PREFACE

The dissertation entitled "Special Congruences in Relativistic Continuum Mechanics" is devoted to the study of geometrical properties of special congruences.

Different tetrad formulations are in vogue in the general theory of relativity. We use the one introduced by Greenberg (1970) in which there is one time-like vector field and other three are space-like vector fields. By using this tetrad formalism we introduce the Ricci Rotation Coefficients (RRCs) and its relationship with tetrad components is enumerated. As the vector field determines a congruence, consequently, the four vectors of the tetrad determine orthogonal quadruple of congruences. A comprehensive account of special types of congruences viz. Tendency of a congruence, Divergence of a congruence, Normal congruence, Geodesic congruence, Irrotational congruence, Canonical congruence, Harmonic congruence, Killing congruence is cited in the **First Chapter.** Accordingly, the following results are discussed in Chapter-I.

1 It is verified that if all the congruences of an orthogonal quadruple are normal, then

$\gamma_{\alpha\beta\sigma} = 0, \quad \alpha \neq \beta \neq \sigma, \ \alpha, \beta, \sigma = 1,2,3,4.$

2 The interdependence of normal congruence, geodesic congruence and irrotational congruence is obtained.

3 It is shown that if any three congruences of an orthogonal quadruple are normal then they are canonical with respect to the remaining congruence of quadruple.

4 It is also shown that if a congruence $e_{(\alpha)}$ of an orthogonal quadruple satisfies any two of the following conditions, then it will satisfy the third condition.

a) The tendencies of a geodesic congruence $e_{(\alpha)}$ along the other three directions of orthogonal quadruple are zero.

b) $e_{(\alpha)}$ is a Killing congruence.

c) Three congruences $e_{(\beta)}$ of an orthogonal quadruple are canonical with respect to $e_{(\alpha)}$.

An exposition of kinematical parameters, natural transport laws and propagation equations of kinematical parameters are the subject matter of the **Chapter-II.** The properties of the time-like congruence and space-like congruences depicted in Chapter-I have been exploited to study the local behaviour of the kinematical parameters and propagation equations of the kinematical parameters. The parameters associated with the orthogonal quadruple of congruences are discussed in terms of RRCs. It is shown that the time-like congruence u^i is

1 expansion-free iff sum of its tendencies along the mutually orthogonally congruences of the tetrad vanishes identically.

2 Shear -free if and only if

a) tendencies of u^i in the direction of space-like congruences p^i,q^i and r^i are equal, and

b) p^i,q^i and r^i are canonical with respect to u^i.

3 rotation-free iff u^i is a normal congruence.

4 a rigid congruence, then the space-like congruences p^i ,q^i, r^i are canonical with respect to the time-like congruence u^i and tendencies of u^i along p^i,q^i and r^i are zero.

5 a essentially expanding then its tendencies along p^i, q^i and r^i are equal and

$$\gamma_{1\alpha\beta} = 0 \ , \ \alpha \neq \beta, \quad \alpha \ , \ \beta = 2,3,4.$$

Similar results are also explored for the space-like congruences.

The RRCs scalar version for kinematical parameters associated with the space-like congruences p^i ,q^i and r^i with their natural transport laws are discussed. It has been observed that all the three natural transport laws are not independent but only two are independent. The RRCs scalar version of natural transport laws have been exploited to study the propagation equations and Lie invariance of kinematical parameters associated with p^i, q^i and r^i. The propagation equations for shear tensor and rotation tensor associated with the time-like congruence u^i and space-like congruences p^i, q^i and r^i are obtained. It is shown that the rotation tensor $\underset{(p)}{W_{ij}}$ of space-like congruence p^i is parallelly propagated in the direction of p^i iff

1 $H_{1,k}\ p^k = 0$,

2 Tendencies of q^i and r^i in the direction of p^i are zero, and

3 $\gamma_{312} = \gamma_{412} = 0$ for $H_1 = 1/2(\gamma_{234} - \gamma_{243}) \neq 0$.

Similar results are also obtained for the kinematical parameters of other congruences.

In **Chapter-III** we utilize the role of Lie derivative to study the Lie invariance of kinematical parameters associated with time-like congruence u^i and space-like congruences p^i, q^i and r^i. It has been established that if the tendencies of time-like normal congruence u^i in the direction of space-like congruences p^i, q^i and r^i are zero, then

$$\underset{u}{\mathcal{L}} \underset{(u)}{\sigma}_{ij} = 0 \iff \underset{(u)}{\sigma}_{ij} = 0$$

The results analogous to this are also obtained for all space-like congruences. In case of rotation tensor of u^i, the necessary and sufficient conditions for the Lie invariance of rotation tensor are discussed. Further, for rotation tensor of space-like congruences it is observed that the following statements are equivalent.

1 $\underset{q}{\mathcal{L}} \underset{(q)}{W}_{ij} = 0$.

2 $H_2 (\gamma_{322} + \gamma_{344}) - H_{2,k} \quad q^k = 0$

where $H_2 = \gamma_{324} - \gamma_{342}$.

Analogue results are also discussed for the kinematical parameters of other congruences.