PREFACE

The dissertation entitled **'A NEWMAN AND PENROSE TYPE TETRAD FORMALISM'**, is devoted to the development of a tetrad formalism similar to that of Newman-Penrose null tetrad formalism and to study the geometrical properties of the vector fields of the tetrad in terms of this new formalism.

Different tetrad formalism are in vogue in the general theory of relativity, viz.

1. A tetrad with one time like and three space like vector fields. *Greenberg (1970).*
2. A tetrad with one null and three space like vector fields. *Synge (1972)*
3. A tetrad with two null vector fields and two space like vector fields. *Hall (1977).*
4. A tetrad with all four vector fields are null. *Newman and Penrose (1962).*
5. A tetrad with single time like vector field and its intrinsic derivatives *Radhakrishna (1993),*

Together with intrinsic scalars viz. Curvature κ, Torsion τ and bitorsion B. The most prominent among all the formalism was the one proposed by *Newman and Penrose in 1962*, which uses four null vector fields. For more than four decades the progress made in understanding of Einstein field equations may be attributed to the Newman Penrose formalism. The equations of motion are given $T^{ik}_{;k} = 0$, where T^{ik} is the stress energy momentum tensor which is the source of gravitation. Unfortunately, they give no information about the interaction of the free-gravitational field characterized by C_{hijk} with the sources. To get this information, we have to explore the twenty four Bianchi identities and not the contracted Binachi identities, which are four in number. An exploitation of every one of the twenty four Bianchi identities is possible only by the technique developed by Newman and Penrose formalism. Analogous to the 'amazingly' useful for Einstein's Theory of Gravitation, *Jogia and Griffiths (1980)* have extended a null tetrad formalism for studying the Einstein – Cartan theory of gravitation in which torsion plays dominant role. During the same time *Gambini and Herrara (1980)* had developed a null tetrad formalism for space- times with torsion. The author is influenced by this extension and thought of developing another spin coefficient formalism for Greenberg's tetrad similar to that of Newman and Penrose whose approach will be useful in dealing with the problems like self gravitating perfect fluids, definite material schemes, magnetofluid schemes shock waves, gravitational and acoustic waves in an elastic medium.

Thus the primary aim of this dissertation is to develop a new formalism analogous to Newman and Penrose formalism for *Greenberg's*

(1970) tetrad, which consists of one time like and three space like vector fields. A comprehensive account of different types of tetrad formalism in relativity is given in Chapter 1. Since our tetrad formalism is similar to that of Newman-Penrose spin coefficient formalism an exhaustive account of Newman – Penrose formalism is delineated in the same chapter. The chapter.1 is introductory and no originality is claimed in this chapter.

The investigation of tetrad formalism analogues to that of Newman and Penrose are the subject matters of chapter.2. Accordingly a tetrad of four basis of vector fields u^i, p^i, q^i, r^i satisfying the following conditions.

$$u^i u_i = 1, \quad p_i p^i = q_i q^i = r_i r^i = -1, \quad \text{and}$$

$$u_i p^i = u_i q^i = u_i r^i = 0,$$

$$p_i q^i = p_i r^i = q_i r^i = 0.$$

is introduced in the section 2. The vector field u^i is the time like vector field, while the vector fields p_i, q_i, r_i are space like vector fields. In the next section the scalar invariants $\gamma_{\alpha\beta\sigma}$ usually referred as the Ricci's rotation coefficients are defined. They satisfy the condition $\gamma_{(\alpha\beta)\sigma} = 0$. Hence it has twenty four independent components. Each one of these twenty four components is defined through the spin coefficients in the same section. The concept of intrinsic derivative of a vector field and curvature tensor is cited in section.4. The trace free part of the curvature tensor is called the Weyl tensor. It has ten independent components in 4-dimensional space time of General Theory of

Gravitation. These ten components of Weyl tensor, and ten components of Ricci tensor are depicted in Section.5. An explicit form of each one of the twenty four Ricci identities and twenty four Bianchi identities in the Newman – Penrose type tetrad formalism are worked and established in Section.6 and Section.7 respectively. These equations are more numerous and more lengthy, we have therefore mainly devoted this dissertation to the development of this formalism.

An exposition of kinematical parameters for each of the vector field of the tetrad and the natural transport laws for each of the space-like vector field in terms of the Newman – Penrose type formalism is depicted in Chapter 3. The section wise investigations are given below.

In section 2 , the kinematical parameters for the time like vector field are constructed in terms of Newman – Penrose type tetrad formalism. Geometrically we have shown that ,

a) The flow of a fluid is expansion free iff $\sigma + \lambda + \gamma = 0$

b) The flow of a fluid in geodesic iff $\kappa = \upsilon = \varepsilon = 0$

c) The flow of a fluid is shear-free iff $\lambda + \gamma = 2\sigma$, $\sigma + \gamma = 2\lambda$, $\sigma + \lambda = 2\gamma$, $\rho + \pi = 0, \tau + \alpha = 0, \mu + \beta = 0$

d) The flow of a fluid is irrotational , iff $\rho - \pi = 0$, $\tau - \alpha = 0$, $\mu - \beta = 0$

e) The gradient of a fluid vanishes identically

$$\text{iff} \quad \kappa = \upsilon = \varepsilon = \rho = \tau = \pi = \lambda = \mu = \alpha = \beta = \gamma = 0$$

Section 3 deals with the kinematical parameters for space-like vector field $p_i(p_i p^i = -1)$. These parameters are subjected to the natural transport laws given by Greenberg (1970). The kinematical parameters and the natural transport laws are accomplished. It is shown that the space like vector field p_i is,

a) Expansion free , iff $(\rho_1 + \mu_1) = 0$

b) The flow of a fluid is geodesic, iff $\kappa = \kappa_1 = \upsilon_1 = 0$

c) The flow of fluid is shear free, iff $\begin{matrix} (\tau_1 + \lambda_1) = 0 \\ (\rho_1 - \mu_1) = 0 \end{matrix}$

d) The flow of fluid is irrotational, iff $(\lambda_1 - \tau_1) = 0$

Subject to the conditions viz, The natural transport laws,

$$\pi = \alpha = \kappa_1 = \alpha_1 = \upsilon_1 = \sigma - \kappa = 0$$

Similar investigation for space like vector field q_i and space like vector r_i are accomplished in the section 4 and section 5. It is cited below ,

a) The flow of a fluid expansion free iff $\sigma_1 - \gamma_1 = 0$

b) The flow of a fluid geodesic iff $\upsilon = \varepsilon_1 = \kappa_1 = 0$

c) The flow of a fluid irrotational iff $\tau_1 + \alpha_1 = 0$

d) The flow of a fluid shear free iff $\alpha_1 - \tau_1 = 0 = \sigma_1 + \gamma_1$

Subject to the condition viz., the natural transport laws are as follows,

$$\sigma_1 = \gamma_1 = \upsilon = \varepsilon_1 = \kappa_1 = \tau_1 = \alpha_1 = 0$$

Similarly for the space like vector field r_i is,

a) The flow of fluid is expansion free, iff $(\pi_1 + B_1) = 0$

b) The flow of a fluid is geodesic, iff $\varepsilon = \upsilon_1 = \varepsilon_1 = 0$

c) The flow of fluid is shear free, iff $(\beta_1 - \pi_1) = 0$ $(\lambda_1 + \alpha_1) = 0$

d) The flow of fluid irrotational, iff $(\alpha_1 - \lambda_1) = 0$

Subject to the condition viz, the natural transport laws are given below.

$$\varepsilon = \upsilon_1 = \varepsilon_1 = \pi_1 = \beta_1 = \lambda_1 = \alpha_1 = 0$$