PREFACE

Almost all astrophysical systems posses strong gravitational field. Moreover recent observational and phenomenal discoveries justify the presence of intense magnetic field in the system like very big magnetic variable stars, Neutron stars, pulsars etc. (Misner, Thorne, Wheeler, 1973). Even the inter galactic medium is supposed to be comprised of self gravitating magneto fluid. The scheme comprising of relativistic charged fluid distribution possessing infinite conductivity and constant magnetic permeability is introduced by Lichnerowicz, 1967. A step to generalise this study to a wider matter distribution known as Relativistic Ferrofluid is a relativistic charged fluid distribution with infinite conductivity and variable magnetic permeability (Ray and Banarji, 1980).

The field equations of gravitation which forms a standard bridge between geometry and dynamics of space-time form a set of highly non-linear differential equations. A step towards simplification of these is the inhabitance of metric symmetries. Apart from these metric symmetries the appliance of isometries in deriving relativistic models play a crucial role in the development of the subject.

The work of the dissertation operates through two separate streams. First gives the exposure of particular choice of matter distribution known as the Relativistic Magneto-Dust Distribution which acts as a source of gravitational and magnetic fields. While the other stream deals with symmetry known as Conformal motions. In particular the study is limited to examine the dynamical implications of Relativistic Magneto-Dust Distribution concomitant with Conharmonic Conformal motions and Conformal motions along flow lines and magnetic lines.

The chapter wise exploration of important results is given below :

- *CHAPTER I :*

Basic concepts and system equations for the dissertation work are introduced. The stress-energy tensor for Magneto-Dust Distribution is developed thereby evaluating its time-like and space-like eigen values. Moreover energy conditions pertaining to this stress-energy tensor are found. Different geometrical symmetries which are used in dissertation work are introduced. Field equations governing the gravitational field and electromagnetic filed are presented.

The stress-energy tensor characterizing Magneto-dust Distribution is described as,

$$T_{ab} = (\rho+\mu h^2)\, u_a u_b - \tfrac{1}{2}\, \mu h^2\, g_{ab} - \mu\, h_a h_b$$

Important results derived in the dissertation are presented below :

A) Maxwell equations imply

$$\mu\, [\, \dot{u}_b h^b + h^b{}_{;b}\,] + \mu_{;b} h^b = 0 .$$

This states that the magnetic permeability is conserved along divergence free magnetic lines iff 4-acceleration is normal to these lines.

B) Equation of continuity provides the result

$$\dot{\rho} + \rho\, \theta = \tfrac{1}{2}\, \dot{\mu} h^2 .$$

This directs that the matter energy density is conserved along the expansion free flow iff the magnetic permeability is kept invariant along these lines.

C) The equation of stream lines yields

$$\rho\, \dot{u}_b h^b = \tfrac{1}{2}\, \mu_{;b} h^b h^2 .$$

This means that the acceleration is normal to magnetic lines iff magnetic permeability is conserved along these lines.

- ***CHAPTER II :***

This chapter deals with several properties of Magneto-Dust Distribution admitting a group of Conharmonic Conformal motions. Some important deductions are shown below :

Theorem : If the flow vector u is Conformal Killing vector then

i) $\theta = 0, \sigma = 0,$

ii) $\psi = 0$ ·

This implies that i) If u is Conformal Killing vector then flow lines are expansion and shear free. ii) Conformal Killing vector is changed into Killing vector.

Theorem : If the flow vector u is Conformal Killing vector then

i) $\dot{u}_b h^b = 0,$

ii) $2\dot{\mu} h^2 + \mu (h^2)^{\cdot} = 0,$

iii) $\dot{\rho} = \frac{1}{2} \dot{\mu} h^2,$

iv) $\mu_{;b} h^b = 0$.

These results can be interpreted as i) 4-acceleration is normal to magnetic lines. ii) Magnitude of the magnetic field is invariant iff magnetic permeability is invariant along flow lines. iii) Matter energy density is invariant iff magnetic permeability is invariant with flow lines. iv) Magnetic permeability is preserved along magnetic lines.

Theorem : If magnetic field vector h is Conformal Killing vector then

i) $h^b{}_{;b} = 4\,\dot{h}_a u^a = (2/h^2)\,\underset{h}{L}\,h^2 = 4\,\psi$,

ii) $\dot{\mu} + \mu\theta = 0$.

Theorem : If magnetic field vector h is Conformal Killing vector then

i) $\dot{\rho} = 0$ iff $\dot{\mu} = 0$,

ii) $\mu_{;b}h^b = -2\,\rho\,\psi/h^2$.

Theorem : If magnetic field vector h is Conformal Killing vector then

i) $\psi = 0$ OR

ii) $\rho = (3/2)\,\mu h^2$ OR

iii) $\psi = 0,\ \rho = (3/2)\,\mu h^2$

We have $\psi \neq 0$ always. Hence only possibility $\rho = (3/2)\,\mu h^2$ is considered.

When the time-like flow vector u and magnetic field vector h both are Conformal then we have proved

a) $\theta = 0,\ \sigma = 0,\ \dot{u}_a = 0$,

b) $h^b;b = 0$,

c) $(h^2{}_{;b})\,h^b = 0,\ \mu_{;b}h^b = 0$,

d) $\dot{\rho} = 0$.

This means that

a) Flow lines are expansion free, shear free and geodesic.

b) Magnetic lines are divergence free.

c) Magnetic permeability and magnitude of the magnetic field are invariant along magnetic lines and flow lines.

d) Matter energy density is conserved along u.

Theorem : If the Magneto-Dust Distribution admits Conformal Killing vector ξ then

$$2 \underset{\xi}{L} \mu h^2 = (4/h^2)\, \psi_{;ab} h^a h^b - 4\, \psi_{;ab} u^a u^b + 2\psi h^2$$

Theorem : If the Magneto-Dust Distribution admits Conformal Killing vector ξ then

$$\psi_{;ab} u^a h^b = (\rho/2)\, h^2.$$

Theorem : If the Magneto-Dust Distribution admits Conformal Killing vector ξ then

$$\psi_{;ab} u^a u^b = -1/4 \underset{\xi}{L} \mu\, h^2.$$

Theorem : If the Magneto-Dust Distribution admits Conformal Killing vector ξ then

$$\psi_{;ab} h^a h^b = (h^2/4) \underset{\xi}{L} \mu\, h^2.$$

If $\xi = u$ is Conformal Killing vector then we have

$$\text{I) } \underset{u}{L}(\mu) = 0 \qquad \text{II) } \underset{u}{L}(h^2) = 0 \qquad \text{III) } \underset{u}{L}(\rho) = 0 .$$

If $\xi = h$ is Conformal Killing vector then we have the result

$$\underset{h}{L}(\rho) = 0 \ \text{ iff } \ \underset{h}{L} h^2 = 0 .$$

If $\xi = u = h$ be Conformal Killing vector then we have the result

$$\rho_{;a} = h^2{}_{;a} = \mu_{;a} = 0 .$$

- ***CHAPTER III :***

The Dynamical Inheritance property is studied in this chapter to examine the effects on dynamical parameters associated with Magneto-Dust Distribution. This study provides some important results regarding the Magneto-Dust Distribution. These results are given below :

Theorem : If the Magneto-Dust Distribution admits Dynamical Inheritance property along flow lines then active gravitational mass density (T) is inherited iff

$$(\mu h^2)^{\bullet} + \dot{\mu} h^2 + \mu h^2 \theta = 0 .$$

Theorem : If the Magneto-Dust Distribution admits Dynamical Inheritance property along flow lines then time-like eigen value $e_1 = \rho + \frac{1}{2} \mu h^2$ is inherited along flow lines.

Theorem : If the Magneto-Dust Distribution admits Dynamical Inheritance property along flow lines then

$$\underset{u}{L} \mu h^2 = 2 \alpha \mu h^2 \quad \Leftrightarrow \quad \dot{\mu} h^2 + 2 \mu h^2 \theta = 0 .$$

Theorem : If the Magneto-Dust Distribution admits Dynamical Inheritance property along flow lines then

$$\dot{u}_b h^b = 0 .$$

This means that the acceleration is orthogonal to magnetic lines.

Theorem : If the Magneto-Dust Distribution admits Dynamical Inheritance property along magnetic lines then

$$\underset{h}{L} e_1 = 2 \alpha e_1 \text{ iff } \dot{h}^d u_d = 0 .$$

This implies that the time-like eigen value e_1 is inherited along magnetic line iff $\dot{h}^d u_d = 0$

Theorem : If the Magneto-Dust Distribution admits Dynamical Inheritance property along magnetic lines then necessary and sufficient condition for Inheritance of active gravitational mass along magnetic lines is given by

$$\mu\,(h^2{}_{;c}h^c) - \mu\, h^2\, h^a{}_{;a} + 2\,(\rho + \mu h^2)\,\dot{h}^d u_d = 0\,.$$

Theorem : If the Magneto-Dust Distribution admits Dynamical Inheritance property along magnetic lines then the following results are equivalent.

I) $\underset{h}{L}\,\mu = 2\,\alpha\,\mu\,,$

II) $\underset{h}{L}\,h^2 = 0\,.$

Theorem : If the Magneto-Dust Distribution admits Dynamical Inheritance property along magnetic lines then $\dot{\mu} + \mu\,\theta = 0\,.$

This means that magnetic permeability is invariant along flow lines iff flow lines are expansion free.

For Conformal Killing vector u, h and u and h both Conformal we have following claims :

Claim : If u is Conformal Killing vector then Dynamical Inheritance is transformed into Dynamical collineation.

Claim : If h is Conformal Killing vector then magnetic field vector becomes Killing vector if $\psi = 0\,.$

Claim : If $\xi = u = h$ are Conformal Killing vectors then Dynamical Inheritance is transformed into Dynamical Collineation.

Some interesting features of Relativistic Magneto-Dust Distribution under prescribed geometrical restrictions are given above. It will be nice to develop a suitable model in which the above results hold which was not possible here due to time constraint.