CHAPTER III

A STUDY OF NON-STATIC SPHERICALLY SYMMETRIC SPACE-TIME IN EINSTEIN-CARTAN THEORY OF GRAVITATION.

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1. Introduction:

Analogous to the 'amazingly useful' Newman-Penrose tetrad formalism for Einstein theory of gravitation, Jogia and Griffiths (1980) have developed a tetrad formalism for the study of torsion in the Einstein-Cartan theory of gravitation. We exploit the Newman-Penrose-Jogia-Griffiths (NPJG) formalism to study the role of torsion on the geometry of the space-time structure. In this Chapter we study the non-static spherically symmetric metric in U_4 theory of gravitation. Accordingly, we basic notions and the tensors that are pertinent to the study of the metric in U_4 theory of gravitation are described in the Section 2. The structure equations in the U_4 theory of gravitation are given by Katkar (2008). We utilize it to find the tetrad components of Connection 1-forms and Curvature 2-forms in Einstein-Cartan theory of gravitation in the Section 3. In the Section 4, the tetrad components of Curvature tensor, Ricci tensor and Weyl tensor are obtained with reference to the non-static spherically symmetric metric in the U_4 theory of gravitation. The roll of Spin components on the geometry of the space-time is observed. It has been observed that the spin tensor influences the space-time geometry of the Einstein-Cartan theory of gravitation. If the Spin tensor component $s_\circ = 0$ and $s_1 \neq 0$, the space-time of U_4 theory of gravitation is shown to be Petro-type I. If however, the tetrad components of the Spin tensor are functions of r and t alone then we have $\psi_\circ = \psi_1 = \psi_3 = \psi_4 = 0$ proving the space-time of U_4 theory of gravitation is Petro-type D. However, if $s_\circ \neq 0$ and $s_1 = 0$, then none of the Weyl tensor vanishes, showing that $s_\circ$ has predominant effect on the space-time geometry of U_4. In the absence of the Spin tensor the space-time reduces to the space-time D of Einstein theory of gravitation. In Section 5, we utilize the results of Section 4 to find the expression for the electric part and the magnetic part of Weyl tensor in U_4 theory of gravitation. It has been noticed that the Spin influences the electric and magnetic parts of Weyl tensor. In the absence of the Spin tensor we notice that the magnetic part of the Weyl tensor vanishes.

2. The Non-Static Spherically Symmetric Space-time:

We consider a line element of spherically symmetric non-static dust distribution

$$ds^2 = dt^2 - P^2 dr^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad ...(2.1)$$

where P and R are functions of r and t and t is a commoving time parameter with fluid vector u^i.

The covariant components of the metric tensor are given by

$$g_{11} = 1, \qquad g_{22} = -P^2,$$
$$g_{33} = -R^2, \qquad g_{44} = -R^2\sin^2\theta. \qquad ...(2.2)$$

The contravariant components of the metric tensor are

$$g^{ij} = \frac{1}{g_{ij}} \quad \forall\, i = j \text{ and } \quad g^{ij} = 0 \quad for\ \ i \neq j \qquad ...(2.3)$$

To simplify the computation we look for a basis 1-forms θ^α such that the coefficients of the metric are constants.

Thus a simple choice is

$$\theta^1 = \frac{1}{\sqrt{2}}(dt + Pdr),$$

$$\theta^2 = \frac{1}{\sqrt{2}}(dt - Pdr),$$

$$\theta^3 = \frac{-R}{\sqrt{2}}(d\theta - i\sin\theta d\phi), \qquad ...(2.4)$$

$$\theta^4 = \frac{-R}{\sqrt{2}}(d\theta + i\sin\theta d\phi),$$

Then the line element (2.1) becomes

$$ds^2 = 2\theta^1\theta^2 - 2\theta^3\theta^4 \qquad ...(2.5)$$

Solving equations (2.4) we obtain

$$dt = \frac{1}{\sqrt{2}}(\theta^1 + \theta^2),$$

$$dr = \frac{1}{\sqrt{2}P}(\theta^1 - \theta^2),$$

$$d\theta = -\frac{1}{\sqrt{2}R}(\theta^3 + \theta^4), \qquad ...(2.6)$$

$$d\phi = -\frac{i\cos ec\theta}{\sqrt{2}R}(\theta^3 - \theta^4).$$

For computation purpose, we obtain the following wedge products:

$$dt \wedge dr = -\frac{1}{P}\theta^{12} ,$$

$$dt \wedge d\theta = -\frac{1}{2R}(\theta^{13} + \theta^{14} + \theta^{23} + \theta^{24}) ,$$

$$dt \wedge d\phi = -\frac{i\cos ec\theta}{2R}(\theta^{13} - \theta^{14} + \theta^{23} - \theta^{24}) ,$$

$$dr \wedge d\theta = -\frac{1}{2PR}(\theta^{13} + \theta^{14} - \theta^{23} - \theta^{24}), \qquad ...(2.7)$$

$$dr \wedge d\phi = -\frac{i\cos ec\theta}{2PR}(\theta^{13} - \theta^{14} - \theta^{23} + \theta^{24}) ,$$

$$d\theta \wedge d\phi = \frac{i\cos ec\theta}{R^2}\theta^{34} ,$$

where $\theta^{\alpha\beta} = \theta^\alpha \wedge \theta^\beta$ is a basis 2-forms.

The definition of basis 1-forms $\theta^\alpha = e_i^{(\alpha)} dx^i$ and the equation (2.4) gives the null vector fields as

$$l_i = \frac{1}{\sqrt{2}}(\delta_i^1 - P\delta_i^2) , \qquad n_i = \frac{1}{\sqrt{2}}(\delta_i^1 + P\delta_i^2) , \qquad m_i = \frac{R}{\sqrt{2}}(\delta_i^3 + i\sin\theta\delta_i^4)$$

...(2.8)

where l_i and n_i are real null vector fields and m_i is a complex null vector field. The complex conjugate $\overline{m}_i$ can be obtained by taking complex conjugate of m_i.

The equation $e^i_{(\alpha)} = g^{ik} e_{(\alpha)k}$ gives

$$l^i = \frac{1}{\sqrt{2}}(\delta_1^i + P^{-1}\delta_2^i),$$

$$n^i = \frac{1}{\sqrt{2}}(\delta_1^i - P^{-1}\delta_2^i),$$

$$m^i = -\frac{1}{\sqrt{2}R}(\delta_3^i + i\,\mathrm{cosec}\,\theta\delta_4^i) \qquad ...(2.9)$$

To find the tetrad components of Connection 1-forms we can take the exterior derivatives of basis 1-forms θ^α described in equations (2.4) are

$$d\theta^1 = \frac{1}{\sqrt{2}}[\dot{P}dt \wedge dr]$$

$$= \frac{\dot{P}}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(\theta^1 + \theta^2) \wedge \frac{1}{\sqrt{2}P}(\theta^1 - \theta^2)\right]$$

$$d\theta^1 = -\frac{\dot{P}}{\sqrt{2}P}\theta^{12}$$

Similarly, we get

$$d\theta^2 = \frac{\dot{P}}{\sqrt{2}P}\theta^{12},$$

$$d\theta^3 = \frac{1}{\sqrt{2}}\left[R^{-1}\left(\dot{R} + \frac{R'}{P}\right)\theta^{13} + R^{-1}\left(\dot{R} - \frac{R'}{P}\right)\theta^{23} + R^{-1}\cot\theta\theta^{34}\right], \qquad ...(2.10)$$

$$d\theta^4 = \frac{1}{\sqrt{2}}\left[R^{-1}\left(\dot{R} + \frac{R'}{P}\right)\theta^{14} + R^{-1}\left(\dot{R} - \frac{R'}{P}\right)\theta^{24} - R^{-1}\cot\theta\theta^{34}\right].$$

In U_4 theory of gravitation space-time is non-Riemannian and is described by torsion tensor given as

$$Q_{ij}{}^{k} = -\frac{1}{2}(K_{ij}{}^{k} - K_{ji}{}^{k}) \qquad ...(2.11)$$

where $K_{ij}{}^{k}$ is the Contortion tensor satisfying the property $K_{i(jk)} = 0$.

Using equation (5.5) of chapter I we find the expression for torsion tensor in terms of its tetrad components in the form

$$\begin{aligned} Q_{ij}{}^{k} = & (\varepsilon_1 + \bar{\varepsilon}_1) l_{[i} n_{j]} n^k + (\gamma_1 + \bar{\gamma}_1) l_{[i} n_{j]} l^k + (\bar{\mu}_1 - \mu_1) m_{[i} \bar{m}_{j]} l^k + (\bar{\rho}_1 - \rho_1) m_{[i} \bar{m}_{j]} n^k + \\ & + \{\bar{\kappa}_1 n_{[i} m_{j]} n^k - (\bar{\tau}_1 + \pi_1) l_{[i} n_{j]} m^k - (\pi_1 - \alpha_1 - \bar{\beta}_1) n_{[i} m_{j]} l^k - \bar{\sigma}_1 n_{[i} m_{j]} m^k + \\ & + (\varepsilon_1 - \bar{\varepsilon}_1 - \rho_1) n_{[i} m_{j]} \bar{m}^k - \nu_1 l_{[i} m_{j]} l^k + (\bar{\tau}_1 - \alpha_1 - \bar{\beta}_1) l_{[i} m_{j]} n^k + \lambda_1 l_{[i} m_{j]} m^k + \\ & + (\gamma_1 - \bar{\gamma}_1 + \bar{\mu}_1) l_{[i} m_{j]} \bar{m}^k + (\alpha_1 - \bar{\beta}_1) m_{[i} \bar{m}_{j]} m^k\} + \{c.c.\} \end{aligned}$$

...(2.12)

Using equation (2.12) in the equation (1.4) of chapter I, one can obtain the expression for Spin angular momentum tensor as

$$S^{ijk} = -\frac{1}{2K}[-(\mu_1+\bar{\mu}_1)l^{[i}n^{j]}l^k - (\rho_1+\bar{\rho})l^{[i}n^{j]}n^k + (\mu_1-\bar{\mu}_1)m^{[i}\,\overline{m}^{j]}l^k +$$
$$+(\rho_1-\bar{\rho}_1)m^{[i}\,\overline{m}^{j]}n^k + \{(\pi_1+\bar{\tau}_1)l^{[i}n^{]}m^k + \nu_1 l^{[i}m^{j]}l^k + (2\alpha_1-\pi_1)l^{[i}m^{j]}n^k -$$
$$-\lambda_1 l^{[i}m^{j]}m^k + (\mu_1-2\gamma_1)l^{[i}m^{j]}\overline{m}^k + (\bar{\tau}_1-2\bar{\beta}_1)n^{[i}m^{j]}l^k - \bar{\kappa}_1 n^{[i}m^{j]}n^k +$$
$$+\bar{\sigma}_1 n^{[i}m^{j]}m^k + (2\bar{\varepsilon}_1-\bar{\rho}_1)n^{[i}m^{j]}\overline{m}^k + (\bar{\tau}_1-\pi_1)m^{[i}\,\overline{m}^{j]}m^k\} + \{c.c.\}]$$

...(2.13)

The Spin angular momentum tensor $S_{ij}{}^{k}$ can be decomposed in terms of the Spin tensor S_{ij} as follows

$$S_{ij}{}^{k} = S_{ij}\,u^k, \qquad ...(2.14)$$

where S_{ij} is anti-symmetric tensor orthogonal to 4-velocity vector u^i . This gives

$$S_{ij}\,u^i = 0.$$

We express S_{ij} in terms of the vectors of the tetrad as

$$S_{ij} = 2[\bar{s}_2\, l_{[i}m_{j]} + s_2\, l_{[i}\overline{m}_{j]} + (s_1+\bar{s}_1)l_{[i}n_{j]} + (s_1-\bar{s}_1)m_{[i}\overline{m}_{j]} + \bar{s}_{\circ}m_{[i}n_{j]} + s_{\circ}\overline{m}_{[i}n_{j]}]$$

...(2.15)

where $s_\circ$, s_1, s_2 are the tetrad components of Spin tensor S_{ij} and are defined by

$$s_\circ = s_{13} = s_{ij}l^i m^j \;,$$

$$s_1 = (s_{12}+s_{43}) = \frac{1}{2}s_{ij}(l^i n^j + \overline{m}^i m^j) \;,$$

$$s_2 = s_{32} = s_{ij}m^i n^j \;.$$

For the choice $\quad u^i = \frac{1}{\sqrt{2}}(l^i + n^i)\,, \qquad ...(2.16)$

the condition $S_{ij}u^i = 0$ gives $s_\circ = s_2$, $s_1 = -\bar{s}_1$

Consequently, we get from equation (2.15)

$$S_{ij} = 2[\bar{s}_2(l_{[i}m_{j]} + m_{[i}n_{j]}) + s_2(l_{[i}\overline{m}_{j]} + \overline{m}_{[i}n_{j]}) + 2\,s_1 m_{[i}\overline{m}_{j]}] \qquad ...(2.17)$$

The tetrad components of the field equation (1.4) of chapter I are given by

$$Q_{\alpha\beta}{}^{\gamma} + \delta^{\gamma}_{\alpha}Q_{\beta\delta}{}^{\delta} - \delta^{\gamma}_{\beta}Q_{\alpha\delta}{}^{\delta} = K\,S_{\alpha\beta}{}^{\gamma} \qquad ...(2.18)$$

Using the tetrad components of the 4-velocity vector (2.16) and the tetrad components of equation (2.18), lengthy but straight forward calculations give

$$\pi_1 = \tau_1 = \sigma_1 = \lambda_1 = 0 \ ,$$
$$\mu_1 + \bar{\mu}_1 = \rho_1 + \bar{\rho}_1 = 0 \ ,$$
$$\rho_1 = 2\varepsilon_1 = \mu_1 = 2\gamma_1 = -\sqrt{2} K s_1 \ , \qquad \ldots(2.19)$$
$$\bar{\alpha}_1 = -\beta_1 = \frac{1}{\sqrt{2}} K s_\circ \ , \qquad \kappa_1 = \bar{\nu}_1 = -K\sqrt{2}\, s_\circ .$$

3. Tetrad Components of Connection 1-forms and Curvature 2-forms:

Connection 1-forms:

We start with the Cartan's first equation of structure given by (Katkar 2008)

$$d\theta^\alpha = -\omega^\alpha{}_\beta \wedge \theta^\beta + T^\alpha \ , \qquad \alpha, \beta = 1, 2, 3, 4 \qquad \ldots(3.1)$$

where $\omega^\alpha{}_\beta = \gamma^\alpha{}_{\beta\gamma}\theta^\gamma$ are components of connection 1-forms , ...(3.2)

$$T^\alpha = K^\alpha{}_{\beta\gamma}\theta^\beta \wedge \theta^\gamma \qquad \text{are 2-form} \qquad \ldots(3.3)$$

and $K^\alpha{}_{\beta\gamma} = K^k{}_{ij} e^i_{(\beta)} e^j_{(\gamma)} e^{(\alpha)}_k$ are tetrad components of Contortion tensor and it has the following symmetry

$$K_{\alpha(\beta\gamma)} = 0 \qquad \ldots(3.4)$$

By giving different values to α, β, γ from 1 to 4 and using definitions of Contortion tensor $K^\alpha{}_{\beta\gamma}$ described in equation (5.4) of Chapter I, we obtain

$$T^1 = K^1{}_{\beta\gamma}\theta^{\beta\gamma}$$
$$= K_{\beta\gamma\sigma}\eta^{\sigma 1}\theta^{\beta\gamma}$$
$$= K_{\beta\gamma 2}\eta^{21}\theta^{\beta\gamma}$$
$$= K_{\beta\gamma 2}\theta^{\beta\gamma}$$
$$= K_{122}\theta^{12} + K_{132}\theta^{13} + K_{142}\theta^{14} + K_{212}\theta^{21} + K_{232}\theta^{23} + K_{242}\theta^{24} + K_{312}\theta^{31} + $$
$$+ K_{322}\theta^{32} + K_{342}\theta^{34} + K_{412}\theta^{41} + K_{422}\theta^{42} + K_{432}\theta^{43}$$

$$T^1 = -K_{212}\theta^{12} + (K_{132} - K_{312})\theta^{13} + (K_{142} - K_{412})\theta^{14} + K_{232}\theta^{23} + K_{242}\theta^{24} +$$
$$+ (K_{342} - K_{432})\theta^{34}$$

$$T^1 = (\gamma_1 + \bar{\gamma}_1)\theta^{12} + (\bar{\alpha}_1 + \beta_1 - \bar{\pi}_1)\theta^{13} - (\alpha_1 + \bar{\beta}_1 - \pi_1)\theta^{14} - \bar{\nu}_1\theta^{23} - \nu_1\theta^{24} -$$
$$-(\mu_1 - \bar{\mu}_1)\theta^{34}$$

Similarly, we obtain

$$T^2 = (\varepsilon_1 + \bar{\varepsilon}_1)\theta^{12} + \kappa_1\theta^{13} + \bar{\kappa}_1\theta^{14} + (\tau_1 - \bar{\alpha}_1 - \beta_1)\theta^{23} + (\bar{\tau}_1 - \alpha_1 - \bar{\beta}_1)\theta^{24} - -(\rho_1 - \bar{\rho}_1)\theta^{34},$$

$$T^3 = -(\bar{\tau}_1 + \pi_1)\theta^{12} + (\bar{\varepsilon}_1 - \varepsilon_1 - \bar{\rho}_1)\theta^{13} - \bar{\sigma}_1\theta^{14} + (\mu_1 - \gamma_1 + \bar{\gamma}_1)\theta^{23} + \lambda_1\theta^{24} + +(\alpha_1 - \bar{\beta}_1)\theta^{34},$$

$$T^4 = -(\tau_1 + \bar{\pi}_1)\theta^{12} - \sigma_1\theta^{13} + (\varepsilon_1 - \bar{\varepsilon}_1 - \rho_1)\theta^{14} + \bar{\lambda}_1\theta^{23} + (\bar{\mu}_1 - \bar{\gamma}_1 + \gamma_1)\theta^{24} - -(\bar{\alpha}_1 - \beta_1)\theta^{34}$$

...(3.5)

Substituting values of spin coefficients from equation (2.19) in the equation (3.5) we get

$$T^1 = \sqrt{2}Ks_\circ\theta^{23} + \sqrt{2}K\bar{s}_\circ\theta^{24} + 2\sqrt{2}Ks_1\theta^{34},$$

$$T^2 = -\sqrt{2}Ks_\circ\theta^{13} - \sqrt{2}K\bar{s}_\circ\theta^{14} + 2\sqrt{2}Ks_1\theta^{34},$$

$$T^3 = \sqrt{2}K\bar{s}_\circ\theta^{34}, \qquad \text{...(3.6)}$$

$$T^4 = -\sqrt{2}Ks_\circ\theta^{34}.$$

Now, from equation (3.1) we have

$$d\theta^\alpha - T^\alpha = -\omega^\alpha{}_\beta \wedge \theta^\beta$$

For $\alpha, \beta = 1, 2, 3, 4$ the equation gives the set of four equations as

$$d\theta^1 - T^1 = -\omega_1^1 \wedge \theta^1 - \omega_2^1 \wedge \theta^2 - \omega_3^1 \wedge \theta^3 - \omega_4^1 \wedge \theta^4,$$

$$d\theta^2 - T^2 = -\omega_1^2 \wedge \theta^1 - \omega_2^2 \wedge \theta^2 - \omega_3^2 \wedge \theta^3 - \omega_4^2 \wedge \theta^4, \qquad \text{...(3.7)}$$

$$d\theta^3 - T^3 = -\omega_1^3 \wedge \theta^1 - \omega_2^3 \wedge \theta^2 - \omega_3^3 \wedge \theta^3 - \omega_4^3 \wedge \theta^4,$$

$$d\theta^4 - T^4 = -\omega_1^4 \wedge \theta^1 - \omega_2^4 \wedge \theta^2 - \omega_3^4 \wedge \theta^3 - \omega_4^4 \wedge \theta^4.$$

Since $\omega^1{}_2 = \omega_{22} = 0, \qquad \omega^2{}_1 = \omega_{11} = 0,$

$\omega^3{}_4 = -\omega_{44} = 0, \qquad \omega^4{}_3 = -\omega_{33} = 0.$

and

$$\omega^1{}_1 = -\omega^2{}_2 = \omega_{12}, \quad \omega^1{}_3 = \omega^4{}_2 = \omega_{23}, \quad \omega^2{}_3 = \omega^4{}_1 = \omega_{13},$$

$$\omega^3{}_3 = -\omega^4{}_4 = \omega_{34}, \quad \omega^1{}_4 = \omega^3{}_2 = \omega_{24}, \quad \omega^2{}_4 = \omega^3{}_1 = \omega_{14}. \qquad \text{...(3.8)}$$

To find the components of Connection 1-form we choose

$$\omega^1{}_1 = A_1\theta^1 + A_2\theta^2 + A_3\theta^3 + A_4\theta^4,$$

$$\omega^1{}_3 = B_1\theta^1 + B_2\theta^2 + B_3\theta^3 + B_4\theta^4,$$

$$\omega^1{}_4 = C_1\theta^1 + C_2\theta^2 + C_3\theta^3 + C_4\theta^4,$$

$$\omega^2{}_3 = D_1\theta^1 + D_2\theta^2 + D_3\theta^3 + D_4\theta^4, \qquad \text{...(3.9)}$$

$$\omega^2{}_4 = E_1\theta^1 + E_2\theta^2 + E_3\theta^3 + E_4\theta^4,$$

$$\omega^3{}_3 = F_1\theta^1 + F_2\theta^2 + F_3\theta^3 + F_4\theta^4.$$

Using equations (3.8) and (3.9) in (3.7) we obtain

$$d\theta^1 - T^1 = A_2\theta^{12} + (A_3 - B_1)\,\theta^{13} + (A_4 - C_1)\theta^{14} - B_2\theta^{23} - C_2\theta^{24} + (B_4 - C_3)\theta^{34}$$

Similarly, we obtain

$$d\theta^2 - T^2 = A_1\theta^{12} - D_1\theta^{13} - E_1\theta^{14} - (A_3 + D_2)\,\theta^{23} - (A_4 + E_2)\theta^{24} + (D_4 - E_3)\theta^{34},$$

$$d\theta^3 - T^3 = (E_2 - C_1)\theta^{12} + (E_3 - F_1)\,\theta^{13} + E_4\,\theta^{14} + (C_3 - F_2)\theta^{23} + C_4\theta^{24} + F_4\,\theta^{34},$$

$$d\theta^4 - T^4 = (D_2 - B_1)\theta^{12} + D_3\,\theta^{13} + (D_4 + F_1)\,\theta^{14} + B_3\theta^{23} + (B_4 + F_2)\theta^{24} + F_3\,\theta^{34} \qquad \text{...(3.10)}$$

Now, using the definitions (2.10), (3.6) in (3.10) and comparing the corresponding coefficients on both sides, we readily obtain the values of constants as

$$A_1 = -A_2 = \frac{\dot{P}}{\sqrt{2}P}\ ; \qquad B_2 = \sqrt{2}Ks_\circ\ ;$$

$$B_4 = -\sqrt{2}Ks_1 + \frac{1}{\sqrt{2}}R^{-1}\left(\dot{R} - \frac{R'}{P}\right); \qquad C_2 = \sqrt{2}K\bar{s}_\circ\ ;$$

$$C_3 = \sqrt{2}Ks_1 + \frac{1}{\sqrt{2}}R^{-1}\left(\dot{R} - \frac{R'}{P}\right); \qquad D_1 = -\sqrt{2}Ks_\circ;$$

$$D_4 = -\sqrt{2}Ks_1 + \frac{1}{\sqrt{2}}R^{-1}\left(\dot{R} + \frac{R'}{P}\right); \qquad E_1 = -\sqrt{2}K\bar{s}_\circ\ ; \qquad \text{...(3.11)}$$

$$E_3 = \sqrt{2}Ks_1 + \frac{1}{\sqrt{2}}R^{-1}\left(\dot{R} + \frac{R'}{P}\right); \qquad F_1 = F_2 = \sqrt{2}Ks_1\ ;$$

$$F_3 = \sqrt{2}Ks_\circ - \frac{1}{\sqrt{2}}R^{-1}\cot\theta\ ; \qquad F_4 = -\sqrt{2}K\bar{s}_\circ + \frac{1}{\sqrt{2}}R^{-1}\cot\theta$$

and all other are zero.

Consequently, we obtain the non-vanishing tetrad components of Connection 1-form as

$$\omega^1{}_1 = -\omega^2{}_2 = \frac{\dot{P}}{\sqrt{2}P}(\theta^1 - \theta^2) ,$$

$$\omega^1{}_3 = \omega^4{}_2 = \sqrt{2}Ks_\circ\theta^2 + \left[\frac{1}{\sqrt{2}}R^{-1}\left(\dot{R} - \frac{R'}{P}\right) - \sqrt{2}Ks_1\right]\theta^4 ,$$

$$\omega^2{}_3 = \omega^4{}_1 = -\sqrt{2}Ks_\circ\theta^1 + \left[\frac{1}{\sqrt{2}}R^{-1}\left(\dot{R} + \frac{R'}{P}\right) - \sqrt{2}Ks_1\right]\theta^4 \qquad ...(3.12)$$

$$\omega^3{}_3 = -\omega^4{}_4 = 2\sqrt{2}Ks_1\theta^1 + \sqrt{2}Ks_1\theta^2 + \left[-\frac{1}{\sqrt{2}}R^{-1}\cot\theta + \sqrt{2}Ks_\circ\right]\theta^3 + \left[\frac{1}{\sqrt{2}}R^{-1}\cot\theta - \sqrt{2}K\bar{s}_\circ\right]\theta^4 .$$

The expressions for $\omega^1{}_4, \omega^2{}_4$, $\omega^3{}_1$, $\omega^3{}_2$ are obtain just by interchanging 3 and 4 and taking complex conjugate of corresponding terms.

In the U_4 theory of gravitation the expression for Connection 1-forms in NP spin coefficients can be written as

$$\omega^1{}_3 = (\bar{\pi}^\circ + \bar{\pi}_1)\theta^1 + (\bar{\nu}^\circ + \bar{\nu}_1)\theta^2 + (\bar{\lambda}^\circ + \bar{\lambda}_1)\theta^3 + (\bar{\mu}^\circ + \bar{\mu}_1)\theta^4 ,$$

$$\omega^1{}_4 = (\pi^\circ + \pi_1)\theta^1 + (\nu^\circ + \nu_1)\theta^2 + (\mu^\circ + \mu_1)\theta^3 + (\lambda^\circ + \lambda_1)\theta^4 ,$$

$$\omega^2{}_3 = -[(\kappa^\circ + \kappa_1)\theta^1 + (\tau^\circ + \tau_1)\theta^2 + (\sigma^\circ + \sigma_1)\theta^3 + (\rho^\circ + \rho_1)\theta^4] , \qquad ...(3.13)$$

$$\omega^2{}_4 = -[(\bar{\kappa}^\circ + \bar{\kappa}_1)\theta^1 + (\bar{\tau}^\circ + \bar{\tau}_1)\theta^2 + (\bar{\rho}^\circ + \bar{\rho}_1)\theta^3 + (\bar{\sigma}^\circ + \bar{\sigma}_1)\theta^4] ,$$

$$\omega^2{}_2 = -[(\varepsilon^\circ + \varepsilon_1 + \bar{\varepsilon}^\circ + \bar{\varepsilon}_1)\theta^1 + (\gamma^\circ + \gamma_1 + \bar{\gamma}^\circ + \bar{\gamma}_1)\theta^2 + (\bar{\alpha}^\circ + \bar{\alpha}_1 + \beta^\circ + \beta_1)\theta^3 + (\alpha^\circ + \alpha_1 + \bar{\beta}^\circ + \bar{\beta}_1)\theta^4] ,$$

$$\omega^4{}_4 = -[(\varepsilon^\circ + \varepsilon_1 - \bar{\varepsilon}^\circ - \bar{\varepsilon}_1)\theta^1 + (\gamma^\circ + \gamma_1 - \bar{\gamma}^\circ - \bar{\gamma}_1)\theta^2 - (\bar{\alpha}^\circ + \bar{\alpha}_1 - \beta^\circ - \beta_1)\theta^3 + (\alpha^\circ + \alpha_1 - \bar{\beta}^\circ - \bar{\beta}_1)\theta^4] .$$

Using equations (2.19) and (3.12), the NP spin coefficients turn out to be

$$\pi^\circ = \lambda^\circ = \tau^\circ = \sigma^\circ = 0 ,$$

$$\varepsilon^\circ = \sqrt{2}Ks_1 + \frac{\dot{P}}{2\sqrt{2}P} ,$$

$$\gamma^\circ = \sqrt{2}Ks_1 - \frac{\dot{P}}{2\sqrt{2}P} ,$$

$$\bar{\nu}^{\circ} = \kappa^{\circ} = 2\sqrt{2}Ks_{\circ},$$

$$\mu^{\circ} = 2\sqrt{2}Ks_1 + \frac{1}{\sqrt{2}}R^{-1}\left(\dot{R} - \frac{R'}{P}\right),$$

$$\rho^{\circ} = 2\sqrt{2}Ks_1 - \frac{1}{\sqrt{2}}R^{-1}\left(\dot{R} + \frac{R'}{P}\right), \qquad \ldots(3.14)$$

$$\bar{\alpha}^{\circ} = -\beta^{\circ} = -\sqrt{2}Ks_{\circ} + \frac{1}{2\sqrt{2}}R^{-1}\cot\theta \quad .$$

Curvature 2-forms:

To find the tetrad components of Curvature 2-forms, we start with Cartan's Second equations of structure

$$\Omega^{\alpha}{}_{\beta} = d\omega^{\alpha}{}_{\beta} + \omega^{\alpha}{}_{\varepsilon} \wedge \omega^{\varepsilon}{}_{\beta} \qquad \ldots(3..15)$$

where $d\omega^{\alpha}{}_{\beta}$, for different values of $\alpha, \beta = 1, 2, 3, 4$ are obtained by taking the exterior derivatives of Connection 1-forms given in equation (3.12).

Then

$$d\omega^1{}_1 = \frac{1}{\sqrt{2}}d\left(\frac{\dot{P}}{P}\right) \wedge (\theta^1 - \theta^2) + \frac{\dot{P}}{\sqrt{2}P}(d\theta^1 - d\theta^2)$$

$$= \frac{1}{\sqrt{2}}\left[\left(\frac{\ddot{P}}{P} - \frac{\dot{P}^2}{P^2}\right)dt + \left(\frac{\dot{P}'}{P} - \frac{\dot{P}P'}{P^2}\right)dr\right] \wedge (\theta^1 - \theta^2) + \frac{\dot{P}}{\sqrt{2}P}\left[-\sqrt{2}\frac{\dot{P}}{P}\theta^{12}\right]$$

$$d\omega^1{}_1 = -\frac{\ddot{P}}{P}\theta^{12},$$

Similarly, we obtain

$$d\omega^1{}_3 = \left[\sqrt{2}KDs_{\circ} + Ks_{\circ}\frac{\dot{P}}{P}\right]\theta^{12} - \sqrt{2}K\delta s_{\circ}\theta^{23} +$$

$$+\left[\frac{\ddot{R}}{2R} + \frac{\dot{P}R'}{2P^2R} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} - Ks_1\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) - \sqrt{2}KDs_1\right]\theta^{14} -$$

$$+\left[\begin{array}{l}\frac{\ddot{R}}{2R} - \frac{\dot{R}'}{PR} + \frac{\dot{P}R'}{2P^2R} - \frac{P'R'}{2P^3R} + \frac{R''}{2P^2R} - \sqrt{2}K\Delta s_1 - \\ -\sqrt{2}K\bar{\delta}s_{\circ} - Ks_1\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right)\end{array}\right]\theta^{24} +$$

$$+\left[-\frac{\dot{R}\cot\theta}{2R^2} + \frac{R'\cot\theta}{2PR^2} - \sqrt{2}K\delta s_1 + Ks_1\frac{\cot\theta}{R}\right]\theta^{34}$$

$$d\omega^2{}_3 = \left[\sqrt{2}K\Delta s_\circ + Ks_\circ \frac{\dot{P}}{P}\right]\theta^{12} + \sqrt{2}K\delta s_\circ \theta^{13} +$$

$$+\left[\frac{\ddot{R}}{2R} + \frac{\dot{R}'}{PR} - \frac{\dot{P}R'}{2P^2R} - \frac{P'R'}{2P^3R} + \frac{R''}{2P^2R} - \sqrt{2}KDs_1 + \sqrt{2}K\bar{\delta}s_\circ - Ks_1\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right)\right]\theta^{14} +$$

$$+\left[\frac{\ddot{R}}{2R} - \frac{\dot{P}R'}{2P^2R} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} - Ks_1\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) - \sqrt{2}K\Delta s_1\right]\theta^{24} -$$

$$+\left[-\frac{\dot{R}\cot\theta}{2R^2} - \frac{R'\cot\theta}{2PR^2} - \sqrt{2}K\delta s_1 + Ks_1\frac{\cot\theta}{R}\right]\theta^{34}$$

,

$$d\omega^3{}_3 = \sqrt{2}K(Ds_1 - \Delta s_1)\theta^{12} + \left[\sqrt{2}K(Ds_\circ - \delta s_1) + Ks_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right)\right]\theta^{13} +$$

$$+\left[-\sqrt{2}K(D\bar{s}_\circ + \bar{\delta}s_1) - K\bar{s}_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right)\right]\theta^{14} +$$

$$+\left[\sqrt{2}K(\Delta s_\circ - \delta s_1) + Ks_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right)\right]\theta^{23} + \qquad \ldots(3.16)$$

$$+\left[-\sqrt{2}K(\Delta\bar{s}_\circ + \bar{\delta}s_1) - K\bar{s}_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right)\right]\theta^{24} +$$

$$+\left[-\sqrt{2}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) - \frac{\cot^2\theta}{R^2} + K(s_\circ + \bar{s}_\circ)\frac{\cot\theta}{R}\right]\theta^{34}$$

and $d\omega^1{}_4$, $d\omega^2{}_4$ are obtain by interchange 3 and 4 and taking complex conjugate of corresponding terms.

Now from Cartan's Second equation of structure (3.15) we have for $\alpha = \beta = 1$,

$$\Omega^1{}_1 = d\omega^1{}_1 + \omega^1{}_\varepsilon \wedge \omega^\varepsilon{}_1$$
$$= d\omega^1{}_1 + \omega^1{}_1 \wedge \omega^1{}_1 + \omega^1{}_3 \wedge \omega^3{}_1 + \omega^1{}_4 \wedge \omega^4{}_1$$

Using values of Connection 1-form from (3.12) into (3.16) we get

$$\Omega^1{}_1 = -\frac{\dot{P}}{P}\theta^{12} + 2K^2 s_\circ \bar{s}_\circ \theta^{12} + \sqrt{2}Ks_\circ\left[\frac{1}{\sqrt{2}}\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) + \sqrt{2}Ks_1\right]\theta^{23} +$$

$$+\sqrt{2}K\bar{s}_\circ\left[\frac{1}{\sqrt{2}}\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) - \sqrt{2}Ks_1\right]\theta^{14} + \sqrt{2}Ks_\circ\left[\frac{1}{\sqrt{2}}\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) + \sqrt{2}Ks_1\right]\theta^{14} -$$

$$-\left[\frac{1}{\sqrt{2}}\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) - \sqrt{2}Ks_1\right]\left[\frac{1}{\sqrt{2}}\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) + \sqrt{2}Ks_1\right]\theta^{34} + 2K^2 s_\circ \bar{s}_\circ \theta^{12} +$$

$$+2K^2 s_\circ \bar{s}_\circ \theta^{12} + \sqrt{2}K\bar{s}_\circ\left[\frac{1}{\sqrt{2}}\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)-\sqrt{2}Ks_1\right]\theta^{24}$$

$$+\left[\frac{1}{\sqrt{2}}\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)+\sqrt{2}Ks_1\right]\left[\frac{1}{\sqrt{2}}\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)-\sqrt{2}Ks_1\right]\theta^{34}$$

On simplifying we get

$$\Omega^1{}_1 = -\Omega^2{}_2 = \left[-\frac{\ddot{P}}{P}+4K^2 s_\circ \bar{s}_\circ\right]\theta^{12} + \left[Ks_\circ\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)+2K^2 s_\circ s_1\right]\theta^{13} +$$

$$+\left[K\bar{s}_\circ\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)-2K^2\bar{s}_\circ s_1\right]\theta^{14} + \left[Ks_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)+2K^2 s_\circ s_1\right]\theta^{23} +$$

$$+\left[K\bar{s}_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)-2K^2\bar{s}_\circ s_1\right]\theta^{24} + \left[4Ks_1\frac{R'}{PR}\right]\theta^{34}$$

Similarly, we obtain

$$\Omega^1{}_3 = \Omega^4{}_2 = \left[\sqrt{2}KDs_\circ - 2K^2 s_\circ s_1 + 2Ks_\circ\frac{\dot{P}}{P}\right]\theta^{12} +$$

$$+\left[-\sqrt{2}K\delta s_\circ - Ks_\circ\frac{\cot\theta}{R}+2K^2 s_\circ{}^2\right]\theta^{23} +$$

$$+\left[\begin{array}{l}-\sqrt{2}KDs_1 - 2Ks_1\frac{\dot{R}}{R}-Ks_1\frac{\dot{P}}{P}+2K^2 s_1{}^2 + \\ +\frac{\ddot{R}}{2R}+\frac{\dot{P}\dot{R}}{2PR}+\frac{P'R'}{2P^3R}-\frac{R''}{2P^2R}\end{array}\right]\theta^{14} +$$

$$+\left[-\sqrt{2}K\delta s_1 + 2K^2 s_\circ s_1 - Ks_\circ\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)\right]\theta^{34} +$$

$$+\left[\begin{array}{l}-\sqrt{2}K(\Delta s_1+\bar{\delta}s_\circ)-2Ks_1\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)+Ks_1\frac{\dot{P}}{P}+ \\ +Ks_\circ\frac{\cot\theta}{R}-2K^2 s_\circ\bar{s}_\circ+2K^2 s_1{}^2+\frac{\ddot{R}}{2R}-\frac{\dot{P}\dot{R}}{2PR}- \\ -\frac{P'R'}{2P^3R}+\frac{R''}{2P^2R}-\frac{\dot{R}'}{PR}+\frac{\dot{P}R'}{P^2R}\end{array}\right]\theta^{24},$$

$$\Omega^2{}_3 = \Omega^4{}_1 = \left[\sqrt{2}K\Delta s_\circ - 2K^2 s_\circ s_1 + 2Ks_\circ\frac{\dot{P}}{P}\right]\theta^{12} +$$

$$+\left[\sqrt{2}K\delta s_\circ - 2K^2 s^2{}_\circ + Ks_\circ\frac{\cot\theta}{R}\right]\theta^{13} +$$

$$+\begin{bmatrix} -\sqrt{2}K(Ds_1-\bar{\delta}s_\circ)-2Ks_1\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)+Ks_1\frac{\dot{P}}{P}- \\ -Ks_\circ\frac{\cot\theta}{R}+2K^2s_\circ\bar{s}_\circ+2K^2s_1{}^2+\frac{\ddot{R}}{2R}-\frac{\dot{P}\dot{R}}{2PR}- \\ -\frac{P'R'}{2P^3R}+\frac{R''}{2P^2R}+\frac{\dot{R}'}{PR}-\frac{\dot{P}R'}{P^2R} \end{bmatrix}\theta^{14}+$$

$$+\begin{bmatrix} -\sqrt{2}K\Delta s_1-2Ks_1\frac{\dot{R}}{R}-Ks_1\frac{\dot{P}}{P}+2K^2s_1{}^2+ \\ +\frac{\ddot{R}}{2R}+\frac{\dot{P}\dot{R}}{2PR}+\frac{P'R'}{2P^3R}-\frac{R''}{2P^2R} \end{bmatrix}\theta^{24}+ \qquad , \qquad \ldots(3.17)$$

$$+\left[-\sqrt{2}K\delta s_1+2K^2s_\circ s_1-Ks_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)\right]\theta^{34}$$

$$\Omega^3{}_3=-\Omega^4{}_4=\left[\sqrt{2}K(Ds_1-\Delta s_1)\right]\theta^{12}+\left[\sqrt{2}K(Ds_\circ-\delta s_1)+2Ks_\circ\frac{\dot{R}}{R}+2K^2s_\circ s_1\right]\theta^{13}+$$

$$+\left[-\sqrt{2}K(D\bar{s}_\circ+\bar{\delta}s_1)-2K\bar{s}_\circ\frac{\dot{R}}{R}+2K^2\bar{s}_\circ s_1\right]\theta^{14}+$$

$$+\left[\sqrt{2}K(\Delta s_\circ-\delta s_1)-2Ks_\circ\frac{R'}{PR}-2K^2s_\circ s_1\right]\theta^{23}+$$

$$+\left[-\sqrt{2}K(\Delta\bar{s}_\circ+\bar{\delta}s_1)+2K\bar{s}_\circ\frac{R'}{PR}-2K^2\bar{s}_\circ s_1\right]\theta^{24}$$

$$+\begin{bmatrix} -\sqrt{2}K(\delta\bar{s}_\circ+\bar{\delta}s_\circ)+K(s_\circ+\bar{s}_\circ)\frac{\cot\theta}{R}- \\ -4K^2s_1{}^2-\frac{\cot^2\theta}{R^2}+\frac{\dot{R}^2}{R^2}-\frac{R'^2}{P^2R^2} \end{bmatrix}\theta^{34}$$

4. Tetrad Components of Curvature tensor, Ricci tensor, Curvature scalar and Weyl tensor:

We know the Curvature 2-form is defined as

$$\Omega^\alpha{}_\beta=\frac{1}{2}R^\alpha{}_{\beta\gamma\delta}\,\theta^{\gamma\delta}\ , \qquad \alpha,\beta,\gamma,\delta=1,2,3,4 \qquad \ldots(4.1)$$

By giving different values to $\alpha,\beta,\gamma,\delta$ from 1 to 4 in equation (4.1) we obtain non-vanishing tetrad components of Curvature tensor as

$$\Omega^1{}_1=\frac{1}{2}R^1{}_{1\gamma\delta}\,\theta^{\gamma\delta}$$

$$\Omega^1{}_1 = \frac{1}{2}\,\eta^{12} R_{21\gamma\delta}\,\theta^{\gamma\delta}$$

$$= \frac{1}{2}[R_{2112}\theta^{12} + R_{2113}\theta^{13} + R_{2114}\theta^{14} + R_{2121}\theta^{21} + R_{2123}\theta^{23} + R_{2124}\theta^{24} + R_{2131}\theta^{31} +$$

$$+ R_{2132}\theta^{32} + R_{2134}\theta^{34} + R_{2141}\theta^{41} + R_{2142}\theta^{42} + R_{2143}\theta^{43}]$$

$$\Omega^1{}_1 = -R_{1212}\,\theta^{12} - R_{1213}\,\theta^{13} - R_{1214}\,\theta^{14} - R_{1223}\,\theta^{23} - R_{1224}\,\theta^{24} - R_{1234}\,\theta^{34} \qquad ...(4.2)$$

Using equation (3.17) in (4.2) and comparing corresponding terms on both sides we obtain the following non-vanishing components of Curvature tensor,

$$R_{1212} = \frac{\ddot{P}}{P} - 4K^2 s_\circ \bar{s}_\circ \,,$$

$$R_{1213} = -Ks_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) - 2K^2 s_\circ s_1 \,,$$

$$R_{1214} = -K\bar{s}_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) + 2K^2 \bar{s}_\circ s_1 \,,$$

$$R_{1223} = -Ks_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) - 2K^2 s_\circ s_1 \,,$$

$$R_{1224} = -K\bar{s}_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) + 2K^2 \bar{s}_\circ s_1 \,,$$

$$R_{1234} = -4Ks_1 \frac{R'}{PR} \,.$$

Similarly, we get

$$R_{1312} = \sqrt{2}K\Delta s_\circ + 2Ks_\circ \frac{\dot{P}}{P} - 2K^2 s_\circ s_1 \,,$$

$$R_{1313} = \sqrt{2}K\delta s_\circ - 2K^2 s^2{}_\circ + Ks_\circ \frac{\cot\theta}{R} \,,$$

$$R_{1314} = -\sqrt{2}K(Ds_1 - \bar{\delta} s_\circ) - 2Ks_1\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) + Ks_1\frac{\dot{P}}{P} - Ks_\circ \frac{\cot\theta}{R} + 2K^2 s_\circ \bar{s}_\circ + $$
$$+ 2K^2 s_1{}^2 + \frac{\ddot{R}}{2R} - \frac{P'R'}{2P^3 R} - \frac{\dot{P}\dot{R}}{2PR} + \frac{R''}{2P^2 R} + \frac{\dot{R}'}{PR} - \frac{\dot{P}R'}{P^2 R} \,,$$

$$R_{1324} = -\sqrt{2}K\Delta s_1 - 2Ks_1\frac{\dot{R}}{R} - Ks_1\frac{\dot{P}}{P} + 2K^2 s_1{}^2 + \frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \frac{P'R'}{2P^3 R} - \frac{R''}{2P^2 R} \,,$$

$$R_{1334}=-\sqrt{2}K\delta s_1+2K^2 s_\circ s_1-Ks_\circ\left(\frac{\dot R}{R}+\frac{R'}{PR}\right)\ ,$$

$$R_{2312}=\sqrt{2}KDs_\circ-2K^2 s_\circ s_1+2Ks_\circ\frac{\dot P}{P}\ ,$$

$$R_{2314}=-\sqrt{2}KDs_1-2Ks_1\frac{\dot R}{R}-Ks_1\frac{\dot P}{P}+2K^2 s_1{}^2+\frac{\ddot R}{2R}+\frac{\dot P\dot R}{2PR}+\frac{P'R'}{2P^3R}-\frac{R''}{2P^2R}\ ,$$

$$R_{2323}=-\sqrt{2}K\delta s_\circ+2K^2 s_\circ{}^2-Ks_\circ\frac{\cot\theta}{R}\ ,$$

$$R_{2324}=-\sqrt{2}K(\Delta s_1+\bar\delta s_\circ)-2Ks_1\left(\frac{\dot R}{R}-\frac{R'}{PR}\right)+Ks_1\frac{\dot P}{P}+Ks_\circ\frac{\cot\theta}{R}-2K^2 s_\circ\bar s_\circ+$$
$$+2K^2 s_1{}^2+\frac{\ddot R}{2R}-\frac{\dot P\dot R}{2PR}-\frac{P'R'}{2P^3R}+\frac{R''}{2P^2R}-\frac{\dot R'}{PR}+\frac{\dot PR'}{P^2R}$$

$$R_{2334}=-\sqrt{2}K\delta s_1+2K^2 s_\circ s_1-Ks_\circ\left(\frac{\dot R}{R}-\frac{R'}{PR}\right)\ ,$$

$$R_{3412}=\sqrt{2}K(Ds_1-\Delta s_1)\ ,$$

$$R_{3413}=\sqrt{2}K(Ds_\circ-\delta s_1)+2K^2 s_\circ s_1+2Ks_\circ\frac{\dot R}{R}\ ,$$

$$R_{3414}=-\sqrt{2}K(D\bar s_\circ+\bar\delta s_1)+2K^2\bar s_\circ s_1-2K\bar s_\circ\frac{\dot R}{R}\ ,$$

$$R_{3423}=\sqrt{2}K(\Delta s_\circ-\delta s_1)-2K^2 s_\circ s_1-2Ks_\circ\frac{R'}{PR},\qquad \dots (4.3)$$

$$R_{3424}=-\sqrt{2}K(\Delta\bar s_\circ+\bar\delta s_1)-2K^2\bar s_\circ s_1+2K\bar s_\circ\frac{R'}{PR}\ ,$$

$$R_{3434}=-\sqrt{2}K(\delta\bar s_\circ+\bar\delta s_\circ)+K(s_\circ+\bar s_\circ)\frac{\cot\theta}{R}-4K^2 s_1{}^2-\frac{\cot^2\theta}{R^2}+\frac{\dot R^2}{R^2}-\frac{R'^2}{P^2R^2}$$

and $R_{1323}=R_{2313}=R_{1424}=R_{2414}=0$

where the terms R_{1412}, R_{1413}, R_{1414}, R_{1423}, R_{1424}, R_{1434}, R_{2412} , R_{2413}, R_{2414}, R_{2423}, R_{2424}, R_{2434} are just complex conjugates of R_{1312}, R_{1313}, R_{1314}, R_{1323}, R_{1324} , R_{1334}, R_{2312} , R_{2313}, R_{2314}, R_{2323}, R_{2324}, R_{2334} and are obtain by interchanging 3 and 4 and taking complex conjugates of the respective expressions and all other are zero. We note that $R_{\alpha\beta\gamma\delta}\neq R_{\gamma\delta\alpha\beta}$.

The tetrad components of Ricci tensor $R_{\alpha\beta}$ and Ricci scalar R are defined as

$$R_{\alpha\beta} = \eta^{\sigma\delta} R_{\sigma\alpha\beta\delta} \qquad \alpha,\beta,\sigma,\delta = 1,2,3,4$$

$$R_{\alpha\beta} = R_{1\alpha\beta 2} + R_{2\alpha\beta 1} - R_{3\alpha\beta 4} - R_{4\alpha\beta 3} \qquad \ldots(4.4)$$

and $\quad R = \eta^{\alpha\beta} R_{\alpha\beta} = R_{12} + R_{21} - R_{34} - R_{43}$

Therefore the non-vanishing tetrad components of Ricci tensor and Ricci scalar are

$$R_{11} = \sqrt{2}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) - K(s_\circ + \bar{s}_\circ)\frac{\cot\theta}{R} + 4K^2 s_\circ\bar{s}_\circ + 4K^2 s_1{}^2 + \frac{\ddot{R}}{R} - \frac{\dot{P}\dot{R}}{PR} - \frac{P'R'}{P^3R} + $$
$$+\frac{R''}{P^2R} + \frac{2\dot{R}'}{PR} - \frac{2\dot{P}R'}{P^2R} ,$$

$$R_{12} = R_{21} = -4K^2 s_\circ\bar{s}_\circ + 4K^2 s_1{}^2 + \frac{\ddot{P}}{P} + \frac{\ddot{R}}{R} + \frac{\dot{P}\dot{R}}{PR} + \frac{P'R'}{P^3R} - \frac{R''}{P^2R} ,$$

$$R_{13} = -\sqrt{2}K\delta s_1 - 2Ks_\circ\frac{\dot{R}}{R} ,$$

$$R_{14} = \sqrt{2}K\bar{\delta}s_1 - 2K\bar{s}_\circ\frac{\dot{R}}{R} ,$$

$$R_{22} = -\sqrt{2}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) + K(s_\circ + \bar{s}_\circ)\frac{\cot\theta}{R} - 4K^2 s_\circ\bar{s}_\circ + 4K^2 s_1{}^2 + \frac{\ddot{R}}{R} - \frac{\dot{P}\dot{R}}{PR} - $$
$$-\frac{P'R'}{P^3R} + \frac{R''}{P^2R} - \frac{2\dot{R}'}{PR} + \frac{2\dot{P}R'}{P^2R} ,$$

$$R_{23} = -\sqrt{2}K\delta s_1 + 4K^2 s_\circ s_1 + 2Ks_\circ\frac{R'}{PR} ,$$

$$R_{24} = \sqrt{2}K\bar{\delta}s_1 - 4K^2\bar{s}_\circ s_1 + 2K\bar{s}_\circ\frac{R'}{PR} ,$$

$$R_{31} = \sqrt{2}K(Ds_\circ + \Delta s_\circ - \delta s_1) + 2Ks_\circ\left(\frac{\dot{P}}{P} + \frac{\dot{R}}{R}\right) , \qquad \ldots(4.5)$$

$$R_{32} = -\sqrt{2}K(Ds_\circ - \Delta s_\circ + \delta s_1) - 2Ks_\circ\left(\frac{\dot{P}}{P} + \frac{R'}{PR}\right),$$

$$R_{34} = \sqrt{2}K(Ds_1 + \Delta s_1 + \delta\bar{s}_\circ + \bar{\delta}s_\circ) - K(s_\circ + \bar{s}_\circ)\frac{\cot\theta}{R} + 4Ks_1\frac{\dot{R}}{R} + 2Ks_1\frac{\dot{P}}{P} - \frac{\ddot{R}}{R} - $$
$$-\frac{\dot{P}\dot{R}}{PR} - \frac{P'R'}{P^3R} + \frac{R''}{P^2R} - \frac{\dot{R}^2}{R^2} + \frac{R'^2}{P^2R^2} + \frac{\cot^2\theta}{R^2}$$

and

$$R=-2\sqrt{2}K(\delta\bar{s}_\circ+\bar{\delta}s_\circ)+2K(s_\circ+\bar{s}_\circ)\frac{\cot\theta}{R}+8K^2s_1{}^2-8K^2s_\circ\bar{s}_\circ+\frac{2\ddot{P}}{P}+\frac{4\ddot{R}}{R}+\frac{4\dot{P}\dot{R}}{PR}+$$
$$+\frac{4P'R'}{P^3R}-\frac{4R''}{P^2R}+\frac{2\dot{R}^2}{R^2}-\frac{2R'^2}{P^2R^2}-\frac{2\cot^2\theta}{R^2}$$

...(4.6)

The tetrad components of Weyl tensor are given by

$$C_{\alpha\beta\gamma\delta}=R_{\alpha\beta\gamma\delta}+\frac{1}{2}(\eta_{\alpha\gamma}R_{\beta\delta}+\eta_{\beta\delta}R_{\alpha\gamma}-\eta_{\alpha\delta}R_{\beta\gamma}-\eta_{\beta\gamma}R_{\alpha\delta})+\frac{R}{6}(\eta_{\alpha\delta}\eta_{\beta\gamma}-\eta_{\alpha\gamma}\eta_{\beta\delta})$$

...(4.7)

By giving different values to α, β, γ, δ from 1 to 4, we obtain the tetrad components of Weyl tensor as

$$C_{1212}=R_{1212}+\frac{1}{2}(-\eta_{12}R_{21}-\eta_{21}R_{12})+\frac{R}{6}\eta_{12}\eta_{21}$$

$$C_{1212}=R_{1212}+\frac{1}{2}(-R_{21}-R_{12})+\frac{R}{6}$$

$$C_{1212}=\frac{\ddot{P}}{P}-4K^2s_\circ\bar{s}_\circ+\frac{1}{2}\begin{pmatrix}8K^2s_\circ\bar{s}_\circ-8K^2s_1{}^2-\frac{2\ddot{P}}{P}-\frac{2\ddot{R}}{R}-\\-\frac{2\dot{P}\dot{R}}{PR}-\frac{2P'R'}{P^3R}+\frac{2R''}{P^2R}\end{pmatrix}+$$
$$+\frac{1}{6}\begin{pmatrix}-2\sqrt{2}K(\delta\bar{s}_\circ+\bar{\delta}s_\circ)+2K(s_\circ+\bar{s}_\circ)\frac{\cot\theta}{R}+8K^2s_1{}^2-8K^2s_\circ\bar{s}_\circ+\\+\frac{2\ddot{P}}{P}+\frac{4\ddot{R}}{R}+\frac{4\dot{P}\dot{R}}{PR}+\frac{4P'R'}{P^3R}-\frac{4R''}{P^2R}+\frac{2\dot{R}^2}{R^2}-\frac{2R'^2}{P^2R^2}-\frac{2\cot^2\theta}{R^2}\end{pmatrix}$$

$$C_{1212}=C_{3434}=\frac{1}{3}\begin{bmatrix}-\sqrt{2}K(\delta\bar{s}_\circ+\bar{\delta}s_\circ)+K(s_\circ+\bar{s}_\circ)\frac{\cot\theta}{R}-4K^2s_\circ\bar{s}_\circ-8K^2s_1{}^2+\\+\frac{\ddot{P}}{P}-\frac{\ddot{R}}{R}-\frac{\dot{P}\dot{R}}{PR}-\frac{P'R'}{P^3R}+\frac{R''}{P^2R}+\frac{\dot{R}^2}{3R^2}-\frac{R'^2}{3P^2R^2}-\frac{\cot^2\theta}{3R^2}\end{bmatrix}$$

Likewise, we get

$$C_{1213}=-C_{1334}=\frac{1}{\sqrt{2}}K\delta s_1-2K^2s_\circ s_1+Ks_\circ\frac{R'}{PR}\quad ,$$

$$C_{1214}=C_{1434}=-\frac{1}{\sqrt{2}}K\bar{\delta}s_1+2K^2\bar{s}_\circ s_1+K\bar{s}_\circ\frac{R'}{PR}\ ,$$

$$C_{1223}=C_{2334}=-\frac{1}{\sqrt{2}}K\delta s_1-Ks_\circ\frac{\dot{R}}{R}\ ,$$

$$C_{1224}=-C_{2434}=\frac{1}{\sqrt{2}}K\bar{\delta}s_1-K\bar{s}_\circ\frac{\dot{R}}{R}\,,$$

$$C_{1234}=-4Ks_1\frac{R'}{PR}\,,$$

$$C_{1312}=-C_{3413}=-\frac{1}{\sqrt{2}}K(Ds_\circ-\Delta s_\circ-\delta s_1)-2K^2s_\circ s_1+Ks_\circ\left(\frac{\dot{P}}{P}-\frac{\dot{R}}{R}\right),$$

$$C_{1313}=-C_{2323}=\sqrt{2}K\delta s_\circ-2K^2s_\circ{}^2+Ks_\circ\frac{\cot\theta}{R}\,,$$

$$C_{1314}=-C_{1413}=-\frac{1}{\sqrt{2}}K(\delta\bar{s}_\circ-\bar{\delta}s_\circ+2Ds_1)-2Ks_1\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)+ +Ks_1\frac{\dot{P}}{P}-\frac{K(s_\circ-\bar{s}_\circ)}{2}\frac{\cot\theta}{R}\,,$$

$$C_{1324}=C_{2413}=\frac{1}{\sqrt{2}}K(Ds_1-\Delta s_1)+\frac{1}{3\sqrt{2}}K(\delta\bar{s}_\circ+\bar{\delta}s_\circ)-\frac{K(s_\circ+\bar{s}_\circ)}{6}\frac{\cot\theta}{R}+ +\frac{4}{3}K^2s_1{}^2+\frac{2}{3}K^2s_\circ\bar{s}_\circ-\frac{\ddot{P}}{6P}+\frac{\ddot{R}}{6R}+\frac{\dot{P}\dot{R}}{6PR}+\frac{P'R'}{6P^3R}-\frac{R''}{6P^2R}- -\frac{\dot{R}^2}{6R^2}+\frac{R'^2}{6P^2R^2}+\frac{\cot^2\theta}{6R^2}\,,$$

$$C_{1414}=-C_{2424}=\sqrt{2}K\bar{\delta}\bar{s}_\circ-2K^2\bar{s}_\circ{}^2+K\bar{s}_\circ\frac{\cot\theta}{R}\,,$$

$$C_{2312}=\frac{1}{\sqrt{2}}K(Ds_\circ-\Delta s_\circ+\delta s_1)-2K^2s_\circ s_1+Ks_\circ\left(\frac{\dot{P}}{P}-\frac{R'}{PR}\right),$$

$$C_{2314}=C_{1423}=-\frac{1}{\sqrt{2}}K(Ds_1-\Delta s_1)+\frac{1}{3\sqrt{2}}K(\delta\bar{s}_\circ+\bar{\delta}s_\circ)-\frac{K(s_\circ+\bar{s}_\circ)}{6}\frac{\cot\theta}{R}+ +\frac{4}{3}K^2s_1{}^2+\frac{2}{3}K^2s_\circ\bar{s}_\circ-\frac{\ddot{P}}{6P}+\frac{\ddot{R}}{6R}+\frac{\dot{P}\dot{R}}{6PR}+\frac{P'R'}{6P^3R}-\frac{R''}{6P^2R}- -\frac{\dot{R}^2}{6R^2}+\frac{R'^2}{6P^2R^2}+\frac{\cot^2\theta}{6R^2}\,,$$

$$C_{2324}=-C_{2423}=\frac{1}{\sqrt{2}}K(\delta\bar{s}_\circ-\bar{\delta}s_\circ-2\Delta s_1)+\frac{K(s_\circ-\bar{s}_\circ)}{2}\frac{\cot\theta}{R}-2Ks_1\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)+Ks_1\frac{\dot{P}}{P}$$

$$C_{2412}=\frac{1}{\sqrt{2}}K(D\bar{s}_\circ-\Delta\bar{s}_\circ-\bar{\delta}s_1)+2K^2\bar{s}_\circ s_1+K\bar{s}_\circ\left(\frac{\dot{P}}{P}-\frac{R'}{PR}\right),$$

$$C_{3412} = \sqrt{2}K(Ds_1 - \Delta s_1),$$

$$C_{3414} = -C_{1412} = -\frac{1}{\sqrt{2}}K(D\bar{s}_\circ - \Delta\bar{s}_\circ + \bar{\delta}s_1) + 2K^2\bar{s}_\circ s_1 + K\bar{s}_\circ\left(\frac{\dot{P}}{P} - \frac{\dot{R}}{R}\right),$$

$$C_{3423} = \frac{1}{\sqrt{2}}K(Ds_\circ + \Delta s_\circ - \delta s_1) - 2K^2 s_\circ s_1 + Ks_\circ\left(\frac{\dot{P}}{P} - \frac{R'}{PR}\right), \quad \ldots(4.8)$$

$$C_{3424} = -\frac{1}{\sqrt{2}}K(D\bar{s}_\circ + \Delta\bar{s}_\circ + \bar{\delta}s_1) - 2K^2\bar{s}_\circ s_1 + K\bar{s}_\circ\left(\frac{R'}{PR} - \frac{\dot{P}}{P}\right)$$

and $C_{1323} = C_{2313} = C_{1424} = C_{2414} = 0$.

From equations (4.8) and equations (5.7) and (5.8) in Chapter 1, we obtain

$$\phi_{\circ\circ} = -\frac{1}{\sqrt{2}}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) + \frac{K(s_\circ + \bar{s}_\circ)}{2}\frac{\cot\theta}{R} - 2K^2 s_\circ\bar{s}_\circ - 2K^2 s_1{}^2 - \frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \\ + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} - \frac{\dot{R}'}{PR} + \frac{\dot{P}R'}{P^2R},$$

$$\phi_{\circ 1} = -\frac{1}{2\sqrt{2}}K(Ds_\circ + \Delta s_\circ - 2\delta s_1) - \frac{1}{2}Ks_\circ\frac{\dot{P}}{P},$$

$$\phi_{\circ 2} = 0,$$

$$\phi_{11} = -\frac{1}{2\sqrt{2}}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) + \frac{K(s_\circ + \bar{s}_\circ)}{4}\frac{\cot\theta}{R} + K^2 s_\circ\bar{s}_\circ - K^2 s_1{}^2 - \frac{\ddot{P}}{4P} + \frac{\dot{R}^2}{4R^2} - \\ - \frac{R'^2}{4P^2R^2} - \frac{\cot^2\theta}{4R^2},$$

$$\phi_{12} = \frac{1}{2\sqrt{2}}K(Ds_\circ - \Delta s_\circ + 2\delta s_1) - K^2 s_\circ s_1 + \frac{1}{2}Ks_\circ\frac{\dot{P}}{P},$$

$$\phi_{22} = \frac{1}{\sqrt{2}}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) - \frac{K(s_\circ + \bar{s}_\circ)}{2}\frac{\cot\theta}{R} + 2K^2 s_\circ\bar{s}_\circ - 2K^2 s_1{}^2 - \frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \\ + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} + \frac{\dot{R}'}{PR} - \frac{\dot{P}R'}{P^2R},$$

$$\phi_\circ = \frac{1}{2\sqrt{2}}K(Ds_\circ + \Delta s_\circ) + Ks_\circ\frac{\dot{R}}{R} + \frac{1}{2}Ks_\circ\frac{\dot{P}}{P},$$

$$\phi_1 = \frac{1}{\sqrt{2}}K(Ds_1 + \Delta s_1) + 2Ks_1\frac{\dot{R}}{R} + Ks_1\frac{\dot{P}}{P},$$

$$\phi_2 = \frac{1}{2\sqrt{2}}K(D\bar{s}_\circ - \Delta\bar{s}_\circ) + K\bar{s}_\circ\frac{R'}{PR} + \frac{1}{2}K\bar{s}_\circ\frac{\dot{P}}{P} - K^2\bar{s}_\circ s_1,$$

$$\psi_\circ = -\sqrt{2}K\delta s_\circ - Ks_\circ\frac{\cot\theta}{R} + 2K^2 s_\circ{}^2 ,$$

$$\psi_1 = \frac{1}{2\sqrt{2}}K(Ds_\circ - \Delta s_\circ - 2\delta s_1) + \frac{1}{2}Ks_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR} - \frac{\dot{P}}{P}\right) + 2K^2 s_\circ s_1 ,$$

$$\psi_2 = \frac{1}{\sqrt{2}}K(Ds_1 - \Delta s_1) + \frac{1}{3\sqrt{2}}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) - \frac{K(s_\circ + \bar{s}_\circ)}{6}\frac{\cot\theta}{R} + \frac{4}{3}K^2 s_1{}^2 +$$
$$+\frac{2}{3}K^2 s_\circ\bar{s}_\circ - \frac{\ddot{P}}{6P} + \frac{\ddot{R}}{6R} + \frac{\dot{P}\dot{R}}{6PR} + \frac{P'R'}{6P^3R} - \frac{R''}{6P^2R} - \frac{\dot{R}^2}{6R^2} + \frac{R'^2}{6P^2R^2} + \frac{\cot^2\theta}{6R^2}$$

$$\psi_3 = \frac{1}{2\sqrt{2}}K(D\bar{s}_\circ + \Delta\bar{s}_\circ + 2\bar{\delta}s_1) - \frac{1}{2}K\bar{s}_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR} - \frac{\dot{P}}{P}\right) + K^2\bar{s}_\circ s_1 ,$$

$$\psi_4 = \sqrt{2}K\bar{\delta}\bar{s}_\circ + K\bar{s}_\circ\frac{\cot\theta}{R} - 2K^2\bar{s}_\circ{}^2 ,$$

$$\Theta_{\circ\circ} = \frac{i}{\sqrt{2}}K(2Ds_1 + \delta\bar{s}_\circ - \bar{\delta}s_\circ) + \frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R} + 2iKs_1\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) - iKs_1\frac{\dot{P}}{P},$$

$$\Theta_{\circ 1} = \frac{i}{2\sqrt{2}}K(Ds_\circ - \Delta s_\circ) + \frac{i}{2}Ks_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR} - \frac{\dot{P}}{P}\right), \qquad ...(4.9)$$

$$\Theta_{\circ 2} = 0 ,$$

$$\Theta_{11} = \frac{i}{2\sqrt{2}}K(Ds_1 - \Delta s_1) + iKs_1\frac{R'}{PR} ,$$

$$\Theta_{12} = -\frac{i}{2\sqrt{2}}K(Ds_\circ - \Delta s_\circ + 2\delta s_1) - \frac{i}{2}Ks_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR} + \frac{\dot{P}}{P}\right) + iK^2 s_\circ s_1 ,$$

$$\Theta_{22} = \frac{i}{\sqrt{2}}K(\delta\bar{s}_\circ - \bar{\delta}s_\circ + 2\Delta s_1) + \frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R} - 2iKs_1\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) + iKs_1\frac{\dot{P}}{P},$$

$$\chi = -\frac{i}{\sqrt{2}}K(Ds_1 - \Delta s_1) .$$

The components of $\phi_{1\circ}, \phi_{21}$ and $\Theta_{1\circ}$, Θ_{21} are obtain by taking the complex conjugate of $\phi_{\circ 1}$, ϕ_{12} and $\Theta_{\circ 1}$, Θ_{12} respectively.

Now, we assume that $s_\circ$ and s_1 are the functions of r and t. Since,

$$D = {}_{;}l^i \qquad \Delta = {}_{;}n^i \qquad \delta = {}_{;}m^i$$

These definitions together with equations (2.9) we obtain

$$D s_{\circ} = \frac{1}{\sqrt{2}}(s_{\circ,t} + \frac{1}{P} s_{\circ,r}) \ ,$$

$$\Delta s_{\circ} = \frac{1}{\sqrt{2}}(s_{\circ,t} - \frac{1}{P} s_{\circ,r}) \ ,$$

$$\delta s_{\circ} = -\frac{1}{\sqrt{2}R}(s_{\circ,\theta} + i s_{\circ,\phi} \cos ec\theta) \ ,$$

$$\bar{\delta} s_{\circ} = -\frac{1}{\sqrt{2}R}(s_{\circ,\theta} - i s_{\circ,\phi} \cos ec\theta) ,$$

$$D \bar{s}_{\circ} = \frac{1}{\sqrt{2}}(\bar{s}_{\circ,t} + \frac{1}{P} \bar{s}_{\circ,r}) ,$$

$$\Delta \bar{s}_{\circ} = \frac{1}{\sqrt{2}}(\bar{s}_{\circ,t} - \frac{1}{P} \bar{s}_{\circ,r}) ,$$

$$\delta \bar{s}_{\circ} = -\frac{1}{\sqrt{2}R}(\bar{s}_{\circ,\theta} + i \bar{s}_{\circ,\phi} \cos ec\theta) \ ,$$

$$\bar{\delta} \bar{s}_{\circ} = -\frac{1}{\sqrt{2}R}(\bar{s}_{\circ,\theta} - i \bar{s}_{\circ,\phi} \cos ec\theta) ,$$

$$D s_1 = \frac{1}{\sqrt{2}}(s_{1,t} + \frac{1}{P} s_{1,r}) ,$$

$$\Delta s_1 = \frac{1}{\sqrt{2}}(s_{1,t} - \frac{1}{P} s_{1,r}) ,$$

$$\delta s_1 = -\frac{1}{\sqrt{2}R}(s_{1,\theta} + i s_{1,\phi} \cos ec\theta) \ ,$$

$$\bar{\delta} s_1 = -\frac{1}{\sqrt{2}R}(s_{1,\theta} - i s_{1,\phi} \cos ec\theta) ,$$

$$D \bar{s}_1 = \frac{1}{\sqrt{2}}(\bar{s}_{1,t} + \frac{1}{P} \bar{s}_{1,r}) ,$$

$$\Delta \bar{s}_1 = \frac{1}{\sqrt{2}}(\bar{s}_{1,t} - \frac{1}{P} \bar{s}_{1,r}) ,$$

$$\delta \bar{s}_1 = -\frac{1}{\sqrt{2}R}(\bar{s}_{1,\theta} + i \bar{s}_{1,\phi} \cos ec\theta) , \qquad ...(4.10)$$

$$\bar{\delta} \bar{s}_1 = -\frac{1}{\sqrt{2}R}(\bar{s}_{1,\theta} - i \bar{s}_{1,\phi} \cos ec\theta) .$$

Using equation (4.10) in (4.9), we obtain

$$\phi_{\circ\circ} = \frac{1}{2R}K(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) - \frac{i}{2R}K\operatorname{cosec}\theta(s_{\circ,\phi} - \bar{s}_{\circ,\phi}) + \frac{K(s_\circ + \bar{s}_\circ)}{2}\frac{\cot\theta}{R} - 2K^2 s_\circ \bar{s}_\circ - 2K^2 s_1{}^2 - \frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} - \frac{\dot{R}'}{PR} + \frac{\dot{P}R'}{P^2R},$$

$$\phi_{\circ 1} = -\frac{1}{2}K\left(s_{\circ,t} + \frac{1}{R}s_{1,\theta} + \frac{i}{R}s_{1,\phi}\operatorname{cosec}\theta\right) - \frac{1}{2}Ks_\circ\frac{\dot{P}}{P},$$

$$\phi_{11} = \frac{1}{4R}K(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) - \frac{i}{4R}K\operatorname{cosec}\theta(s_{\circ,\phi} - \bar{s}_{\circ,\phi}) + \frac{K(s_\circ + \bar{s}_\circ)}{4}\frac{\cot\theta}{R} + K^2 s_\circ \bar{s}_\circ - K^2 s_1{}^2 - \frac{\ddot{P}}{4P} + \frac{\dot{R}^2}{4R^2} - \frac{R'^2}{4P^2R^2} - \frac{\cot^2\theta}{4R^2},$$

$$\phi_{12} = \frac{1}{2}K\left(\frac{1}{P}s_{\circ,r} - \frac{1}{R}s_{1,\theta} - \frac{i}{R}s_{1,\phi}\operatorname{cosec}\theta\right) - K^2 s_\circ s_1 + \frac{1}{2}Ks_\circ\frac{\dot{P}}{P},$$

$$\phi_{22} = -\frac{1}{2R}K(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) + \frac{i}{2R}K\operatorname{cosec}\theta(s_{\circ,\phi} - \bar{s}_{\circ,\phi}) - \frac{K(s_\circ + \bar{s}_\circ)}{2}\frac{\cot\theta}{R} + 2K^2 s_\circ \bar{s}_\circ - 2K^2 s_1{}^2 - \frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} + \frac{\dot{R}'}{PR} - \frac{\dot{P}R'}{P^2R},$$

$$\phi_\circ = \frac{1}{2}Ks_{\circ,t} + Ks_\circ\frac{\dot{R}}{R} + \frac{1}{2}Ks_\circ\frac{\dot{P}}{P},$$

$$\phi_1 = \frac{1}{2}Ks_{1,t} + 2Ks_1\frac{\dot{R}}{R} + Ks_1\frac{\dot{P}}{P},$$

$$\phi_2 = \frac{1}{2P}K\bar{s}_{\circ,r} + K\bar{s}_\circ\frac{R'}{PR} + \frac{1}{2}K\bar{s}_\circ\frac{\dot{P}}{P} - K^2\bar{s}_\circ s_1,$$

$$\psi_\circ = \frac{1}{R}K(s_{\circ,\theta} + i s_{\circ,\phi}\operatorname{cosec}\theta) - Ks_\circ\frac{\cot\theta}{R} + 2K^2 s_\circ{}^2,$$

$$\psi_1 = \frac{1}{2}K\left(\frac{1}{P}s_{\circ,r} + \frac{1}{R}s_{1,\theta} + \frac{i}{R}s_{1,\phi}\operatorname{cosec}\theta\right) + \frac{1}{2}Ks_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR} - \frac{\dot{P}}{P}\right) + 2K^2 s_\circ s_1,$$

$$\psi_2 = K\left[\frac{1}{P}s_{1,r} - \frac{1}{6R}(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) + \frac{i}{6R}\operatorname{cosec}\theta(s_{\circ,\phi} - \bar{s}_{\circ,\phi})\right] - \frac{K(s_\circ + \bar{s}_\circ)}{6}\frac{\cot\theta}{R} + \frac{4}{3}K^2 s_1{}^2 + \frac{2}{3}K^2 s_\circ \bar{s}_\circ - \frac{\ddot{P}}{6P} + \frac{\ddot{R}}{6R} + \frac{\dot{P}\dot{R}}{6PR} + \frac{P'R'}{6P^3R} - \frac{R''}{6P^2R} - \frac{\dot{R}^2}{6R^2} + \frac{R'^2}{6P^2R^2} - + \frac{\cot^2\theta}{6R^2},$$

$$\psi_3 = \frac{1}{2}K\left(\bar{s}_{\circ,t} - \frac{1}{R}s_{1,\theta} + \frac{i}{R}s_{1,\phi}\cos ec\theta\right) - \frac{1}{2}K\bar{s}_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR} - \frac{\dot{P}}{P}\right) + K^2\bar{s}_\circ s_1 \,,$$

$$\psi_4 = -\frac{K}{R}(\bar{s}_{\circ,\theta} - i\bar{s}_{\circ,\phi}\cos ec\theta) + K\bar{s}_\circ\frac{\cot\theta}{R} - 2K^2\bar{s}_\circ^{\,2} \,,$$

$$\Theta_{\circ\circ} = \frac{i}{2R}K(s_{\circ,\theta} - \bar{s}_{\circ,\theta}) + \frac{1}{2R}K\cos ec\theta(s_{\circ,\phi} + \bar{s}_{\circ,\phi}) + iK(s_{1,t} + \frac{1}{P}s_{1,r}) + $$
$$+\frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R} + 2iKs_1\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) - iKs_1\frac{\dot{P}}{P} \,,$$

$$\Theta_{\circ 1} = \frac{i}{2P}Ks_{\circ,r} + \frac{i}{2}Ks_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR} - \frac{\dot{P}}{P}\right),$$

$$\Theta_{11} = \frac{i}{2P}Ks_{1,r} + iKs_1\frac{R'}{PR} \,, \qquad ...(4.11)$$

$$\Theta_{12} = -\frac{i}{2}K\left(\frac{1}{P}s_{\circ,r} - \frac{1}{R}s_{1,\theta} - \frac{i}{R}s_{1,\phi}\cos ec\theta\right) + iK^2 s_\circ s_1 - \frac{i}{2}Ks_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR} + \frac{\dot{P}}{P}\right),$$

$$\Theta_{22} = \frac{i}{2R}K(s_{\circ,\theta} - \bar{s}_{\circ,\theta}) + \frac{1}{2R}K\cos ec\theta(s_{\circ,\phi} + \bar{s}_{\circ,\phi}) + iK(s_{1,t} - \frac{1}{P}s_{1,r}) + $$
$$+\frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R} - 2iKs_1\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) + iKs_1\frac{\dot{P}}{P} \,,$$

$$\chi = -\frac{i}{P}Ks_{1,r} \,.$$

Corollary (1): If $s_\circ = 0$ and $s_1 \neq 0$ then we have

$$\psi_\circ = \psi_4 = 0 \,,$$

$$\psi_1 = \frac{1}{2R}K\left(s_{1,\theta} + is_{1,\phi}\cos ec\theta\right),$$

$$\psi_2 = K\frac{1}{P}s_{1,r} + \frac{4}{3}K^2 s_1^{\,2} - \frac{\ddot{P}}{6P} + \frac{\ddot{R}}{6R} + \frac{\dot{P}\dot{R}}{6PR} + \frac{P'R'}{6P^3R} - \frac{R''}{6P^2R} - \frac{\dot{R}^2}{6R^2} + \frac{R'^2}{6P^2R^2} + \frac{\cot^2\theta}{6R^2}$$

$$\psi_3 = \frac{1}{2R}K\left(-s_{1,\theta} + is_{1,\phi}\cos ec\theta\right).$$

Vanishing of $\psi_\circ$ and ψ_4 shows that the space-time of U_4 theory of gravitation is Petro-type I. We also notice here that if the tetrad components of the Spin tensor S_{ij} are functions of r and t alone then we have in this case

$$\psi_\circ = \psi_1 = \psi_3 = \psi_4 = 0,$$

proving that the space-time structure of U_4 theory of gravitation is of Petro-type D. Similarly, we obtain

$$\Theta_{\circ 1} = \Theta_{\circ 2} = 0 ,$$

$$\Theta_{\circ\circ} = iK(s_{1,t} + \frac{1}{P}s_{1,r}) + 2iKs_1\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) - iKs_1\frac{\dot{P}}{P} ,$$

$$\Theta_{11} = \frac{i}{2P}Ks_{1,r} + iKs_1\frac{R'}{PR} ,$$

$$\Theta_{12} = \frac{i}{2R}K\left(s_{1,\theta} + is_{1,\phi}\cos ec\theta\right) ,$$

$$\Theta_{22} = iK(s_{1,t} - \frac{1}{P}s_{1,r}) - 2iKs_1\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) + iKs_1\frac{\dot{P}}{P} ,$$

$$\chi = -\frac{i}{P}K\,s_{1,r},$$

$$\phi_\circ = \phi_2 = 0 ,$$

$$\phi_1 = \frac{1}{2}Ks_{1,t} + 2Ks_1\frac{\dot{R}}{R} + Ks_1\frac{\dot{P}}{P} ,$$

$$\phi_{\circ\circ} = -2K^2{s_1}^2 - \frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} - \frac{\dot{R}'}{PR} + \frac{\dot{P}R'}{P^2R} ,$$

$$\phi_{\circ 1} = \phi_{12} = -\frac{1}{2R}K\left(s_{1,\theta} + i\,s_{1,\phi}\cos ec\theta\right),$$

$$\phi_{11} = -K^2{s_1}^2 - \frac{\ddot{P}}{4P} + \frac{\dot{R}^2}{4R^2} - \frac{R'^2}{4P^2R^2} - \frac{\cot^2\theta}{4R^2} ,$$

$$\phi_{22} = -2K^2{s_1}^2 - \frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} + \frac{\dot{R}'}{PR} - \frac{\dot{P}R'}{P^2R} .$$

Corollary (2): If $s_\circ \neq 0$ and $s_1 = 0$, then we have

$$\psi_\circ = \frac{1}{R}K\,(s_{\circ,\theta} + i\,s_{\circ,\phi}\cos ec\theta) - Ks_\circ\frac{\cot\theta}{R} + 2K^2 s_\circ^{\,2}\ ,$$

$$\psi_1 = \frac{1}{2}K\left(\frac{1}{P}s_{\circ,r}\right) + \frac{1}{2}Ks_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR} - \frac{\dot{P}}{P}\right),$$

$$\psi_2 = K\left[-\frac{1}{6R}(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) + \frac{i}{6R}\cos ec\theta\,(s_{\circ,\phi} - \bar{s}_{\circ,\phi})\right] - \frac{K\,(s_\circ + \bar{s}_\circ)}{6}\frac{\cot\theta}{R} + \frac{2}{3}K^2 s_\circ \bar{s}_\circ -$$
$$-\frac{\ddot{P}}{6P} + \frac{\ddot{R}}{6R} + \frac{\dot{P}\dot{R}}{6PR} + \frac{P'R'}{6P^3R} - \frac{R''}{6P^2R} - \frac{\dot{R}^2}{6R^2} + \frac{R'^2}{6P^2R^2} - + \frac{\cot^2\theta}{6R^2}\ ,$$

$$\psi_3 = \frac{1}{2}K\,\bar{s}_{\circ,t} - \frac{1}{2}K\bar{s}_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR} - \frac{\dot{P}}{P}\right),$$

$$\psi_4 = -\frac{K}{R}(\bar{s}_{\circ,\theta} - i\,\bar{s}_{\circ,\phi}\cos ec\theta) + K\bar{s}_\circ\frac{\cot\theta}{R} - 2K^2\bar{s}_\circ^{\,2}\ ,$$

We see that none of the components of the Weyl tensor vanishes, showing that the component $s_\circ$ of the Spin angular momentum tensor has predominant effect on the space-time geometry of U_4.

$$\Theta_{11} = 0\ ,$$

$$\Theta_{\circ\circ} = \frac{i}{2R}K\,(s_{\circ,\theta} - \bar{s}_{\circ,\theta}) + \frac{1}{2R}K\cos ec\theta\,(s_{\circ,\phi} + \bar{s}_{\circ,\phi}) + \frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R}\ ,$$

$$\Theta_{\circ 1} = \frac{i}{2P}Ks_{\circ,r} + \frac{i}{2}Ks_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR} - \frac{\dot{P}}{P}\right),$$

$$\Theta_{12} = -\frac{i}{2P}K\,s_{\circ,r} - \frac{i}{2}Ks_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR} + \frac{\dot{P}}{P}\right),$$

$$\Theta_{22} = \frac{i}{2R}K\,(s_{\circ,\theta} - \bar{s}_{\circ,\theta}) + \frac{1}{2R}K\cos ec\theta\,(s_{\circ,\phi} + \bar{s}_{\circ,\phi}) + \frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R}\ ,$$

$$\phi_1 = \phi_{\circ 2} = \chi = 0\ ,$$

$$\phi_\circ = \frac{1}{2}Ks_{\circ,t} + Ks_\circ\frac{\dot{R}}{R} + \frac{1}{2}Ks_\circ\frac{\dot{P}}{P}\ ,$$

$$\phi_2 = \frac{1}{2P}K\,\bar{s}_{\circ,r} + K\bar{s}_\circ\frac{R'}{PR} + \frac{1}{2}K\bar{s}_\circ\frac{\dot{P}}{P}\ ,$$

$$\phi_{\circ\circ} = \frac{1}{2R}K(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) - \frac{i}{2R}K\operatorname{cosec}\theta(s_{\circ,\phi} - \bar{s}_{\circ,\phi}) + \frac{K(s_\circ + \bar{s}_\circ)}{2}\frac{\cot\theta}{R} - 2K^2 s_\circ \bar{s}_\circ$$
$$- \frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} - \frac{\dot{R}'}{PR} + \frac{\dot{P}R'}{P^2R},$$

$$\phi_{\circ 1} = -\frac{1}{2}K s_{\circ,t} - \frac{1}{2}K s_\circ \frac{\dot{P}}{P},$$

$$\phi_{11} = \frac{1}{4R}K(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) - \frac{i}{4R}K\operatorname{cosec}\theta(s_{\circ,\phi} - \bar{s}_{\circ,\phi}) + \frac{K(s_\circ + \bar{s}_\circ)}{4}\frac{\cot\theta}{R} + K^2 s_\circ \bar{s}_\circ$$
$$- \frac{\ddot{P}}{4P} + \frac{\dot{R}^2}{4R^2} - \frac{R'^2}{4P^2R^2} - \frac{\cot^2\theta}{4R^2},$$

$$\phi_{12} = \frac{1}{2P}K s_{\circ,r} + \frac{1}{2}K s_\circ \frac{\dot{P}}{P},$$

$$\phi_{22} = -\frac{1}{2R}K(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) + \frac{i}{2R}K\operatorname{cosec}\theta(s_{\circ,\phi} - \bar{s}_{\circ,\phi}) - \frac{K(s_\circ + \bar{s}_\circ)}{2}\frac{\cot\theta}{R} + 2K^2 s_\circ \bar{s}_\circ -$$
$$- \frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} + \frac{\dot{R}'}{PR} - \frac{\dot{P}R'}{P^2R}.$$

Corollary (3): If $s_\circ = 0$ and $s_1 = 0$ then we have

$$\psi_\circ = \psi_1 = \psi_3 = \psi_4 = 0,$$

$$\psi_2 = -\frac{\ddot{P}}{6P} + \frac{\ddot{R}}{6R} + \frac{\dot{P}\dot{R}}{6PR} + \frac{P'R'}{6P^3R} - \frac{R''}{6P^2R} - \frac{\dot{R}^2}{6R^2} + \frac{R'^2}{6P^2R^2} - + \frac{\cot^2\theta}{6R^2},$$

$$\Theta_{\circ\circ} = \Theta_{\circ 1} = \Theta_{11} = \Theta_{12} = \Theta_{22} = 0.$$

This shows that in the absence of Spin, the result reduces to Petro-type D space-time of Einstein theory of gravitation.

$$\phi_\circ = \phi_1 = \phi_2 = \chi = 0,$$

$$\phi_{\circ 1} = \phi_{\circ 2} = \phi_{12} = 0,$$

$$\phi_{\circ\circ} = -\frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} - \frac{\dot{R}'}{PR} + \frac{\dot{P}R'}{P^2R},$$

$$\phi_{11} = -\frac{\ddot{P}}{4P} + \frac{\dot{R}^2}{4R^2} - \frac{R'^2}{4P^2R^2} - \frac{\cot^2\theta}{4R^2},$$

$$\phi_{22} = -\frac{\ddot{R}}{2R} + \frac{\dot{P}\dot{R}}{2PR} + \frac{P'R'}{2P^3R} - \frac{R''}{2P^2R} + \frac{\dot{R}'}{PR} - \frac{\dot{P}R'}{P^2R}.$$

Conclusion: It has been observed that the Spin tensor influences the space-time geometry of the Einstein-Cartan theory of gravitation. If the Spin tensor component $s_\circ = 0$ and $s_1 \neq 0$, the space-time of U_4 theory of gravitation is shown to be Petro-type I. If however, the tetrad components of the Spin tensor are functions of r and t alone then we have $\psi_\circ = \psi_1 = \psi_3 = \psi_4 = 0$ proving the space-time of U_4 theory of gravitation is Petro-type D. However, if $s_\circ \neq 0$ and $s_1 = 0$, then none of the Weyl tensor vanishes, showing that $s_\circ$ has predominant effect on the space-time geometry of U_4. In the absence of the Spin tensor the space-time reduces to the space-time D of Einstein theory of gravitation.

5. Electric and Magnetic Parts of the Weyl tensor:

Let E_{ik} and H_{ik} be the electric and magnetic parts of the Weyl tensor C_{ijkl} and are defined by

$$E_{ik} = C_{ijkl} u^j u^l \qquad ...(5.1)$$

and $$H_{ik} = C^*_{ijkl} u^j u^l \qquad ...(5.2)$$

where C^*_{ijkl} is the dual of C_{ijkl} and is given by

$$C^*_{ijkl} = \frac{1}{2} \varepsilon_{kl}{}^{mn} C_{ijmn} \qquad ...(5.3)$$

where $\varepsilon_{kl}{}^{mn}$ is the Levi-civita permutation symbol.

The time-like vector field u^i can be expressed as

$$u^i = \frac{1}{\sqrt{2}}(l^i + n^i) \qquad ...(5.4)$$

Then equations (6.1) and (6.2) becomes,

$$E_{hj} = \frac{1}{2} C_{hijk} U^{ik}$$

and $$H_{hj} = C^*_{hijk} U^{ik} \qquad ...(5.5)$$

where $U^{ik} = (l^i l^k + l^i n^k + n^i l^k + n^i n^k)$.

Unlike the tetrad components of electric part of Weyl tensor E_{hj} in Einstein gravitation theory, they are not symmetric in Einstein-Cartan theory of gravitation. These tetrad components are defined as

$$E_{11} = E_{hj} l^h l^j \;; \qquad E_{12} = E_{hj} l^h n^j ; \qquad E_{13} = E_{hj} l^h m^j ;$$

$$E_{21} = E_{hj} n^h l^j ; \qquad E_{22} = E_{hj} n^h n^j ; \qquad E_{23} = E_{hj} n^h m^j ;$$

$$E_{31} = E_{hj} m^h l^j ; \qquad E_{32} = E_{hj} m^h n^j ; \qquad E_{33} = E_{hj} m^h m^j ;$$

$$E_{34} = E_{hj} m^h \overline{m}^j . \qquad \ldots(5.6)$$

Similarly, the tetrad components of the magnetic parts of Weyl tensor H_{hj} are obtained from equation (5.6) just by replacing E by H.

Now, Weyl tensor can be expressed as composition of electric and magnetic parts as follows

$$C_{hijk} = E_{ik} V_{hj} + E_{hj} V_{ik} - E_{ij} V_{hk} - E_{hk} V_{ij} + \frac{1}{2} \eta_{hi}^{pq} H_{qj} U_{kp} - \frac{1}{2} \eta_{hi}^{pq} H_{qk} U_{pj} + + \frac{1}{2} \eta_{jk}^{pq} H_{hq} U_{ip} - \frac{1}{2} \eta_{jk}^{pq} H_{iq} U_{hp} \qquad \ldots(5.7)$$

where $V_{ij} = l_i l_j + 2 m_{(i} \overline{m}_{j)} + n_i n_j$

From equation (5.5) and (5.6) we determine

$$E_{11} = -E_{12} = -E_{21} = E_{22} = \frac{1}{2} C_{1212} \,,$$

$$E_{13} = -E_{23} = -\frac{1}{2} (C_{1213} + C_{1223}) \,,$$

$$E_{31} = -E_{32} = -\frac{1}{2} (C_{1312} + C_{2312}) \,,$$

$$E_{33} = \frac{1}{2} (C_{1313} + C_{1323} + C_{2313} + C_{2323}) \,,$$

$$E_{34} = \frac{1}{2} (C_{1314} + C_{1324} + C_{2314} + C_{2324}) . \qquad \ldots(5.8)$$

We express electric part of Weyl tensor in terms of its tetrad components as

$$E_{ij} = E_{\alpha\beta} e_i^{(\alpha)} e_j^{(\beta)} \,, \qquad \alpha, \beta = 1,2,3,4 \qquad \ldots(5.9)$$

Consequently, this can also be written as

$$E_{ij} = E_{22}l_il_j + E_{21}l_in_j - E_{24}l_im_j - E_{23}l_i\overline{m}_j + E_{12}n_il_j + E_{11}n_in_j - E_{14}n_im_j - E_{13}n_i\overline{m}_j - \\ - E_{42}m_il_j - E_{41}m_in_j + E_{44}m_im_j + E_{43}m_i\overline{m}_j - E_{32}\overline{m}_il_j - E_{31}\overline{m}_in_j + E_{34}\overline{m}_im_j + \\ + E_{33}\overline{m}_i\overline{m}_j \qquad \ldots(5.10)$$

Equation (5.8) and (5.10) give the expression for the electric part of Weyl tensor in terms of the linear combination of the basis of the tetrad as

$$E_{ij} = \frac{1}{2}C_{1212}[l_i l_j - l_{(i} n_{j)} + n_i n_j] - \frac{1}{2}(C_{1214} + C_{1224})[l_i m_j - n_i m_j] - \\ - \frac{1}{2}(C_{1213} + C_{1223})[l_i \overline{m}_j - n_i \overline{m}_j] - \frac{1}{2}(C_{1412} + C_{2412})[m_i l_j - m_i n_j] - \\ - \frac{1}{2}(C_{1312} + C_{2312})[\overline{m}_i l_j - \overline{m}_i n_j] + \frac{1}{2}(C_{1413} + C_{1423} + C_{2413} + C_{2423}) m_i \overline{m}_j + \\ + \frac{1}{2}(C_{1314} + C_{1324} + C_{2314} + C_{2324}) \overline{m}_i m_j + \frac{1}{2}(C_{1414} + C_{1424} + C_{2414} + C_{2424}) m_i m_j + \\ + \frac{1}{2}(C_{1313} + C_{1323} + C_{2313} + C_{2323}) \overline{m}_i \overline{m}_j \qquad \ldots(5.11)$$

Likewise, the tetrad components of the magnetic part H_{ij} and its expression in terms of basis of the tetrad are obtained from equation (5.8) and (5.11) just by replacing the tetrad components of the Weyl tensor by its dual.

However, the tetrad components of the dual of Weyl tensor are given by

$$C^{*}{}_{\alpha\beta\gamma\delta} = \frac{i}{2}\varepsilon_{\gamma\delta}{}^{\sigma\rho}C_{\alpha\beta\sigma\rho} \qquad \ldots(5.12)$$

This can also be written as

$$C^{*}_{\alpha\beta\gamma\delta} = \frac{i}{2}[C_{\alpha\beta21}\varepsilon_{\gamma\delta12} + C_{\alpha\beta12}\varepsilon_{\gamma\delta21} - C_{\alpha\beta24}\varepsilon_{\gamma\delta13} - C_{\alpha\beta42}\varepsilon_{\gamma\delta31} - C_{\alpha\beta23}\varepsilon_{\gamma\delta14} - C_{\alpha\beta32}\varepsilon_{\gamma\delta41} - \\ - C_{\alpha\beta14}\varepsilon_{\gamma\delta23} - C_{\alpha\beta41}\varepsilon_{\gamma\delta32} - C_{\alpha\beta13}\varepsilon_{\gamma\delta24} - C_{\alpha\beta31}\varepsilon_{\gamma\delta42} + C_{\alpha\beta43}\varepsilon_{\gamma\delta34} + C_{\alpha\beta34}\varepsilon_{\gamma\delta43}] \qquad \ldots(5.13)$$

By assigning different values to indices $\alpha, \beta, \gamma, \delta$ from 1 to 4 we can establish the relation between the tetrad components of the Weyl tensor and its dual and using these relations we can readily obtain non-vanishing components of the dual of Weyl tensor as follows,

$$C^{*}_{1212} = -iC_{1234} = 4iKs_1\frac{R'}{PR} ,$$

$$C^{*}_{1213} = iC_{1213} = \frac{i}{\sqrt{2}} K\delta s_1 - 2iK^2 s_\circ s_1 + iKs_\circ \frac{R'}{PR} \ ,$$

$$C^{*}_{1214} = -iC_{1214} = \frac{i}{\sqrt{2}} K\bar{\delta} s_1 - 2iK^2 \bar{s}_\circ s_1 - iK\bar{s}_\circ \frac{R'}{PR} \ ,$$

$$C^{*}_{1223} = -iC_{1223} = \frac{i}{\sqrt{2}} K\delta s_1 + iKs_\circ \frac{\dot{R}}{R} \ ,$$

$$C^{*}_{1224} = iC_{1224} = \frac{i}{\sqrt{2}} K\bar{\delta} s_1 - iK\bar{s}_\circ \frac{\dot{R}}{R} \ ,$$

$$C^{*}_{1234} = -iC_{1212} = -\frac{i}{3}\left[\begin{array}{l} -\sqrt{2}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) + K(s_\circ + \bar{s}_\circ)\dfrac{\cot\theta}{R} - 4K^2 s_\circ \bar{s}_\circ - 8K^2 s_1{}^2 + \\ + \dfrac{\ddot{P}}{P} - \dfrac{\ddot{R}}{R} - \dfrac{\dot{P}\dot{R}}{PR} - \dfrac{P'R'}{P^3 R} + \dfrac{R''}{P^2 R} + \dfrac{\dot{R}^2}{3R^2} - \dfrac{R'^2}{3P^2R^2} - \dfrac{\cot^2\theta}{3R^2} \end{array}\right] ,$$

$$C^{*}_{1312} = -ic_{1334} = \frac{i}{\sqrt{2}} K\delta s_1 - 2iK^2 s_\circ s_1 + iKs_\circ \frac{R'}{PR} \ ,$$

$$C^{*}_{1313} = iC_{1313} = \sqrt{2} iK\delta s_\circ - 2iK^2 s_\circ{}^2 + iKs_\circ \frac{\cot\theta}{R} \ ,$$

$$C^{*}_{1314} = -iC_{1314} = \frac{i}{\sqrt{2}} K(\delta\bar{s}_\circ - \bar{\delta}s_\circ + 2Ds_1) + 2iKs_1\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) - iKs_1 \frac{\dot{P}}{P} + + \frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R} \ ,$$

$$C^{*}_{1324} = iC_{1324} = \frac{i}{\sqrt{2}} K(Ds_1 - \Delta s_1) + \frac{i}{3\sqrt{2}} K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) + \frac{4i}{3}K^2 s_1{}^2 + \frac{2i}{3}K^2 s_\circ \bar{s} - \frac{iK(s_\circ + \bar{s}_\circ)}{6}\frac{\cot\theta}{R} + + \frac{i}{6}\left[-\frac{\ddot{P}}{P} + \frac{\ddot{R}}{R} + \frac{\dot{P}\dot{R}}{PR} + \frac{P'R'}{P^3R} - \frac{R''}{P^2R} - \frac{\dot{R}^2}{R^2} + \frac{R'^2}{P^2R^2} + \frac{\cot^2\theta}{R^2}\right] ,$$

$$C^{*}_{1334} = -iC_{1312} = \frac{i}{\sqrt{2}} K(Ds_\circ - \Delta s_\circ - \delta s_1) + 2iK^2 s_\circ s_1 - iKs_\circ\left(\frac{\dot{P}}{P} - \frac{\dot{R}}{R}\right) ,$$

$$C^{*}_{1412} = -iC_{1434} = \frac{i}{\sqrt{2}} K\bar{\delta} s_1 - 2iK^2 \bar{s}_\circ s_1 - iK\bar{s}_\circ \frac{R'}{PR} \ ,$$

$$C^*_{1413} = iC_{1413} = \frac{i}{\sqrt{2}} K(\delta\bar{s}_\circ - \bar{\delta}s_\circ + 2Ds_1) + 2iKs_1\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right) - iKs_1\frac{\dot{P}}{P} + \\ + \frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R},$$

$$C^*_{1414} = -iC_{1414} = -\sqrt{2}iK\delta\bar{s}_\circ + 2iK^2\bar{s}_\circ^{\,2} - iK\bar{s}_\circ\frac{\cot\theta}{R},$$

$$C^*_{1423} = -iC_{1423} = \frac{i}{\sqrt{2}}K(Ds_1 - \Delta s_1) - \frac{i}{3\sqrt{2}}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) - \frac{4i}{3}K^2 s_1^{\,2} - \\ - \frac{2i}{3}K^2 s_\circ\bar{s}_\circ + \frac{iK(s_\circ + \bar{s}_\circ)}{6}\frac{\cot\theta}{R} - \\ - \frac{i}{6}\left[-\frac{\ddot{P}}{6P} + \frac{\ddot{R}}{6R} + \frac{\dot{P}\dot{R}}{6PR} + \frac{P'R'}{6P^3R} - \frac{R''}{6P^2R} - \frac{\dot{R}^2}{6R^2} + \frac{R'^2}{6P^2R^2} + \frac{\cot^2\theta}{6R^2}\right],$$

$$C^*_{1434} = -iC_{1412} = \frac{i}{\sqrt{2}}K(D\bar{s}_\circ - \Delta\bar{s}_\circ + \bar{\delta}s_1) - 2iK^2\bar{s}_\circ s_1 - iK\bar{s}_\circ\left(\frac{\dot{P}}{P} - \frac{\dot{R}}{R}\right),$$

$$C^*_{2312} = -iC_{2334} = \frac{i}{\sqrt{2}}K\delta s_1 + iKs_\circ\frac{\dot{R}}{R},$$

$$C^*_{2314} = -iC_{2314} = \frac{i}{\sqrt{2}}K(Ds_1 - \Delta s_1) - \frac{i}{3\sqrt{2}}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) - \frac{4i}{3}K^2 s_1^{\,2} + \\ - \frac{2i}{3}K^2 s_\circ\bar{s}_\circ + \frac{iK(s_\circ + \bar{s}_\circ)}{6}\frac{\cot\theta}{R} - \\ - \frac{i}{6}\left[-\frac{\ddot{P}}{6P} + \frac{\ddot{R}}{6R} + \frac{\dot{P}\dot{R}}{6PR} + \frac{P'R'}{6P^3R} - \frac{R''}{6P^2R} - \frac{\dot{R}^2}{6R^2} + \frac{R'^2}{6P^2R^2} + \frac{\cot^2\theta}{6R^2}\right]$$

$$C^*_{2323} = -iC_{2323} = \sqrt{2}iK\delta s_\circ - 2iK^2 s_\circ^{\,2} + iKs_\circ\frac{\cot\theta}{R},$$

$$C^*_{2324} = iC_{2324} = \frac{i}{\sqrt{2}}K(\delta\bar{s}_\circ - \bar{\delta}s_\circ - 2\Delta s_1) + \frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R} - 2iKs_1\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) + \\ + iKs_1\frac{\dot{P}}{P},$$

$$C^*_{2334} = -iC_{2312} = -\frac{i}{\sqrt{2}}K(Ds_\circ - \Delta s_\circ + \delta s_1) + 2iK^2 s_\circ s_1 - iKs_\circ\left(\frac{\dot{P}}{P} - \frac{R'}{PR}\right),$$

$$C^*_{2412} = -iC_{2434} = \frac{i}{\sqrt{2}}K\bar{\delta}s_1 - iK\bar{s}_\circ\frac{\dot{R}}{R},$$

$$C^{*}_{2413} = iC_{2413} = \frac{i}{\sqrt{2}}K(Ds_1 - \Delta s_1) + \frac{i}{3\sqrt{2}}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) + \frac{4i}{3}K^2 s_1{}^2 + + \frac{2i}{3}K^2 s_\circ \bar{s}_\circ - \frac{iK(s_\circ + \bar{s}_\circ)}{6}\frac{\cot\theta}{R} + + \frac{i}{6}\left[-\frac{\ddot{P}}{6P} + \frac{\ddot{R}}{6R} + \frac{\dot{P}\dot{R}}{6PR} + \frac{P'R'}{6P^3R} - \frac{R''}{6P^2R} - \frac{\dot{R}^2}{6R^2} + \frac{R'^2}{6P^2R^2} + \frac{\cot^2\theta}{6R^2}\right],$$

$$C^{*}_{2423} = -iC_{2423} = \frac{i}{\sqrt{2}}K(\delta\bar{s}_\circ - \bar{\delta}s_\circ - 2\Delta s_1) + \frac{iK(s_\circ - \bar{s}_\circ)}{2}\frac{\cot\theta}{R} - 2iKs_1\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right) + + iKs_1\frac{\dot{P}}{P},$$

$$C^{*}_{2424} = iC_{2424} = -\sqrt{2}iK\delta\bar{s}_\circ + 2iK^2\bar{s}_\circ{}^2 - iK\bar{s}_\circ\frac{\cot\theta}{R},$$

$$C^{*}_{2434} = -iC_{2412} = -\frac{i}{\sqrt{2}}K(D\bar{s}_\circ - \Delta\bar{s}_\circ - \bar{\delta}s_1) - 2iK^2\bar{s}_\circ s_1 - iK\bar{s}_\circ\left(\frac{\dot{P}}{P} - \frac{R'}{PR}\right),$$

$$C^{*}_{3412} = -iC_{3434} = -\frac{i}{3}\left[\begin{array}{l} -\sqrt{2}K(\delta\bar{s}_\circ + \bar{\delta}s_\circ) + K(s_\circ + \bar{s}_\circ)\dfrac{\cot\theta}{R} - 8K^2 s_1{}^2 - 4K^2 s_\circ\bar{s}_\circ + \\ + \dfrac{\ddot{P}}{P} - \dfrac{\ddot{R}}{R} - \dfrac{\dot{P}\dot{R}}{PR} - \dfrac{P'R'}{P^3R} + \dfrac{R''}{P^2R} + \dfrac{\dot{R}^2}{3R^2} - \dfrac{R'^2}{3P^2R^2} - \dfrac{\cot^2\theta}{3R^2} \end{array}\right],$$

$$C^{*}_{3413} = iC_{3413} = \frac{i}{\sqrt{2}}K(Ds_\circ - \Delta s_\circ - \delta s_1) + 2iK^2 s_\circ s_1 - iKs_\circ\left(\frac{\dot{P}}{P} - \frac{\dot{R}}{R}\right),$$

$$C^{*}_{3414} = -iC_{3414} = \frac{i}{\sqrt{2}}K(D\bar{s}_\circ - \Delta\bar{s}_\circ + \bar{\delta}s_1) - 2iK^2\bar{s}_\circ s_1 - iK\bar{s}_\circ\left(\frac{\dot{P}}{P} - \frac{\dot{R}}{R}\right),$$

$$C^{*}_{3423} = -iC_{3423} = -\frac{i}{\sqrt{2}}K(Ds_\circ + \Delta s_\circ - \delta s_1) + 2iK^2 s_\circ s_1 - iKs_\circ\left(\frac{\dot{P}}{P} - \frac{R'}{PR}\right),$$

$$C^{*}_{3424} = iC_{3424} = -\frac{i}{\sqrt{2}}K(D\bar{s}_\circ + \Delta\bar{s}_\circ + \bar{\delta}s_1) - 2iK^2\bar{s}_\circ s_1 + iK\bar{s}_\circ\left(\frac{R'}{PR} - \frac{\dot{P}}{P}\right),$$

$$C^{*}_{3434} = -iC_{3412} = -\sqrt{2}iK(Ds_1 - \Delta s_1) \qquad \ldots(5.14)$$

Now, the expression for E_{ij} and H_{ij} with respect to the given metric become

$$E_{ij}=\frac{1}{6}\begin{bmatrix}-\sqrt{2}K(\delta\bar{s}_\circ+\bar{\delta}s_\circ)+K(s_\circ+\bar{s}_\circ)\dfrac{\cot\theta}{R}-\\ -4K^2s_\circ\bar{s}_\circ-8K^2{s_1}^2+\dfrac{\ddot{P}}{P}-\dfrac{\ddot{R}}{R}-\dfrac{\dot{P}\dot{R}}{PR}-\dfrac{P'R'}{P^3R}+\\ +\dfrac{R''}{P^2R}+\dfrac{\dot{R}^2}{3R^2}-\dfrac{R'^2}{3P^2R^2}-\dfrac{\cot^2\theta}{3R^2}\end{bmatrix}[l_il_j-l_{(i}n_{j)}+n_in_j]-$$

$$-\frac{1}{2}\left[K\bar{s}_\circ\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)-2K^2\bar{s}_\circ s_1\right][l_im_j-n_im_j]-$$

$$-\frac{1}{2}\left[Ks_\circ\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)-2K^2s_\circ s_1\right][l_i\overline{m}_j-n_i\overline{m}_j]-$$

$$-\frac{1}{2}\left[-\sqrt{2}K\bar{\delta}s_1+4K^2\bar{s}_\circ s_1+2K\bar{s}_\circ\frac{\dot{P}}{P}-K\bar{s}_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)\right][m_il_j-m_in_j]-$$

$$-\frac{1}{2}\left[\sqrt{2}K\delta s_1-4K^2s_\circ s_1+2Ks_\circ\frac{\dot{P}}{P}-Ks_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)\right][\overline{m}_il_j-\overline{m}_in_j]+$$

$$+\frac{1}{2}\begin{bmatrix}\sqrt{2}K(Ds_1-\Delta\bar{s}_\circ)+\dfrac{\sqrt{2}}{3}K(\delta\bar{s}_\circ+\bar{\delta}s_\circ)-\dfrac{K(s_\circ+\bar{s}_\circ)\cot\theta}{3R}-\\ -2Ks_1\dfrac{\dot{P}}{P}+4Ks_1\dfrac{\dot{R}}{R}+\dfrac{4}{3}K^2s_\circ\bar{s}_\circ+\dfrac{8}{3}K^2{s_1}^2-\dfrac{\ddot{P}}{3P}+\dfrac{\ddot{R}}{3R}+\\ +\dfrac{\dot{P}\dot{R}}{3PR}+\dfrac{P'R'}{3P^3R}-\dfrac{R''}{3P^2R}-\dfrac{\dot{R}^2}{3R^2}+\dfrac{R'^2}{3P^2R^2}+\dfrac{\cot^2\theta}{3R^2}\end{bmatrix}m_i\overline{m}_j+$$

$$+\frac{1}{2}\begin{bmatrix}-\sqrt{2}K(Ds_1+\Delta s_\circ)+\dfrac{\sqrt{2}}{3}K(\delta\bar{s}_\circ+\bar{\delta}s_\circ)-\dfrac{K(s_\circ+\bar{s}_\circ)\cot\theta}{3R}+\\ +2Ks_1\dfrac{\dot{P}}{P}-4Ks_1\dfrac{\dot{R}}{R}+\dfrac{4}{3}K^2s_\circ\bar{s}_\circ+\dfrac{8}{3}K^2{s_1}^2-\dfrac{\ddot{P}}{3P}+\dfrac{\ddot{R}}{3R}+\\ +\dfrac{\dot{P}\dot{R}}{3PR}+\dfrac{P'R'}{3P^3R}-\dfrac{R''}{3P^2R}-\dfrac{\dot{R}^2}{3R^2}+\dfrac{R'^2}{3P^2R^2}+\dfrac{\cot^2\theta}{3R^2}\end{bmatrix}\overline{m}_im_j$$

...(5.15)

and

$$H_{ij}=\left(2iKs_1\frac{R'}{PR}\right)[l_il_j-l_{(i}n_{j)}+n_in_j]-$$

$$-\frac{1}{2}\left(\sqrt{2}iK\bar{\delta}s_1-iK\bar{s}_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)-2iK^2\bar{s}_\circ s_1\right)[l_{(i}m_{j)}-n_{(i}m_{j)}]-$$

$$-\frac{1}{2}\left(\sqrt{2}iK\delta s_1+iKs_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)-2iK^2s_\circ s_1\right)[l_{(i}\overline{m}_{j)}-n_{(i}\overline{m}_{j)}]+$$

$$+\frac{1}{2}\left(2\sqrt{2}iK(Ds_1-\Delta s_1)+iK(s_\circ-\bar{s}_\circ)\frac{\cot\theta}{R}+4iKs_1\frac{R'}{PR}\right)m_{(i}\overline{m}_{j)}+$$

$$+\frac{1}{2}\left(-2\sqrt{2}iK\bar{\delta}\bar{s}_\circ-2iK\bar{s}_\circ\frac{\cot\theta}{R}+4iK^2\bar{s}_\circ^{\,2}\right)m_i m_j+$$

$$+\frac{1}{2}\left(2\sqrt{2}iK\delta s_\circ+2iKs_\circ\frac{\cot\theta}{R}-4iK^2s_\circ^{\,2}\right)\overline{m}_i\overline{m}_j$$

...(5.16)

respectively.

Using equations (4.10) in (5.15) and (5.16) we obtain

$$E_{ij}=\frac{1}{6}\begin{bmatrix}\frac{K}{R}(s_{\circ,\theta}+\bar{s}_{\circ,\theta})-\frac{iK}{R}\operatorname{cosec}\theta(s_{\circ,\phi}-\bar{s}_{\circ,\phi})-4K^2s_\circ\bar{s}_\circ-\\-8K^2s_1^{\,2}+K(s_\circ+\bar{s}_\circ)\frac{\cot\theta}{R}+\frac{\ddot{P}}{P}-\frac{\ddot{R}}{R}-\frac{\dot{P}\dot{R}}{PR}-\frac{P'R'}{P^3R}+\\+\frac{R''}{P^2R}+\frac{\dot{R}^2}{3R^2}-\frac{R'^2}{3P^2R^2}-\frac{\cot^2\theta}{3R^2}\end{bmatrix}[l_il_j-l_{(i}n_{j)}+n_in_j]-$$

$$-\frac{1}{2}\left[K\bar{s}_\circ\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)-2K^2\bar{s}_\circ s_1\right][l_im_j-n_im_j-$$

$$-\frac{1}{2}\left[Ks_\circ\left(\frac{\dot{R}}{R}-\frac{R'}{PR}\right)-2K^2s_\circ s_1\right][l_i\overline{m}_j-n_i\overline{m}_j]-$$

$$-\frac{1}{2}\left[\frac{K}{R}(s_{1,\theta}-is_{1,\phi}\operatorname{cosec}\theta)+4K^2\bar{s}_\circ s_1+2K\bar{s}_\circ\frac{\dot{P}}{P}-K\bar{s}_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)\right][m_il_j-m_in_j]-$$

$$-\frac{1}{2}\begin{bmatrix}-\frac{K}{R}(s_{1,\theta}+is_{1,\phi}\operatorname{cosec}\theta)-4K^2s_\circ s_1+\\+2Ks_\circ\frac{\dot{P}}{P}-Ks_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)\end{bmatrix}[\overline{m}_il_j-\overline{m}_in_j]+$$

$$+\frac{1}{2}\begin{bmatrix}\frac{iK}{3R}\operatorname{cosec}\theta(s_{\circ,\phi}-\bar{s}_{\circ,\phi})+\frac{K}{P}(s_{1,r}+\bar{s}_{\circ,r})-\frac{K}{3R}(s_{\circ,\theta}+\bar{s}_{\circ,\theta})+\\+K(s_{1,t}-\bar{s}_{\circ,t})-\frac{K(s_\circ+\bar{s}_\circ)\cot\theta}{3R}-2Ks_1\frac{\dot{P}}{P}+4Ks_1\frac{\dot{R}}{R}+\\+\frac{4}{3}K^2s_\circ\bar{s}_\circ+\frac{8}{3}K^2s_1^{\,2}-\frac{\ddot{P}}{3P}+\frac{\ddot{R}}{3R}++\frac{\dot{P}\dot{R}}{3PR}+\frac{P'R'}{3P^3R}-\\-\frac{R''}{3P^2R}-\frac{\dot{R}^2}{3R^2}+\frac{R'^2}{3P^2R^2}+\frac{\cot^2\theta}{3R^2}\end{bmatrix}m_i\overline{m}_j+$$

$$+\frac{1}{2}\begin{bmatrix} -\frac{K}{P}(s_{1,r}-s_{\circ,r})-\frac{K}{3R}(s_{\circ,\theta}+\bar{s}_{\circ,\theta})+\frac{iK}{3R}\operatorname{cosec}\theta(s_{\circ,\phi}-\bar{s}_{\circ,\phi})- \\ -\frac{K(s_\circ+\bar{s}_\circ)\cot\theta}{3R}-K(s_{\circ,t}+s_{1,t})+2Ks_1\frac{\dot{P}}{P}-4Ks_1\frac{\dot{R}}{R}+ \\ +\frac{4}{3}K^2s_\circ\bar{s}_\circ+\frac{8}{3}K^2s_1{}^2-\frac{\ddot{P}}{3P}+\frac{\ddot{R}}{3R}+\frac{\dot{P}\dot{R}}{3PR}+\frac{P'R'}{3P^3R}- \\ -\frac{R''}{3P^2R}-\frac{\dot{R}^2}{3R^2}+\frac{R'^2}{3P^2R^2}+\frac{\cot^2\theta}{3R^2} \end{bmatrix}\bar{m}_i m_j \qquad ...(5.17)$$

and

$$H_{ij}=\left(2iKs_1\frac{R'}{PR}\right)[l_il_j-l_{(i}n_{j)}+n_in_j]-$$
$$-\frac{1}{2}\left(-\frac{i}{R}K(s_{1,\theta}-is_{1,\phi}\operatorname{cosec}\theta)-iK\bar{s}_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)-2iK^2\bar{s}_\circ s_1\right)[l_{(i}m_{j)}-n_{(i}m_{j)}]-$$
$$-\frac{1}{2}\left(-\frac{i}{R}K(s_{1,\theta}+is_{1,\phi}\operatorname{cosec}\theta)+iKs_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)-2iK^2s_\circ s_1\right)[l_{(i}\bar{m}_{j)}-n_{(i}\bar{m}_{j)}]+$$
$$+\frac{1}{2}\left(\frac{4i}{P}Ks_{1,r}+iK(s_\circ-\bar{s}_\circ)\frac{\cot\theta}{R}+4iKs_1\frac{R'}{PR}\right)m_{(i}\bar{m}_{j)}+$$
$$+\frac{1}{2}\left(\frac{2i}{R}K(\bar{s}_{\circ,\theta}-i\bar{s}_{\circ,\phi}\operatorname{cosec}\theta)-2iK\bar{s}_\circ\frac{\cot\theta}{R}+4iK^2\bar{s}_\circ{}^2\right)m_im_j+$$
$$+\frac{1}{2}\left(-\frac{2i}{R}K(s_{\circ,\theta}+is_{\circ,\phi}\operatorname{cosec}\theta)+2iKs_\circ\frac{\cot\theta}{R}-4iK^2s_\circ{}^2\right)\bar{m}_i\bar{m}_j$$

...(5.18)

Case (1): If $s_\circ=0$ and $s_1\neq 0$ then we have

$$E_{ij}=\frac{1}{6}\begin{bmatrix} -8K^2s_1{}^2+\frac{\ddot{P}}{P}-\frac{\ddot{R}}{R}-\frac{\dot{P}\dot{R}}{PR}-\frac{P'R'}{P^3R}+ \\ +\frac{R''}{P^2R}+\frac{\dot{R}^2}{3R^2}-\frac{R'^2}{3P^2R^2}-\frac{\cot^2\theta}{3R^2} \end{bmatrix}[l_il_j-l_{(i}n_{j)}+n_in_j]-$$
$$-\frac{1}{2}\left[\frac{K}{R}(s_{1,\theta}-is_{1,\phi}\operatorname{cosec}\theta)\right][m_il_j-m_in_j]-$$
$$-\frac{1}{2}\left[-\frac{K}{R}(s_{1,\theta}+is_{1,\phi}\operatorname{cosec}\theta)\right][\bar{m}_il_j-\bar{m}_in_j]+$$
$$+\frac{1}{2}\begin{bmatrix} Ks_{1,t}+\frac{K}{P}s_{1,r}-2Ks_1\frac{\dot{P}}{P}+4Ks_1\frac{\dot{R}}{R}+\frac{4}{3}K^2s_\circ\bar{s}_\circ+\frac{8}{3}K^2s_1{}^2-\frac{\ddot{P}}{3P}+ \\ +\frac{\ddot{R}}{3R}+\frac{\dot{P}\dot{R}}{3PR}+\frac{P'R'}{3P^3R}-\frac{R''}{3P^2R}-\frac{\dot{R}^2}{3R^2}+\frac{R'^2}{3P^2R^2}+\frac{\cot^2\theta}{3R^2} \end{bmatrix}m_i\bar{m}_j+$$

$$+\frac{1}{2}\begin{bmatrix} -K\, s_{1,t} - \frac{K}{P}s_{1,r} + 2Ks_1\frac{\dot{P}}{P} - 4Ks_1\frac{\dot{R}}{R} + \frac{4}{3}K^2 s_\circ \bar{s}_\circ + \frac{8}{3}K^2 {s_1}^2 - \frac{\ddot{P}}{3P} + \\ + \frac{\ddot{R}}{3R} + \frac{\dot{P}\dot{R}}{3PR} + \frac{P'R'}{3P^3R} - \frac{R''}{3P^2R} - \frac{\dot{R}^2}{3R^2} + \frac{R'^2}{3P^2R^2} + \frac{\cot^2\theta}{3R^2} \end{bmatrix} \overline{m}_i m_j$$

and

$$H_{ij} = \left(2iKs_1\frac{R'}{PR}\right)[l_i l_j - l_{(i}n_{j)} + n_i n_j] -$$

$$-\frac{1}{2}\left(-\frac{i}{R}K(s_{1,\theta} - is_{1,\phi}\operatorname{cosec}\theta)\right)[l_{(i}m_{j)} - n_{(i}m_{j)}] -$$

$$-\frac{1}{2}\left(-\frac{i}{R}K(s_{1,\theta} + is_{1,\phi}\operatorname{cosec}\theta)\right)[l_{(i}\overline{m}_{j)} - n_{(i}\overline{m}_{j)}] +$$

$$+\frac{1}{2}\left(\frac{4i}{P}Ks_{1,r} + 4iKs_1\frac{R'}{PR}\right)m_{(i}\overline{m}_{j)}$$

Case (2): If $s_\circ \neq 0$ and $s_1 = 0$ then we have

$$E_{ij} = \frac{1}{6}\begin{bmatrix} \frac{K}{R}(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) - \frac{iK}{R}\operatorname{cosec}\theta(s_{\circ,\phi} - \bar{s}_{\circ,\phi}) - 4K^2 s_\circ \bar{s}_\circ + \\ + K(s_\circ + \bar{s}_\circ)\frac{\cot\theta}{R} + \frac{\ddot{P}}{P} - \frac{\ddot{R}}{R} - \frac{\dot{P}\dot{R}}{PR} - \frac{P'R'}{P^3R} + \frac{R''}{P^2R} + \\ + \frac{\dot{R}^2}{3R^2} - \frac{R'^2}{3P^2R^2} - \frac{\cot^2\theta}{3R^2} \end{bmatrix} [l_i l_j - l_{(i}n_{j)} + n_i n_j] -$$

$$-\frac{1}{2}\left[K\bar{s}_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right)\right][l_i m_j - n_i m_j] - \frac{1}{2}\left[Ks_\circ\left(\frac{\dot{R}}{R} - \frac{R'}{PR}\right)\right][l_i \overline{m}_j - n_i \overline{m}_j] -$$

$$-\frac{1}{2}\left[2K\bar{s}_\circ\frac{\dot{P}}{P} - K\bar{s}_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right)\right][m_i l_j - m_i n_j] -$$

$$-\frac{1}{2}\left[2Ks_\circ\frac{\dot{P}}{P} - Ks_\circ\left(\frac{\dot{R}}{R} + \frac{R'}{PR}\right)\right][\overline{m}_i l_j - \overline{m}_i n_j] +$$

$$+\frac{1}{2}\begin{bmatrix} -K\,\bar{s}_{\circ,t} + \frac{K}{P}\,\bar{s}_{\circ,r} - \frac{K}{3R}(s_{\circ,\theta} + \bar{s}_{\circ,\theta}) + \frac{iK}{3R}\operatorname{cosec}\theta(s_{\circ,\phi} - \bar{s}_{\circ,\phi}) - \\ -\frac{K(s_\circ + \bar{s}_\circ)\cot\theta}{3R} + \frac{4}{3}K^2 s_\circ \bar{s}_\circ - \frac{\ddot{P}}{3P} + \frac{\ddot{R}}{3R} + \frac{\dot{P}\dot{R}}{3PR} + \frac{P'R'}{3P^3R} - \\ -\frac{R''}{3P^2R} - \frac{\dot{R}^2}{3R^2} + \frac{R'^2}{3P^2R^2} + \frac{\cot^2\theta}{3R^2} \end{bmatrix} m_i \overline{m}_j +$$

$$+\frac{1}{2}\begin{bmatrix} -Ks_{\circ,t}+\frac{K}{P}s_{\circ,r}-\frac{K}{3R}(s_{\circ,\theta}+\bar{s}_{\circ,\theta})+\frac{iK}{3R}\cos ec\theta(s_{\circ,\phi}-\bar{s}_{\circ,\phi})- \\ -\frac{K(s_\circ+\bar{s}_\circ)\cot\theta}{3R}+\frac{4}{3}K^2 s_\circ\bar{s}_\circ-\frac{\ddot{P}}{3P}+\frac{\ddot{R}}{3R}+\frac{\dot{P}\dot{R}}{3PR}+\frac{P'R'}{3P^3R}- \\ -\frac{R''}{3P^2R}-\frac{\dot{R}^2}{3R^2}+\frac{R'^2}{3P^2R^2}+\frac{\cot^2\theta}{3R^2} \end{bmatrix}\bar{m}_i m_j$$

and

$$H_{ij}=-\frac{1}{2}\left(-iK\bar{s}_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)\right)[l_{(i}m_{j)}-n_{(i}m_{j)}]-\frac{1}{2}\left(iKs_\circ\left(\frac{\dot{R}}{R}+\frac{R'}{PR}\right)\right)[l_{(i}\bar{m}_{j)}-n_{(i}\bar{m}_{j)}]+$$

$$+\frac{1}{2}\left(iK(s_\circ-\bar{s}_\circ)\frac{\cot\theta}{R}\right)m_{(i}\bar{m}_{j)}+$$

$$+\frac{1}{2}\left(\frac{2i}{R}K(\bar{s}_{\circ,\theta}-i\bar{s}_{\circ,\phi}\cos ec\theta)-2iK\bar{s}_\circ\frac{\cot\theta}{R}+4iK^2\bar{s}_\circ{}^2\right)m_i m_j+$$

$$+\frac{1}{2}\left(-\frac{2i}{R}K(s_{\circ,\theta}+is_{\circ,\phi}\cos ec\theta)+2iKs_\circ\frac{\cot\theta}{R}-4iK^2 s_\circ{}^2\right)\bar{m}_i\bar{m}_j$$

Case (3): If $s_\circ=0$ and $s_1=0$ then we have

$$E_{ij}=\frac{1}{6}\left[\frac{\ddot{P}}{P}-\frac{\ddot{R}}{R}-\frac{\dot{P}\dot{R}}{PR}-\frac{P'R'}{P^3R}+\frac{R''}{P^2R}+\frac{\dot{R}^2}{3R^2}-\frac{R'^2}{3P^2R^2}-\frac{\cot^2\theta}{3R^2}\right][l_i l_j-l_{(i}n_{j)}+n_i n_j]-$$

$$+\frac{1}{2}\left[-\frac{\ddot{P}}{3P}+\frac{\ddot{R}}{3R}+\frac{\dot{P}\dot{R}}{3PR}+\frac{P'R'}{3P^3R}-\frac{R''}{3P^2R}-\frac{\dot{R}^2}{3R^2}+\frac{R'^2}{3P^2R^2}+\frac{\cot^2\theta}{3R^2}\right]m_{(i}\bar{m}_{j)}+$$

and

$H_{ij}=0$.

We notice from the above cases that the Spin influences the electric and magnetic part of Weyl tensor. In the absence of the Spin we see that the magnetic part of Weyl tensor vanishes.