tfrac{1}{2}\mu h^2\, \dot{u}^b h_b. \qquad 43$$

Further consider,

$$u^a h^b\, T_{ad}\, u^d{}_{;b} = u^a h^b \left[(\rho + \mu h^2)\, u_a u_d - \tfrac{1}{2}\mu h^2 g_{ad} - \mu h_a h_d \right] u^d{}_{;b},$$
$$= (\rho + \mu h^2)\, u^d{}_{;b} u^a h^b u_a u_d$$
$$- \tfrac{1}{2}\mu h^2\, u^d{}_{;b} u^a h^b g_{ad} - \mu\, u^d{}_{;b} u^a h^b h_a h_d, \qquad 44$$

We have $u^a u_a = 1$, $u^a h_a = 0$, $u^a{}_{;b} u_a = 0$. Hence from (44), we get

$$u^a h^b\, T_{ad}\, u^d{}_{;b} = 0. \qquad 45$$

Hence from (41), (43), and (45), equation (39) yields as,

$$\rho\, \dot{u}_b h^b + \tfrac{1}{2}\mu h^2\, \dot{u}^b h_b = 0.$$

i.e. $\rho\, \dot{u}_b h^b + \tfrac{1}{2}\mu h^2\, \dot{u}_b h^b = 0.$

i.e. $(\rho + \tfrac{1}{2}\mu h^2)\, \dot{u}_b h^b = 0.$

This implies that

$$\dot{u}_b h^b = 0. \qquad \because \rho + \tfrac{1}{2}\mu h^2 \neq 0 \qquad 46$$

This completes the proof.

CASE II : If $\xi^a = h^a$.

For this choice equation (2) can be written as

$$(T_{ab\,;\,c})\, h^c + (T_{db})(h^d{}_{;\,a}) + (T_{ad})(h^d{}_{;\,b}) \;=\; 2\,\alpha\, T_{ab}\,. \qquad 47$$

Theorem 5 : If the Magneto-Dust Distribution admits Dynamical Inheritance property along magnetic lines then the time eigen value is inherited along magnetic lines iff $\dot{h}^d u_d = 0$.

Proof : Multiplying equation (47) by $u^a u^b$, we get

$$u^a u^b (T_{ab\,;\,c})\, h^c + u^a u^b (T_{db})(h^d{}_{;\,a}) + (T_{ad})(h^d{}_{;\,b})$$
$$= 2\,\alpha\, u^a u^b\, T_{ab}\,. \qquad 48$$

The three terms on left hand side of (48) are simplified as follows :

Consider

$$\begin{aligned} u^a u^b (T_{ab\,;\,c})\, h^c &= u^a u^b \,[\, (\rho + \mu h^2)\, u_a u_b \\ &\quad - \tfrac{1}{2}\, \mu h^2 g_{ab} - \mu h_a h_b \,]_{\,;\,c}\, h^c , \\ &= [\, (\rho + \mu h^2)_{\,;\,c}\, u^a u^b u_a u_b + (\rho + \mu h^2)\, u_{a\,;\,c} \\ &\quad u_b u^a u^b + (\rho + \mu h^2)\, u_{b\,;\,c}\, u_a u^a u^b]\, h^c \\ &\quad - \tfrac{1}{2}\, [\, \mu_{\,;\,c}\, h^2 g_{ab}\, u^a u^b + h^2{}_{;\,c}\, \mu\, g_{ab}\, u^a h^b \,]\, h^c \\ &\quad - [\, \mu_{\,;\,c}\, h_a h_b u^a u^b + \mu h_{a\,;c} h_b u^a u^b + \\ &\quad + \mu\, h_{b\,;\,c} u^a u^b \,]\, h^c . \end{aligned} \qquad 49$$

If we replace $u^a u_a = 1$, $u^a h_a = 0$, $\dot{u}_a u^a = 0$ in (49), we get

$$\begin{aligned} u^a u^b (T_{ab\,;\,c})\, h^c &= (\rho + \mu h^2)_{\,;\,c} h^c \\ &\quad - \tfrac{1}{2}\, (\mu_{\,;\,c}\, h^2 h^c + h^2{}_{;\,c}\, \mu\, h^c) \\ &= [\, (\rho + \mu h^2)_{\,;\,c} - \tfrac{1}{2}\, (\mu h^2)_{\,;\,c} \,]\, h^c \\ &= [\, \rho + \tfrac{1}{2}\, \mu h^2)_{\,;\,c} h^c\,. \end{aligned} \qquad 50$$

Now consider

$$\begin{aligned} u^a u^b (h^d{}_{;\,a})\, T_{db} &= u^a u^b\, h^d{}_{;\,a} \,[\, (\rho + \mu h^2)\, u_d u_b \\ &\quad - \tfrac{1}{2}\, \mu h^2 g_{db} - \mu h_d h_b \,] , \end{aligned}$$

$$= (\rho + \mu h^2)\, h^d{}_{;a}\, u^a u^b u_d u_b$$
$$- \tfrac{1}{2}\mu h^2\, h^d{}_{;a}\, u^a u^b g_{db} - \mu\, h^d{}_{;a} u^a u^b u_d h_b \quad 51$$

Using $u^a u_a = 1$, $u^a h_a = 0$, $h^d{}_{;a} u^a = \dot{h}^d$, in (51), we get

$$u^a u^b (h^d{}_{;a})\, T_{db} = (\rho + \mu h^2)\, \dot{h}^d u_d - \tfrac{1}{2}\mu h^2\, \dot{h}^d u_d,$$
$$= (\rho + \tfrac{1}{2}\mu h^2)\, \dot{h}^d u_d \quad 52$$

Further consider,

$$u^a u^b (h^d{}_{;b})\, T_{ad} = u^a u^b (h^d{}_{;b})\, [\, (\rho + \mu h^2)\, u_a u_d - \tfrac{1}{2}\mu h^2 g_{ad} - \mu h_a h_d\,]$$
$$= (\rho + \mu h^2)\, h^d{}_{;b} u^a u^b u_a u_d$$
$$- \tfrac{1}{2}\mu h^2\, h^d{}_{;b} u^a u^b g_{ad} - \mu\, h^d{}_{;b} u^a u^b h_a h_d, \quad 53$$

We have $u^a u_a = 1$, $u^a h_a = 0$, $h^d{}_{;a} u^a = \dot{h}^d$ Hence from (53), we get

$$u^a u^b (h^d{}_{;b})\, T_{ad} = (\rho + \mu h^2)\, \dot{h}^d u_d - \tfrac{1}{2}\mu h^2\, \dot{h}^d u_d.$$
$$= (\rho + \tfrac{1}{2}\mu h^2)\, \dot{h}^d u_d\,. \quad 54$$

Hence from (50), (52), and (54), equation (48) reduces to,

$$(\rho + \tfrac{1}{2}\mu h^2)_{;c} h^c + (\rho + \tfrac{1}{2}\mu h^2)\, \dot{h}^d u_d$$
$$+ (\rho + \tfrac{1}{2}\mu h^2)\, \dot{h}^d u_d = 2\alpha\, u^a u^b T_{ab}\,.$$

i.e. $\frac{L}{h}(\rho + \tfrac{1}{2}\mu h^2) + 2(\rho + \tfrac{1}{2}\mu h^2)\, \dot{h}^d u_d = 2\alpha(\rho + \tfrac{1}{2}\mu h^2).$

vide I (4.2)

i.e. $\frac{L}{h} e_1 + 2(\rho + \tfrac{1}{2}\mu h^2)\, \dot{h}^d u_d = 2\alpha\, e_1\,.$ 55

Hence the theorem.

Theorem 6 : If the Magneto-Dust Distribution admits Dynamical Inheritance property along magnetic lines then the necessary and sufficient condition for the inheritance of active gravitational mass along magnetic lines is given by $\mu\,(h^2{}_{;c} h^c) - \mu h^2\, h_{;}{}^{a}{}_{a} + 2\,(\rho + \mu h^2)\, \dot{h}^d u_d = 0$

Proof : On contracting equation (47) with g^{ab}, we get

$$(g^{ab}\, T_{ab;c})\, h^c + g^{ab}\, T_{db}\, (h^d{}_{;a}) + g^{ab}\, T_{ad}\, (h^d{}_{;b}) = 2\,\alpha\, g^{ab}\, T_{ab} \,, \qquad 56$$

i.e. $$(\rho_{;c})\, h^c + T^a{}_d\, (h^d{}_{;a}) + T^b{}_d\, (h^d{}_{;b}) = 2\,\alpha\,\rho \,, \qquad 57$$

vide I (4.7)

i.e. $$(\rho_{;c})\, h^c + 2\, T^{ad}\, h_{d;a} = 2\,\alpha\,\rho \,. \qquad 58$$

Therefore, by using I (3.4), equation (58) can be written as

$$(\rho_{;c})\, h^c + 2\, [\, (\rho + \mu h^2)\, u^a u^d - \tfrac{1}{2}\, \mu h^2\, g^{ad} - \mu\, h^a h^d\,]\, h_{d;a} = 2\,\alpha\,\rho \,. \qquad 59$$

We have $h_{d;a} u^a = \dot{h}^d$, $h^d{}_{;a} h^d = -\tfrac{1}{2}\, h^2{}_{;a}$. Hence from (59), we get

$$(\rho_{;c})\, h^c + 2\, [\, (\rho + \mu h^2)\, h_{d;a}\, u^a u^d - \tfrac{1}{2}\, \mu h^2\, h^d{}_{;a}\, g^{ad} - \mu\, h_{d;a}\, h^a h^d\,] = 2\,\alpha\,\rho \qquad 60$$

i.e. $$(\rho_{;c})\, h^c + 2\, [\, (\rho + \mu h^2)\, \dot{h}_d u^d - \tfrac{1}{2}\, \mu h^2\, h^d{}_{;a} - \mu\, h_{d;a}\, h^a h^d\,] = 2\,\alpha\,\rho$$

i.e. $$(\rho_{;c})\, h^c + 2\, [\, (\rho + \mu h^2)\, \dot{h}_d u^d - \tfrac{1}{2}\, \mu h^2\, h^a{}_{;a} - \mu\, h_{d;a}\, h^a h^d\,] = 2\,\alpha\,\rho$$

i.e. $$\underset{h}{L}\,\rho - 2\,\alpha\,\rho = \mu h^2\, h^a{}_{;a} + 2\,\mu\, (- \tfrac{1}{2}\, h^2{}_{;a})\, h^a - 2\, (\rho + \mu h^2)\, \dot{h}_d u^d \,,$$

i.e. $$\underset{h}{L}\,\rho - 2\,\alpha\,\rho = \mu h^2\, h^a{}_{;a} - \mu\, h^2{}_{;a}\, h^a - 2\, (\rho + \mu h^2)\, \dot{h}_d u^d \,, \qquad 61$$

i.e. $$\underset{h}{L}\,\rho - 2\,\alpha\,\rho = \mu h^2\, h^a{}_{;a} - \mu\, \underset{h}{L}\, h^2 - 2\, (\rho + \mu h^2)\, \dot{h}_d u^d \,,$$

i.e. $\underset{h}{L} T - 2\alpha T = \mu h^2 h^a{}_{;a} - \mu \underset{h}{L} h^2 - 2(\rho + \mu h^2) \dot{h}_d u^d$,

Hence

$$\underset{h}{\pounds} T = 2\alpha T \Leftrightarrow \mu h^2 h^a{}_{;a} - \mu \underset{h}{\pounds} h^2 - 2(\rho + \mu h^2) \dot{h}_d u^d = 0 .$$

Hence the proof is complete.

Theorem 7 : If the Magneto-Dust Distribution admits Dynamical Inheritance property along magnetic lines then the following results are equivalent.

I) $\underset{h}{L} \mu = 2\alpha\mu$ II) $\underset{h}{L} h^2 = 0$.

Proof : If transvect equation (47) by $h^a h^b$, we get

$$h^a h^b (T_{ab;c}) h^c + h^a h^b T_{db} (h^d{}_{;a}) + T_{ad} h^a h^b (h^d{}_{;b}) = 2\alpha h^a h^b T_{ab} . \qquad 63$$

On simplifying the terms on left hand side of (63), we have

Consider

$$h^a h^b (T_{ab;c}) h^c = h^a h^b [(\rho + \mu h^2) u_a u_b - \tfrac{1}{2} \mu h^2 g_{ab} - \mu h_a h_b]_{;c} h^c ,$$

vide I (3.4)

$$= [(\rho + \mu h^2)_{;c} u_a u_b h^a h^b + (\rho + \mu h^2) u_{a;c} u_b h^a h^b + (\rho + \mu h^2) u_{b;c} u_a h^a h^b] h^c - \tfrac{1}{2}[\mu_{;c} h^2 g_{ab} h^a h^b + \mu h^2{}_{;c} g_{ab} h^a h^b] h^c - [\mu_{;c} h_a h_b h^a h^b + \mu h_{a;c} h_b h^a h^b + + \mu h_{b;c} h_a h^a h^b] h^c . \qquad 64$$

If we substitute $u^a h_a = 0$, $h^a h_a = -h^2$, $u^a u_a = 1$, (64) reduces to

$$h^a h^b (T_{ab\,;\,c}) h^c = -\tfrac{1}{2} [\mu_{\,;\,c} h^2 (-h^2) + \mu h^2{}_{;\,c} (-h^2)] h^c - [\mu_{\,;\,c} h^4 - \mu h^2 h_{a\,;\,c} h^a - \mu h^2 h_{b\,;\,c} h^b] h^c, \quad 65$$

$$\begin{aligned} h^a h^b (T_{ab\,;\,c}) h^c &= -\tfrac{1}{2} [\mu_{\,;\,c} h^4 - \mu h^2 h^2{}_{;\,c}] h^c \\ &\quad (- \mu_{\,;\,c} h^4 + \mu h^2 h_{a\,;\,c} h^a + \\ &\quad + \mu h^2 h_{b\,;\,c} h^b) h^c \quad , \qquad 66 \\ &= (\tfrac{1}{2} \mu_{\,;\,c} h^4 + \tfrac{1}{2} \mu h^2 h^2{}_{;\,c}) h^c + \\ &\quad + (- \mu_{\,;\,c} h^4 + 2 \mu h^2 h_{a\,;\,c} h^a] h^c, \\ &= \tfrac{1}{2} h^2 [\mu_{\,;\,c} h^2 + \mu h^2{}_{;\,c}) h^c - \\ &\quad - h^2 \mu_{\,;\,c} h^2 h^c - 2 \mu h^2 h_{a\,;\,c} h^a h^c , \\ &= \tfrac{1}{2} h^2 [(\mu h^2)_{;\,c} h^c] - [\mu_{\,;\,c} h^c] h^4 \\ &\quad - 2 \mu h^2 [h_{a\,;\,c} h^a] h^c, \\ &= \tfrac{1}{2} h^2 [\underset{h}{L} \mu h^2] - h^2 [\underset{h}{L} \mu] h^2 \\ &\quad - 2 \mu h^2 [h_{a\,;\,c} h^a h^c], \\ &= \tfrac{1}{2} h^2 [\underset{h}{L} \mu h^2] - h^2 [\underset{h}{L} \mu] h^2 \\ &\quad - 2 \mu h^2 [- \tfrac{1}{2} h^2{}_{;\,c}] h^c , \\ &= (h^2/2) \{ \underset{h}{L} \mu h^2 - 2 h^2 \underset{h}{L} \mu + 2 \mu \underset{h}{L} h^2 \}, \\ &= (h^2/2) \{ \mu \underset{h}{L} h^2 + h^2 \underset{h}{L} \mu - 2 h^2 \underset{h}{L} \mu , \\ &\quad + 2 \mu \underset{h}{L} h^2 \} \\ &= (h^2/2) [3 \mu \underset{h}{L} h^2 - h^2 \underset{h}{L} \mu] . \qquad 67 \end{aligned}$$

Now consider

$$h^a h^b (h^d{}_{;a}) T_{db} = h^a h^b [(\rho + \mu h^2) u_d u_b - \tfrac{1}{2} \mu h^2 g_{db} - \mu h_d h_b] h^d{}_{;a} , \qquad 68$$

$$= (\rho + \mu h^2) h^d{}_{;a} h^a h^b u_d u_b - \tfrac{1}{2} \mu h^2 g_{db} h^a h^b h^d{}_{;a} - \mu h^d{}_{;a} h^a h^b h_d h_b . \qquad 69$$

Using $h^d{}_{;a} h_d = - \tfrac{1}{2} h^2{}_{;a}$, $h^a h_a = - h^2$, $u^a h_a = 0$, $u^a u_a = 1$ in (69), we get

$$h^a h^b (h^d{}_{;a}) T_{db} = - \tfrac{1}{2} \mu h^2 h^a h_d h^d{}_{;a} - \mu (- h^2) h^d{}_{;a} h_d h^a ,$$

$$= - \tfrac{1}{2} \mu h^2 (- \tfrac{1}{2} h^2{}_{;a}) h^a + \mu h^2 (- \tfrac{1}{2} h^2{}_{;a}) h^a$$

$$= 1/4 \, \mu h^2 h^2{}_{;a} h^a - \tfrac{1}{2} \mu h^2 h^2{}_{;a} h^a$$

$$= 1/4 \, \mu h^2 [h^2{}_{;a} h^a - 2 h^2{}_{;a} h^a]$$

$$= - 1/4 \, \mu h^2 h^2{}_{;a} h^a$$

$$= - 1/4 \, \mu h^2 \underset{h}{L} (h^2) . \qquad 70$$

Further consider,

$$h^a h^b (T_{ad}) (h^d{}_{;b}) = h^a h^b [(\rho + \mu h^2) u_a u_d - \tfrac{1}{2} \mu h^2 g_{ad} - \mu h_a h_d] h^d{}_{;b} \qquad 71$$

$$= (\rho + \mu h^2) h^d{}_{;b} h^a h^b u_a u_d - \tfrac{1}{2} \mu h^2 h^d{}_{;b} h^a h^b g_{ad} - \mu h^d{}_{;b} h^a h^b h_a h_d , \qquad 72$$

We have $u^a h_a = 0$, $h^a h_a = - h^2$, $h^d{}_{;a} h_d = - \tfrac{1}{2} h^2{}_{;a}$.

Hence from (72), we get

$$h^a h^b (T_{ad}) (h^d{}_{;b}) = - \tfrac{1}{2} \mu h^2 h^d{}_{;b} h^b h_d + \mu h^2 h^d{}_{;b} h^b h_d$$

$$= - \tfrac{1}{2} \mu h^2 (- \tfrac{1}{2} h^2{}_{;b}) h^b + \mu h^2 (- \tfrac{1}{2} h^2{}_{;b}) h^b$$

$$= 1/4 \, \mu h^2 \underset{h}{L} h^2 - \tfrac{1}{2} \mu h^2 \underset{h}{L} h^2$$

$$= - 1/4 \, \mu h^2 \underset{h}{L} (h^2) \qquad 73$$

Substituting (67), (70), and (73), in equation (63), we get

$$(h^2/2)\,[\,3\,\mu \underset{h}{L} h^2 - h^2 \underset{h}{L} \mu\,] - \tfrac{1}{2}\,\mu h^2 \underset{h}{L} h^2 = $$
$$= 2\,\alpha\, h^a h^b\, T_{ab} \qquad 74$$

$$(h^2/2)\,[\,3\,\mu \underset{h}{L} h^2 - h^2 \underset{h}{L} \mu\,] - \tfrac{1}{2}\,\mu h^2 \underset{h}{L} h^2 = $$
$$= 2\,\alpha\,(-\tfrac{1}{2}\,\mu h^4).$$

$$\tfrac{1}{2}\,[\,3\,\mu \underset{h}{L} h^2 - h^2 \underset{h}{L} \mu\,] - \tfrac{1}{2}\,\mu \underset{h}{L} h^2 = $$
$$= -\,\alpha\,\mu h^2.$$

i.e. $$3\,\mu \underset{h}{L} h^2 - h^2 \underset{h}{L} \mu = -2\,\alpha\,\mu h^2, \qquad 75$$

i.e. $$2\,\mu \underset{h}{L} h^2 - h^2 \underset{h}{L} \mu = -2\,\alpha\,\mu h^2,$$

i.e. $$h^2 \underset{h}{L} \mu - 2\,\mu \underset{h}{L} h^2 = 2\,\alpha\,\mu h^2, \qquad 76$$

i.e. $$\underset{h}{L} \mu - (2/h^2)\,\mu \underset{h}{L} h^2 = 2\,\alpha\,\mu. \qquad 77$$

This implies that $\underset{h}{L}(\mu) = 2\,\alpha\,\mu$ iff $\underset{h}{L} h^2 = 0$.

Hence the proof.

Theorem 8 : If the Magneto-Dust Distribution admits Dynamical Inheritance property along magnetic lines then the magnetic permeability is invariant along flow lines iff the flow lines are expansion free.

Proof : On transvecting equation (47) by $u^a h^b$, we get

$$u^a h^b\,(T_{ab\,;\,c})\,h^c + u^a h^b\,T_{db}\,(h^d{}_{;\,a}) + T_{ad}\,u^a h^b\,(h^d{}_{;\,b})$$
$$= 0 \qquad \text{vide I (4.5)} \qquad 78$$

The three terms on left hand side of (78) are simplified as follows :

Consider,

$$u^a h^b (T_{ab;c}) h^c = u^a h^b [(\rho + \mu h^2) u_a u_b - \tfrac{1}{2} \mu h^2 g_{ab} - \mu h_a h_b]_{;c} h^c ,$$

$$= [(\rho + \mu h^2)_{;c} u_a u_b u^a h^b + (\rho + \mu h^2) u_{a;c} u_b u^a h^b + (\rho + \mu h^2) u_{b;c} \; u_a u^a h^b] h^c$$

$$- \tfrac{1}{2} [\mu_{;c} h^2 g_{ab} u^a h^b + h^2{}_{;c} \mu \, g_{ab} u^a h^b] h^c$$

$$- [\mu_{;c} h_a h_b u^a h^b + \mu h_{a;c} h_b u^a h^b + + \mu \, h_{b;c} u^a h^b h^a] h^c . \qquad 79$$

$$\therefore \quad u^a h^b (T_{ab;c}) h^c = (\rho + \mu h^2) u_{b;c} h^b h^c + \mu h^2 h_{a;c} h^a h^c$$

$$\because \; u^a u_a = 1, h^a h_a = - h^2 , h_{a;c} u^a h^c = - u_{b;c} h^b h^c$$

$$= \rho \, u_{b;c} h^b h^c + \mu h^2 u_{b;c} h^b h^c - \mu h^2 u_{b;c} h^b h^c$$

$$u^a h^b (T_{ab;c}) h^c = \rho \, u_{b;c} h^b h^c \qquad 80$$

Now consider,

$$u^a h^b T_{db} (h^d{}_{;a}) = u^a h^b [(\rho + \mu h^2) u_d u_b - \tfrac{1}{2} \mu h^2 g_{db} - \mu \, h_d h_b] h^d{}_{;a} , \qquad 81$$

$$= (\rho + \mu h^2) u^a h^b u_a u_b h^d{}_{;a} - \tfrac{1}{2} \mu h^2 u^a h^b g_{db} h^d{}_{;a} - - \mu \, h_d h_b u^a h^b h^d{}_{;a} ,$$

$$= - \tfrac{1}{2} \mu h^2 u^a h_d h^d{}_{;a} + \mu h^2 h_d u^a h^d{}_{;a} ,$$

$$= \tfrac{1}{2} \mu h^2 h^d{}_{;a} h_d u^a \qquad \because h^a h_a = - h^2$$

$$= \tfrac{1}{2} \mu h^2 (- \tfrac{1}{2} h^2{}_{;a}) u^a ,$$

$$= - 1/4 \, \mu h^2 (h^2{}_{;a}) u^a .$$

$$\therefore \quad u^a h^b T_{db} (h^d{}_{;c}) = - 1/4 \, \mu h^2 (h^2)^{\bullet} \qquad \because h^2{}_{;a} u^a = (h^2)^{\bullet} . \qquad 82$$

Further consider,

$$u^a h^b (T_{ad}) (h^d{}_{;b}) = u^a h^b [(\rho + \mu h^2) u_a u_d - \tfrac{1}{2} \mu h^2 g_{ad} - \mu \, h_a h_d] h^d{}_{;b} , \qquad 83$$

$$= u^a h^b h^d{}_{;b} (\rho + \mu h^2) u_a u_d - \tfrac{1}{2} \mu h^2 u^a h^b g_{ad} h^d{}_{;b} - \mu \, u^a h^b h_a h_d h^d{}_{;b} , \qquad 84$$

$$= (\rho + \mu h^2) h^d{}_{;b} u_d h^b - \tfrac{1}{2} \mu h^2 h^b u_d h^d{}_{;b} ,$$

$$= (\rho + \tfrac{1}{2} \mu h^2) h^d{}_{;b} h^b u_d . \qquad 85$$

Hence from (85), (82) and (80), we can write equation (78) as

$$\rho [u_{b;c} h^b h^c] - 1/4 \, \mu h^2 (h^2)^{\bullet} + [\rho + \tfrac{1}{2} \mu h^2] [h^d{}_{;b} h^b u_d] , = 0 \qquad 86$$

i.e. $- \rho \, h_{a;c} u^a h^c - 1/4 \, \mu h^2 (h^2)^{\bullet} + \rho \, h_{d;b} h^b u^d + \tfrac{1}{2} \mu h^2 h_{d;b} h^b u^d , = 0$

i.e. $- 1/4 \, \mu h^2 (h^2)^{\bullet} + \tfrac{1}{2} \mu h^2 [h_{d;b} h^b u^d] = 0$,

i.e. $- 1/4 \, \mu h^2 (h^2)^{\bullet} + \tfrac{1}{2} \mu h^2 [- u_{d;b} h^b u^d] = 0$, 87

i.e. $- 1/4 \, \mu h^2 (h^2)^{\bullet} - \tfrac{1}{2} \mu h^2 [- \dot{\mu} h^2 - \mu h^2 \theta - (\mu/2) (h^2)^{\bullet}] = 0$, vide I (8.7)

i.e. $- 1/4 \, \mu h^2 (h^2)^{\bullet} + (\mu/2) h^4 + (\dot{\mu}/2) h^4 \theta + (\mu/4) h^2 (h^2)^{\bullet} = 0$,

i.e. $(\dot{\mu}/2) h^4 + (\mu/2) h^4 \theta = 0$, 88

i.e. $\dot{\mu} + \mu \, \theta = 0$. 89

Hence the proof.

3. DYNAMICAL INHERITANCE FOR CONFORMAL KILLING VECTORS :

Subcase I :

Claim 1 : If u is the Conformal Killing vector then Dynamical Inheritance is transferred into Dynamical Collineation.

Proof : Let u be the Conformal Killing vector

Then we have

$$\theta = 0 \qquad \text{vide II (2.6)}$$

$$2\,\dot{\mu}h^2 + \mu\,(h^2)^{\bullet} = 0 \qquad \text{vide II (2.15)}$$

$$\dot{\rho} = \tfrac{1}{2}\,\dot{\mu}h^2 \qquad \text{vide II (2.16)}$$

Using these results in equation (9), we get

$$\dot{\rho} = 2\,\alpha\,\rho,$$

$$\text{i.e.} \quad \underset{u}{L}\,\rho = 2\,\alpha\,\rho. \qquad 90$$

If $\underset{u}{L}\,\rho = 2\,\alpha\,\rho$ [vide (90)], equation (20), yields

$$\tfrac{1}{2}\,\underset{u}{L}\,(\mu h^2) = \alpha\,\mu h^2,$$

$$\text{i.e.} \quad \underset{u}{L}\,(\mu h^2) = 2\,\alpha\,\mu h^2. \qquad 91$$

Using result (91) in (37), we get

$$2\,\underset{u}{L}\,(\mu h^2) = \underset{h}{L}\,(\mu h^2) - \dot{\mu}h^2, \qquad \because \theta = 0$$

$$\text{i.e.} \quad \underset{u}{L}\,(\mu h^2) = -\,\dot{\mu}h^2,$$

$$\text{i.e.} \quad \underset{u}{L}\,(\mu h^2) = -\,2\,\dot{\rho}, \qquad \because \dot{\rho} = \tfrac{1}{2}\,\dot{\mu}h^2$$

i.e. $\underset{u}{L}(\rho)+L(\mu h^2) = 0$, 92

i.e. $2[2\alpha\rho]+2\alpha(\mu h^2) = 0$, vide (90), (91)

i.e. $2\alpha\rho+\alpha\mu h^2 = 0$,

i.e. $\alpha(2\rho+\mu h^2) = 0$, $\because 2\rho+\mu h^2 \neq 0$

i.e. $\alpha = 0$.

This means that the Dynamical Inheritance is transformed into Dynamical Collineation.

i.e. $\underset{u}{L} T_{ab} = 0$. 93

Subcase II :

Claim 2 : If h is the Conformal Killing vector then magnetic field vector becomes Killing vector if $\psi = 0$.

Proof : Let h be the Conformal Killing vector

Then we have

$h^b{}_{;b} = 4\psi$ vide II (2.19).

$\dot{h}_a u^a = \psi$ vide II (2.21).

$\underset{h}{L} h^2 = 2\psi h^2$ vide II (2.23).

$\dot{\mu}+\mu\theta = 0$ vide II (2.27).

$\dot{\rho}+(\rho+\tfrac{1}{2}\mu h^2)\theta = 0$ vide II (2.28).

$\rho\dot{u}_b h^b - \tfrac{1}{2}\mu_{;b} h^b h^2 = 0$ vide I (9.16).

$\psi[2\rho-3\mu h^2] = 0$ vide II (2.30).

We have equation (62), viz.

$$\underset{h}{L} T - 2\alpha T = \mu h^2 h^a{}_{;a} - \mu \underset{h}{L} h^2 - 2(\rho+\mu h^2)\dot{h}_d u^d, \quad 94$$

If $h^a{}_{;a} = 4\psi$, $\underset{H}{L} h^2 = 2\psi h^2$, $\dot{h}_d u^d = 2\psi$, we have from (94)

$$\underset{h}{L} T - 2\alpha T = \mu h^2 4\psi - \mu 2\psi h^2 - 2(\rho+\mu h^2)2\psi.$$

i.e. $\underset{h}{L} T - 2\alpha T = 4\mu h^2 \psi - 2\mu h^2 \psi - 4\rho\psi - 4\mu h^2 \psi,$

i.e. $\underset{h}{L} T - 2\alpha T = -2\mu h^2 \psi - 4\rho\psi,$

i.e. $\underset{h}{L} T - 2\alpha T = -2[2\rho\psi] - 4\rho\psi, \quad \because \psi[2\rho - \mu h^2] = 0$

$\underset{h}{L} T - 2\alpha T = -4\rho\psi - 4\rho\psi,$

$\underset{h}{L} T - 2\alpha T = -8\rho\psi.$ 95

If $\psi = 0$ then magnetic field vector becomes Killing vector.

Hence the claim.

Subcase III:

Claim 3 : If u = h both are Conformal Killing vectors then DI is transformed into Dynamical Collineation.

Proof : Let $u = h = \xi$ be Conformal Killing vectors

Then we have

$\theta = 0$	vide II (2.32).
$\psi = 0$	vide II (2.33).
$h^b{}_{;b} = 0$	vide II (2.38).
$\underset{h}{L} h^2 = 0$	vide II (2.39). 96
$\dot{\mu} = 0$	vide II (2.42).
$\dot{\rho} = 0$	vide II (2.40).
$\mu_{;b} h^b = 0$	vide II (2.41).
$\mu (h^2)^{\cdot} = 0$	vide II (2.43).

If we use these results (96), in equations (9), (22) and (37), we get

$$2\,\alpha\,\rho = 0,\;\; 2\,\alpha\,\rho + \alpha\,\mu h^2 = 0,\;\; 2\,\alpha\,\mu h^2 = 0. \qquad 97$$

This implies that $\alpha = 0$. 98

Again, if we use results (96) in (55), (61) and (76), we get

$$2\,\alpha\,\rho + \alpha\,\mu h^2 = 0,\;\; 2\,\alpha\,\rho = 0,\;\; 2\,\alpha\,\mu = 0. \qquad 99$$

This implies that $\alpha = 0$.

This means that Dynamical Inheritance is transformed into Dynamical Collineation. vide (98), (99)

i.e. $\underset{\xi}{L}\, T_{ab} = 0$. vide (1) 100

Hence the claim.