CHAPTER II

A STUDY OF ELECTRIC AND MAGNETIC PARTS OF WEYL TENSOR IN KERR-NEWMAN SPACE-TIME.

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A STUDY OF ELECTRIC AND MAGNETIC PARTS OF WEYL TENSOR IN KERR-NEWMAN SPACE-TIME.

1. Introduction:

The purpose of this chapter is

(i) To exploit the NP tetrad formalism for the analysis of electric and magnetic parts of the Weyl tensor.

(ii) To determine explicitly the electric part and magnetic part of Weyl tensor in terms of angular momentum and charge of the gravitating body in the Kerr-Newman space-time.

(iii) To identify the cause of electric field and magnetic field in the Kerr-Newman space-time.

All static space-times are known to be purely electric space-times. There are very few known examples of purely magnetic space-times; one such example is due to Misra et al. (1968). McIntosh et al. (1994) have proclaimed that there may not be purely magnetic non-flat vacuum solution. Haddow (1995) has shown that all vacuum purely magnetic solutions are of Petro type I.

By noting the important role of purely electric and purely magnetic space-times, Ahsan (1999) has investigated the relativistic problems having purely magnetic or electric part of Weyl tensor. He has shown that the Weyl tensor for Gödel Universe is purely electric but not magnetic. Hasmani et al. (2008) have recently shown that the parameter related to the vorticity of the fluid with reference to the Gödel universe causes the electric field.

The material of this chapter is organized as follows: In the Section 2, we exploit the 'amazingly' useful Newman-Penrose tetrad formalism to study the Kerr-Newman space-time. The tetrad components of Connection 1-forms, Curvature 2-forms with respect to the chosen basis vectors are determined. In the next two Sections, the tetrad components of Curvature tensor and Weyl tensor that are pertinent to study the electric and the magnetic parts of Weyl tensor are described. The expressions for the electric and the magnetic parts of the Weyl tensor with reference to the Kerr-Newman space-time are obtained in terms of basis of the tetrad in the Section 5. It has been shown that both the angular momentum per unit mass and the electric charge of the gravitating body are the sources of the electric part and the magnetic part of the Weyl tensor. We have seen that if the angular momentum per unit mass of a gravitating body is zero, the magnetic part of the Weyl tensor ceases to be zero in the Kerr-Newman space-time while the electric part still exists.

2. The Kerr-Newman space-time:

In 1965, Ezra Newman found the axisymmetric solution of Einstein's field equation for black hole which is both rotating and electrically charged. It is generalization of Kerr metric for an uncharged spinning point mass which had been discovered by Roy Kerr (1963). It describes the geometry of space-time in the vicinity of rotating mass 'm' with charge 'e'. It is known as the Kerr-Newman space-time and it is characterized by the metric

$$ds^2 = [1 - R^{-2}(2mr - e^2)]dt^2 + 2aR^{-2}(2mr - e^2)\sin^2\theta\, dt\, d\phi - \frac{R^2}{\Delta}dr^2 - R^2 d\theta^2 - $$
$$ - [(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta] R^{-2} \sin^2\theta\, d\phi^2 \qquad \ldots(2.1)$$

where, $R^2 = \overline{R}\,\overline{R}^* = r^2 + a^2\cos^2\theta$, $\overline{R} = r + ia\cos\theta$,

$\Delta = r^2 - 2mr + a^2 + e^2$, $\overline{R}^* = r - ia\cos\theta$.

and m = mass,

a = angular momentum per unit mass,

e = the charge of the gravitating body.

The covariant components of the metric tensor are given by

$$g_{11} = 1 - R^{-2}(2mr - e^2) \; ; \quad g_{14} = aR^{-2}(2mr - e^2)\sin^2\theta \; ;$$
$$g_{22} = -\frac{R^2}{\Delta} \; ; \quad g_{33} = -R^2 \; ; \qquad \ldots(2.2)$$
$$g_{44} = -[(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta] R^{-2} \sin^2\theta .$$

To obtain the tetrad vectors with respect to the given space-time, we express the metric (2.1) in terms of the basis 1-forms as

$$ds^2 = 2\theta^1\theta^2 - 2\theta^3\theta^4 \qquad \ldots(2.3)$$

where the basis 1-forms θ^α are

$$\theta^1 = \frac{\Delta}{2R^2}dt + \frac{1}{2}dr - \frac{a\Delta}{2R^2}\sin^2\theta\, d\phi \ ,$$

$$\theta^2 = dt - \frac{R^2}{\Delta}dr - a\sin^2\theta\, d\phi \ ,$$

$$\theta^3 = \frac{ia\sin\theta}{\sqrt{2}\overline{R}^*}dt + \frac{R^2}{\sqrt{2}\overline{R}^*}d\theta - \frac{i(r^2+a^2)}{\sqrt{2}\overline{R}^*}\sin\theta\, d\phi \ , \qquad \ldots(2.4)$$

$$\theta^4 = -\frac{ia\sin\theta}{\sqrt{2}\overline{R}}dt + \frac{R^2}{\sqrt{2}\overline{R}}d\theta + \frac{i(r^2+a^2)}{\sqrt{2}\overline{R}}\sin\theta\, d\phi \, .$$

Now, the definition of basis 1-forms $\theta^\alpha = e_i^{(\alpha)}dx^i$ and the equation (2.4) gives the vector fields of the NP complex null tetrad as

$$l^i = \frac{1}{\Delta}((r^2+a^2)\delta_1^i + \Delta\delta_2^i + a\delta_4^i) \ ,$$

$$n^i = \frac{1}{2R^2}((r^2+a^2)\delta_1^i - \Delta\delta_2^i + a\delta_4^i) \ , \qquad \ldots(2.5)$$

$$m^i = \frac{1}{\sqrt{2}\overline{R}}(ia\sin\theta\delta_1^i + \delta_3^i + i\operatorname{cosec}\theta\delta_4^i)$$

where l^i and n^i are real null vector fields and m^i is a complex null vector field. The complex conjugate $\overline{m}^i$ can be obtained by taking the complex conjugate of m^i. These vector fields of the tetrad satisfy the orthonormal conditions

$$l_i n^i = -m_i\overline{m}^i = 1 \qquad \ldots(2.6)$$

and all other inner products are zero.

Solving four equations in (2.4) we obtain,

$$dt = \frac{(r^2+a^2)}{\Delta}\theta^1 + \frac{(r^2+a^2)}{2R^2}\theta^2 + \frac{ia\sin\theta}{\sqrt{2}\overline{R}}\theta^3 - \frac{ia\sin\theta}{\sqrt{2}\overline{R}^*}\theta^4 \ ,$$

$$dr = \theta^1 - \frac{\Delta}{2R^2}\theta^2 \ ,$$

$$d\theta = \frac{1}{\sqrt{2}}\left(\frac{1}{\overline{R}}\theta^3 + \frac{1}{\overline{R}^*}\theta^4\right) , \qquad \ldots(2.7)$$

$$d\phi = \frac{a}{\Delta}\theta^1 + \frac{a}{2R^2}\theta^2 + \frac{i\operatorname{cosec}\theta}{\sqrt{2}\overline{R}}\theta^3 - \frac{i\operatorname{cosec}\theta}{\sqrt{2}\overline{R}^*}\theta^4 \, .$$

Taking the wedge product of equation (2.7) we get

$$dr \wedge dt = \frac{(r^2+a^2)}{R^2}\theta^{12} + \frac{ia\sin\theta}{\sqrt{2}\bar{R}}\theta^{13} - \frac{ia\sin\theta}{\sqrt{2}\bar{R}^*}\theta^{14} - \frac{ia\Delta\sin\theta}{2\sqrt{2}R^2\bar{R}}\theta^{23} + \frac{ia\Delta\sin\theta}{2\sqrt{2}R^2\bar{R}^*}\theta^{24},$$

$$d\theta \wedge dt = -\frac{(r^2+a^2)}{\sqrt{2}\Delta\bar{R}}\theta^{13} - \frac{(r^2+a^2)}{\sqrt{2}\Delta\bar{R}^*}\theta^{14} - \frac{(r^2+a^2)}{2\sqrt{2}R^2\bar{R}}\theta^{23} - \frac{(r^2+a^2)}{2\sqrt{2}R^2\bar{R}^*}\theta^{24} - \frac{ia\sin\theta}{R^2}\theta^{34}$$

$$dr \wedge d\theta = \frac{1}{\sqrt{2}\bar{R}}\theta^{13} + \frac{1}{\sqrt{2}\bar{R}^*}\theta^{14} - \frac{\Delta}{2\sqrt{2}R^2\bar{R}}\theta^{23} - \frac{\Delta}{2\sqrt{2}R^2\bar{R}^*}\theta^{24},$$

$$dr \wedge d\phi = \frac{a}{R^2}\theta^{12} + \frac{i\operatorname{cosec}\theta}{\sqrt{2}\bar{R}}\theta^{13} - \frac{i\operatorname{cosec}\theta}{\sqrt{2}\bar{R}^*}\theta^{14} - \frac{i\Delta\operatorname{cosec}\theta}{2\sqrt{2}R^2\bar{R}}\theta^{23} + \frac{i\Delta\operatorname{cosec}\theta}{2\sqrt{2}R^2\bar{R}^*}\theta^{24},$$

$$d\theta \wedge d\phi = -\frac{a}{\sqrt{2}\Delta\bar{R}}\theta^{13} - \frac{a}{\sqrt{2}\Delta\bar{R}^*}\theta^{14} - \frac{a}{2\sqrt{2}R^2\bar{R}}\theta^{23} - \frac{a}{2\sqrt{2}R^2\bar{R}^*}\theta^{24} - \frac{i\operatorname{cosec}\theta}{R^2}\theta^{34}$$

$$dt \wedge d\phi = -\frac{i(a^2\sin\theta+(r^2+a^2)\operatorname{cosec}\theta)}{\sqrt{2}\Delta\bar{R}}\theta^{13} + \frac{i(a^2\sin\theta+(r^2+a^2)\operatorname{cosec}\theta)}{\sqrt{2}\Delta\bar{R}^*}\theta^{14} - $$
$$-\frac{i(a^2\sin\theta-(r^2+a^2)\operatorname{cosec}\theta)}{2\sqrt{2}R^2\bar{R}}\theta^{23} + \frac{i(a^2\sin\theta-(r^2+a^2)\operatorname{cosec}\theta)}{2\sqrt{2}R^2\bar{R}^*}\theta^{24} +$$
$$+\frac{a}{R^2}\theta^{34}$$

...(2.8)

These equations will be used for computation purpose in the sequel.

Taking the exterior derivative of the basis 1-forms θ^α in equation (2.4) one can obtain

$$d\theta^1 = \frac{\partial}{\partial r}\left(\frac{\Delta}{2R^2}\right)dr\wedge dt + \frac{\partial}{\partial\theta}\left(\frac{\Delta}{2R^2}\right)d\theta\wedge dt - \frac{\partial}{\partial r}\left(\frac{a\Delta\sin^2\theta}{2R^2}\right)dr\wedge d\phi -$$
$$-\frac{\partial}{\partial\theta}\left(\frac{a\Delta\sin^2\theta}{2R^2}\right)d\theta\wedge d\phi$$

Using equations (2.8) and on simplifying we get

$$d\theta^1 = R^{-4}[R^2(r-m)-\Delta r]\theta^{12} + iaos\theta\Delta R^{-4}\theta^{34},$$

Similarly,

$$d\theta^2 = \frac{\sqrt{2}a^2\sin\theta\cos\theta}{R^2\bar{R}}\theta^{23} + \frac{\sqrt{2}a^2\sin\theta\cos\theta}{R^2\bar{R}^*}\theta^{24} + \frac{2ia\cos\theta}{R^2}\theta^{34},$$

$$d\theta^3 = -\frac{\sqrt{2}iar\sin\theta}{R^2\bar{R}^*}\theta^{12} + \frac{1}{\bar{R}}\theta^{13} - \frac{\Delta}{2R^2\bar{R}}\theta^{23} - \frac{(\bar{R}^*\cot\theta - ia\sin\theta)}{\sqrt{2}(\bar{R}^*)^2}\theta^{34},$$

$$d\theta^4 = \frac{\sqrt{2}\, iar\sin\theta}{R^2 \overline{R}}\theta^{12} + \frac{1}{\overline{R}^*}\theta^{14} - \frac{\Delta}{2R^2\overline{R}^*}\theta^{24} + \frac{(\overline{R}\cot\theta + ia\sin\theta)}{\sqrt{2}(\overline{R})^2}\theta^{34} . \quad \ldots(2.9)$$

3. Tetrad Components of Connection 1-Forms $\omega^\alpha{}_\beta$ and Curvature 2-Forms $\Omega^\alpha{}_\beta$:

To find tetrad components of Connection 1-forms, we start with Cartan's first equation of structure

$$d\theta^\alpha = -\omega^\alpha{}_\beta \wedge \theta^\beta \qquad \alpha, \beta = 1, 2, 3, 4. \quad \ldots(3.1)$$

where $\omega^\alpha{}_\beta$ are the components of connection 1- forms such that

$$\omega_{\alpha\beta} + \omega_{\beta\alpha} = 0$$

$$\Rightarrow \qquad \omega_{11} = \omega_{22} = \omega_{33} = \omega_{44} = 0$$

Also $\qquad \omega^\alpha{}_\beta = \eta^{\alpha\sigma}\,\omega_{\sigma\beta}$

This gives $\qquad \omega^1{}_2 = \eta^{1\sigma}\,\omega_{\sigma 2} = \omega_{22}$

$$\Rightarrow \qquad \omega^1{}_2 = \omega_{22} = 0$$

Similarly, we obtain

$$\begin{aligned}
&\omega^1{}_1 = -\omega^2{}_2 = \omega_{12} \ , \\
&\omega^1{}_3 = \omega^4{}_2 = \omega_{23} \ , \\
&\omega^2{}_3 = \omega^4{}_1 = \omega_{13} \ , \\
&\omega^3{}_3 = -\omega^4{}_4 = \omega_{34} \ , \\
&\omega^1{}_4 = \omega^3{}_2 = \omega_{24} \ , \\
&\omega^2{}_4 = \omega^3{}_1 = \omega_{14} \, .
\end{aligned} \quad \ldots(3.2)$$

These Connection 1-forms are defined by

$$\omega^\alpha{}_\beta = \gamma^\alpha{}_{\beta\delta}\,\theta^\delta$$

where $\qquad \gamma^\alpha{}_{\beta\delta} = -e^{(\alpha)}_{i;j}\, e^{\,i}_{(\beta)}\, e^{\,j}_{(\delta)}$.

It can be shown that

$$\omega_{12} = -[(\varepsilon + \overline{\varepsilon})\theta^1 + (\gamma + \overline{\gamma})\theta^2 + (\overline{\alpha} + \beta)\theta^3 + (\alpha + \overline{\beta})\theta^4] \ ,$$

$$\omega_{13} = -(\kappa\theta^1 + \tau\theta^2 + \sigma\theta^3 + \rho\theta^4) ,$$

$$\omega_{14} = -(\bar{\kappa}\theta^1 + \bar{\tau}\theta^2 + \bar{\sigma}\theta^3 + \bar{\rho}\theta^4) ,$$

$$\omega_{23} = \bar{\pi}\theta^1 + \bar{\nu}\theta^2 + \bar{\lambda}\theta^3 + \bar{\mu}\theta^4 ,$$

$$\omega_{24} = \pi\theta^1 + \nu\theta^2 + \lambda\theta^3 + \mu\theta^4 , \qquad \ldots(3.3)$$

$$\omega_{34} = (\varepsilon - \bar{\varepsilon})\theta^1 + (\gamma - \bar{\gamma})\theta^2 - (\bar{\alpha} - \beta)\theta^3 + (\alpha - \bar{\beta})\theta^4 .$$

Also from equation (3.1) we obtain

$$d\theta^1 = -\omega^1{}_1 \wedge \theta^1 - \omega^1{}_2 \wedge \theta^2 - \omega^1{}_3 \wedge \theta^3 - \omega^1{}_4 \wedge \theta^4 ,$$

$$d\theta^2 = -\omega^2{}_1 \wedge \theta^1 - \omega^2{}_2 \wedge \theta^2 - \omega^2{}_3 \wedge \theta^3 - \omega^2{}_4 \wedge \theta^4 ,$$

$$d\theta^3 = -\omega^3{}_1 \wedge \theta^1 - \omega^3{}_2 \wedge \theta^2 - \omega^3{}_3 \wedge \theta^3 - \omega^3{}_4 \wedge \theta^4 , \qquad \ldots(3.4)$$

$$d\theta^4 = -\omega^4{}_1 \wedge \theta^1 - \omega^4{}_2 \wedge \theta^2 - \omega^4{}_3 \wedge \theta^3 - \omega^4{}_4 \wedge \theta^4 .$$

To obtain non-vanishing components of Connection 1-forms we choose

$$\omega^1{}_1 = A_1\theta^1 + A_2\theta^2 + A_3\theta^3 + A_4\theta^4 ,$$

$$\omega^1{}_3 = B_1\theta^1 + B_2\theta^2 + B_3\theta^3 + B_4\theta^4 ,$$

$$\omega^1{}_4 = C_1\theta^1 + C_2\theta^2 + C_3\theta^3 + C_4\theta^4 ,$$

$$\omega^2{}_3 = D_1\theta^1 + D_2\theta^2 + D_3\theta^3 + D_4\theta^4 , \qquad \ldots(3.5)$$

$$\omega^2{}_4 = E_1\theta^1 + E_2\theta^2 + E_3\theta^3 + E_4\theta^4 ,$$

$$\omega^3{}_3 = F_1\theta^1 + F_2\theta^2 + F_3\theta^3 + F_4\theta^4 .$$

where all the coefficients are to be determined. Substituting these values in equations (3.4) we obtain

$$d\theta^1 = A_2\theta^{12} + (A_3 - B_1)\theta^{13} + (A_4 - C_1)\theta^{14} - B_2\theta^{23} - C_2\theta^{24} + (B_4 - C_3)\theta^{34} ,$$

Similarly, we obtain

$$d\theta^2 = A_1\theta^{12} - D_1\theta^{13} - E_1\theta^{14} - (A_3 + D_2)\theta^{23} - (A_4 + E_2)\theta^{24} + (D_4 - E_3)\theta^{34} ,$$

$$d\theta^3 = (E_2 - C_1)\theta^{12} + (E_3 - F_1)\theta^{13} + E_4\theta^{14} + (C_3 - F_2)\theta^{23} + C_4\theta^{24} + F_4\theta^{34} ,$$

$$d\theta^4 = (D_2 - B_1)\theta^{12} + D_3\theta^{13} + (D_4 + F_1)\theta^{14} + B_3\theta^{23} + (B_4 + F_2)\theta^{24} + F_3\theta^{34} .$$

$$\ldots(3.6)$$

Now using the equations (2.9) in (3.6) and comparing the corresponding coefficients on both sides, we readily obtain

$$A_1 = B_2 = B_3 = C_4 = C_2 = D_1 = D_3 = E_1 = E_4 = F_1 = 0 ,$$

$$A_2 = R^{-4}[R^2(r-m) - \Delta r] ; \qquad A_3 = B_1 = -\frac{ia\sin\theta}{\sqrt{2}(\overline{R})^2} ;$$

$$A_4 = C_1 = \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2} ; \qquad B_4 = -\frac{\Delta}{2R^2\overline{R}} ;$$

$$C_3 = -\frac{\Delta}{2R^2\overline{R}^*} ; \qquad D_2 = E_2 = -\frac{ia\sin\theta}{\sqrt{2}R^2} ;$$

$$D_4 = \frac{1}{\overline{R}^*} ; \qquad E_3 = \frac{1}{\overline{R}} ;$$

$$F_2 = -\frac{ia\Delta\cos\theta}{R^4} ; \qquad F_3 = \frac{(\overline{R}\cot\theta + ia\sin\theta)}{\sqrt{2}(\overline{R})^2} ;$$

$$F_4 = -\frac{(\overline{R}^*\cot\theta - ia\sin\theta)}{\sqrt{2}(\overline{R}^*)^2} .$$

Consequently, the non-vanishing tetrad components of Connection 1-forms are:

$$\omega^1{}_1 = -\omega^2{}_2 = R^{-4}[R^2(r-m) - \Delta r]\theta^2 - \frac{ia\sin\theta}{\sqrt{2}(\overline{R})^2}\theta^3 + \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\theta^4 ,$$

$$\omega^1{}_3 = \omega^4{}_2 = -\frac{ia\sin\theta}{\sqrt{2}(\overline{R})^2}\theta^1 - \frac{\Delta}{2R^2\overline{R}}\theta^4 ,$$

$$\omega^1{}_4 = \omega^3{}_2 = \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\theta^1 - \frac{\Delta}{2R^2\overline{R}^*}\theta^3 ,$$

$$\omega^2{}_3 = \omega^4{}_1 = \frac{ia\sin\theta}{\sqrt{2}R^2}\theta^2 + \frac{1}{\overline{R}^*}\theta^4 , \qquad \ldots(3.7)$$

$$\omega^2{}_4 = \omega^3{}_1 = -\frac{ia\sin\theta}{\sqrt{2}R^2}\theta^2 + \frac{1}{\overline{R}}\theta^3 ,$$

$$\omega^3{}_3 = -\omega^4{}_4 = -\frac{ia\cos\theta\Delta}{R^4}\theta^2 + \left(\frac{\cot\theta}{\sqrt{2}\overline{R}} + \frac{ia\sin\theta}{\sqrt{2}(\overline{R})^2}\right)\theta^3 - \left(\frac{\cot\theta}{\sqrt{2}\overline{R}^*} - \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\right)\theta^4 .$$

Now comparing the coefficients of basis 1-forms θ^α of equations (3.3) and (3.7) we obtain the spin coefficients as

$$\kappa = \nu = \lambda = \sigma = \varepsilon = 0 ,$$

$$\rho = -\frac{1}{\overline{R}^*}, \qquad \mu = -\frac{\Delta}{2R^2\overline{R}^*}, \qquad \alpha = \pi - \overline{\beta}, \qquad \gamma = \mu + \frac{r-m}{2R^2},$$

$$\tau = -\frac{ia\sin\theta}{\sqrt{2}R^2}, \qquad \pi = \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}, \qquad \beta = \frac{\cot\theta}{2\sqrt{2}\overline{R}} \qquad \ldots(3.8)$$

Similarly, to find the tetrad components of Curvature 2-forms $\Omega^\alpha{}_\beta$, we start with Cartan's Second equations of structure

$$\Omega^\alpha{}_\beta = d\omega^\alpha{}_\beta + \omega^\alpha{}_\gamma \wedge \omega^\gamma{}_\beta, \qquad \alpha, \beta, \gamma = 1, 2, 3, 4. \qquad \ldots(3.9)$$

where from equations (3.7), by taking the exterior derivatives of Connection 1-forms we obtain

$$d\omega^1{}_1 = d[R^{-2}(r-m) - R^{-4}\Delta r] \wedge \theta^2 + R^{-4}[R^2(r-m) - \Delta r]d\theta^2 - \frac{ia\sin\theta}{\sqrt{2}(\overline{R})^2}d\theta^3 -$$
$$-\frac{ia}{\sqrt{2}}d\left[\frac{\sin\theta}{(\overline{R})^2}\right] \wedge \theta^3 + \frac{ia}{\sqrt{2}}d\left[\frac{\sin\theta}{(\overline{R}^*)^2}\right] \wedge \theta^4 + \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}d\theta^4$$

On simplifying, we get

$$d\omega^1{}_1 = \{R^{-2} - [4r(r-m) + \Delta]R^{-4} + [4\Delta r^2 - 2a^2r^2\sin^2\theta]R^{-6}\}\theta^{12} + \frac{ia\sin\theta}{\sqrt{2}(\overline{R})^3}\theta^{13} -$$
$$-\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^3}\theta^{14} + \left[\sqrt{2}a^2\sin\theta\cos\theta\frac{\Delta r}{R^6\overline{R}} - \frac{ia\Delta\sin\theta}{2\sqrt{2}R^2(\overline{R})^3}\right]\theta^{23} +$$
$$+\left[\sqrt{2}a^2\sin\theta\cos\theta\frac{\Delta r}{R^6\overline{R}^*} + \frac{ia\Delta\sin\theta}{2\sqrt{2}R^2(\overline{R}^*)^3}\right]\theta^{24} +$$
$$+\left[2ia\cos\theta R^{-6}[R^2(r-m) - \Delta r] + 2iar\cos\theta R^{-4} + 4ia^3r\sin^2\theta\cos\theta R^{-6}\right]\theta^{34}$$

Similarly, we obtain

$$d\omega^1{}_3 = \left[\frac{ia\Delta\sin\theta}{\sqrt{2}R^2(\overline{R})^3} - \frac{ia(r-m)\sin\theta}{\sqrt{2}R^2(\overline{R})^2}\right]\theta^{12} + \left[\frac{ia\cos\theta}{2(\overline{R})^3} - \frac{a^2\sin^2\theta}{(\overline{R})^4}\right]\theta^{13} +$$
$$+\left[\frac{ia\cos\theta}{2R^2\overline{R}} - \frac{a^2\sin^2\theta}{R^2(\overline{R})^2} - \frac{(r-m)}{R^2\overline{R}} + \frac{\Delta r}{R^4\overline{R}} + \frac{\Delta}{2R^2(\overline{R})^2} - \frac{\Delta}{2R^4}\right]\theta^{14} +$$
$$+\left[\frac{\Delta(r-m)}{2R^4\overline{R}} - \frac{\Delta^2 r}{2R^6\overline{R}} - \frac{\Delta^2}{4R^4(\overline{R})^2} + \frac{\Delta^2}{4R^6}\right]\theta^{24} +$$
$$+\left[-\frac{\Delta\cot\theta}{2\sqrt{2}R^2(\overline{R})^2} - \frac{2ia\Delta\sin\theta}{2\sqrt{2}R^2(\overline{R})^3}\right]\theta^{34},$$

$$d\omega^2{}_3 = -\frac{ia\cos\theta}{2R^2\overline{R}}\theta^{23} - \frac{ia\cos\theta}{2R^2\overline{R}^*}\theta^{24} +$$

$$+\left[\frac{ia\sin\theta}{\sqrt{2}R^2\overline{R}} - \frac{ia\sin\theta}{\sqrt{2}R^2\overline{R}^*} - \frac{\sqrt{2}a^2\sin\theta\cos\theta}{R^4} + \frac{\cot\theta}{\sqrt{2}R^2}\right]\theta^{34},$$

$$d\omega^3{}_3 = \left[\frac{a^2 r\sin^2\theta}{R^4\overline{R}} - \frac{a^2 r\sin^2\theta}{R^4\overline{R}^*} + \frac{4iar\Delta\cos\theta}{R^6} - \frac{2ia(2r-m)\cos\theta}{R^4}\right]\theta^{12} +$$

$$+\left[\frac{ia\Delta\sin\theta}{\sqrt{2}R^4\overline{R}} + \frac{ia\Delta\sin\theta}{2\sqrt{2}R^2(\overline{R})^3} - \frac{3\sqrt{2}ia\Delta\sin\theta\cos^2\theta}{R^6\overline{R}}\right]\theta^{23} +$$

$$+\left[-\frac{ia\Delta\sin\theta}{\sqrt{2}R^4\overline{R}^*} + \frac{ia\Delta\sin\theta}{2\sqrt{2}R^2(\overline{R}^*)^3} + \frac{\sqrt{2}ia^3\Delta\sin\theta\cos^2\theta}{R^6\overline{R}^*}\right]\theta^{24} + \qquad \ldots(3.10)$$

$$+\left[\begin{array}{l}\frac{1}{R^2} - \frac{2ia\cos\theta}{R^2\overline{R}} + \frac{a^2\sin^2\theta}{R^2(\overline{R})^2} + \frac{2ia\cos\theta}{R^2\overline{R}^*} + \\ + \frac{a^2\sin^2\theta}{R^2(\overline{R}^*)^2} - \frac{a^2\sin^2\theta}{R^4} + \frac{2a^2\cos^2\theta}{R^6}\end{array}\right]\theta^{34}$$

From equations (3.9) we obtain for $\alpha = \beta = 1$,

$$\Omega^1{}_1 = d\omega^1{}_1 + \omega^1{}_\gamma \wedge \omega^\gamma{}_1$$

$$\Omega^1{}_1 = d\omega^1{}_1 + \omega^1{}_3 \wedge \omega^3{}_1 + \omega^1{}_4 \wedge \omega^4{}_1$$

Using equations (3.7) and (3.10) we obtain

$$\Omega^1{}_1 = \{R^{-2} - [4r(r-m)+\Delta]R^{-4} + [4\Delta r^2 - 2a^2r^2\sin^2\theta]R^{-6}\}\theta^{12} - \frac{ia\sin\theta}{\sqrt{2}(\overline{R})^3}\theta^{13} +$$

$$+\frac{ia\sin\theta}{\sqrt{2}(\overline{R})^3}\theta^{13} - \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^3}\theta^{14} + \left[\sqrt{2}a^2\sin\theta\cos\theta\frac{\Delta r}{R^6\overline{R}} - \frac{ia\Delta\sin\theta}{2\sqrt{2}R^2(\overline{R})^3}\right]\theta^{23} +$$

$$+\left[\sqrt{2}a^2\sin\theta\cos\theta\frac{\Delta r}{R^6\overline{R}^*} + \frac{ia\Delta\sin\theta}{2\sqrt{2}R^2(\overline{R}^*)^3}\right]\theta^{24} + \left[\frac{\Delta}{2R^2(\overline{R})^2} - \frac{\Delta}{2R^2(\overline{R}^*)^2}\right]\theta^{34} +$$

$$+\left[2ia\cos\theta R^{-6}[R^2(r-m) - \Delta r] + 2iar\cos\theta R^{-4} + 4ia^3 r\sin^2\theta\cos\theta R^{-6}\right]\theta^{34}$$

$$+\left[-\frac{a^2\sin^2\theta}{2R^2(\overline{R})^2} - \frac{a^2\sin^2\theta}{2R^2(\overline{R}^*)^2}\right]\theta^{12} + \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^3}\theta^{14} - \frac{ia\Delta\sin\theta}{2\sqrt{2}R^4\overline{R}}\theta^{24} + \frac{ia\Delta\sin\theta}{2\sqrt{2}R^4\overline{R}^*}\theta^{23}$$

On simplifying we get

$$\Omega^1{}_1 = -\Omega^2{}_2 = \left[\frac{4mr}{R^4} - \frac{(3r^2 - a^2\cos^2\theta)}{R^6}(2mr - e^2)\right]\theta^{12} +$$

$$+\frac{2ia\cos\theta}{R^6}\left[2r(2mr - e^2) - mR^2\right]\theta^{34}$$

Similarly, we obtain

$$\Omega^1{}_3 = \Omega^4{}_2 = \left[\frac{m}{R^2\,\overline{R}} - \frac{(2mr-e^2)}{R^2\,(\overline{R})^2}\right]\theta^{14}\ ,$$

$$\Omega^1{}_4 = \Omega^3{}_2 = \left[\frac{m}{R^2\,\overline{R}^*} - \frac{(2mr-e^2)}{R^2\,(\overline{R}^*)^2}\right]\theta^{13}\ ,$$

$$\Omega^2{}_3 = \Omega^4{}_1 = \left[\frac{m}{R^2\,\overline{R}^*} - \frac{(2mr-e^2)}{R^2\,(\overline{R}^*)^2}\right]\theta^{24}\ ,$$

$$\Omega^2{}_4 = \Omega^3{}_1 = \left[\frac{m}{R^2\,\overline{R}} - \frac{(2mr-e^2)}{R^2\,(\overline{R})^2}\right]\theta^{23}\ , \qquad \ldots(3.11)$$

$$\Omega^3{}_3 = -\,\Omega^4{}_4 = \frac{2ia\cos\theta}{R^6}\left[mR^2 - 2r(2mr-e^2)\right]\theta^{12} + \frac{(r^2-3a^2\cos^2\theta)}{R^6}(2mr-e^2)\,\theta^{34}$$

and all other are zero.

4. Tetrad components of Curvature tensor, Ricci tensor and Weyl Tensor:

To find tetrad components of Curvature tensor we start with the definition of Curvature 2-forms as

$$\Omega^\alpha{}_\beta = \frac{1}{2}R^\alpha{}_{\beta\gamma\delta}\,\theta^{\gamma\delta} \qquad \ldots(4.1)$$

By giving different values to $\alpha, \beta, \gamma, \delta$ from 1 to 4, equation (4.1) gives

$$\Omega^1{}_1 = \frac{1}{2}\eta^{1\sigma}\,R_{\sigma 1\gamma\delta}\,\theta^{\gamma\delta}$$

$$\Omega^1{}_1 = \frac{1}{2}\eta^{12}\,R_{21\gamma\delta}\,\theta^{\gamma\delta}$$

$$\Omega^1{}_1 = -R_{1212}\theta^{12} - R_{1213}\theta^{13} - R_{1214}\theta^{14} - R_{1223}\theta^{23} - R_{1224}\theta^{24} - R_{1234}\theta^{34}$$

Comparing the coefficients of basis 2-forms of this equation with the equation (3.11) we get

$$R^1{}_{112} = -R_{1212} = \frac{4mr}{R^4} - \frac{(3r^2-a^2\cos^2\theta)}{R^6}(2mr-e^2)\,,$$

$$R^1{}_{134} = -R^3{}_{312} = -R_{1234} = \frac{2ia\cos\theta}{R^6}[2r(2mr-e^2) - mR^2]\ ,$$

Similarly, we obtain

$$R^1{}_{314} = R^2{}_{423} = R_{2314} = \frac{m}{R^2\overline{R}} - \frac{(2mr - e^2)}{R^2(\overline{R})^2},$$

$$R^1{}_{413} = R^2{}_{324} = R_{2413} = \frac{m}{R^2\overline{R}^*} - \frac{(2mr - e^2)}{R^2(\overline{R}^*)^2},$$

$$R^3{}_{334} = R_{3434} = R^{-6}(2mr - e^2)(r^2 - 3a^2\cos^2\theta) \qquad ...(4.2)$$

and all other are zero.

Also, we know tetrad components of Ricci tensor $R_{\alpha\beta}$ and Ricci scalar R can be defined as

$$R_{\alpha\beta} = \eta^{\sigma\delta} R_{\sigma\alpha\beta\delta}$$

$$R_{\alpha\beta} = R_{1\alpha\beta 2} + R_{2\alpha\beta 1} - R_{3\alpha\beta 4} - R_{4\alpha\beta 3}$$

and $$R = R_{\alpha\beta}\,\eta^{\alpha\beta} \qquad ...(4.3)$$

$$R = R_{12} + R_{21} - R_{34} - R_{43}$$

Therefore,

$$R_{12} = R_{1122} + R_{2121} - R_{3124} - R_{4123}$$

$$= R_{1212} + R_{1324} + R_{1423}$$

$$R_{12} = -\frac{4mr}{R^4} + \frac{6mr^3}{R^6} - \frac{3r^2e^2}{R^6} - \frac{2mra^2\cos^2\theta}{R^6} + \frac{a^2e^2\cos^2\theta}{R^6} + \frac{2mr}{R^4} - \\ - \frac{2(2mr - e^2)}{R^6}(r^2 - a^2\cos^2\theta)$$

$$R_{12} = -\frac{e^2}{R^4}$$

Similarly, we get

$$R_{34} = R_{21} = R_{43} = -\frac{e^2}{R^4} \qquad ...(4.4)$$

and all other are zero.

Thus, Ricci scalar R = 0.

Now the tetrad components of Weyl tensor are given by

$$C_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} + \frac{1}{2}(\eta_{\alpha\gamma}R_{\beta\delta} + \eta_{\beta\delta}R_{\alpha\gamma} - \eta_{\alpha\delta}R_{\beta\gamma} - \eta_{\beta\gamma}R_{\alpha\delta}) + \frac{R}{6}(\eta_{\alpha\delta}\eta_{\beta\gamma} - \eta_{\alpha\gamma}\eta_{\beta\delta})$$

...(4.5)

This gives the non-vanishing tetrad components of Weyl tensor as follows

$$C_{1212} = R_{1212} + \frac{1}{2}(\eta_{11}R_{22} + \eta_{22}R_{11} - \eta_{12}R_{21} - \eta_{21}R_{12}) - \frac{R}{6}(\eta_{12}\eta_{21} - \eta_{11}\eta_{22})$$

$$C_{1212} = R_{1212} + \frac{1}{2}(-R_{21} - R_{12})$$

$$C_{1212} = R_{1212} - R_{12}$$

$$C_{1212} = -\frac{4mr}{R^4} + \frac{(3r^2 - a^2\cos^2\theta)}{R^6}(2mr - e^2) + \frac{e^2}{R^4}$$

Similarly,

$$C_{1234} = -\frac{2ia\cos\theta}{R^6}[2r(2mr - e^2) - mR^2],$$

$$C_{2314} = \frac{m}{R^2\overline{R}} - \frac{(2mr - e^2)}{R^2(\overline{R})^2},$$

$$C_{2413} = \frac{m}{R^2\overline{R}^*} - \frac{(2mr - e^2)}{R^2(\overline{R}^*)^2}, \qquad \text{...(4.6)}$$

$$C_{3434} = R^{-6}(2mr - e^2)(r^2 - 3a^2\cos^2\theta) - \frac{e^2}{R^4}.$$

We exploit these results for construction of the electric part and the magnetic part of the Weyl tensor and study the consequences of angular momentum and charge of the gravitating body in causing electric and magnetic parts of the Weyl tensor in the following section.

5. Electric and Magnetic Parts of the Weyl tensor:

Consider an observer with a time-like 4-velocity vector. Such an observer will measure the electric and magnetic components E_{hi} and H_{hi} respectively, of the Weyl tensor through the equations

$$E_{hi} + iH_{hi} = Q_{hi} = \overline{C}_{hijk}u^j u^k \qquad \text{...(5.1)}$$

where $\quad Q_{hi} = Q_{ih}\,; \quad Q^h_{\ h} = Q_{hi}u^i = 0\,;$

and $\overline{C}_{hijk} = C_{hijk} + iC^*_{hijk}$

and $$C^*_{hijk} = \frac{1}{2}\varepsilon_{jk}{}^{mn} C_{himn} \qquad \ldots(5.2)$$

where $\varepsilon_{jk}{}^{mn}$ is the Levi-Civita permutation symbol.

Here, C_{hijk} is the Weyl Curvature tensor and C^*_{hijk} is it's dual.

Also the electric and magnetic parts of the Weyl tensor C_{ijkl} are respectively defined by

$$E_{ik} = C_{ijkl} u^j u^l \qquad \ldots (5.3)$$

and $$H_{ik} = C^*_{ijkl} u^j u^l \qquad \ldots (5.4)$$

The electric and magnetic parts of Weyl tensor are symmetric, u^i - orthogonal and traceless i.e.

$$E_{ik} = E_{ki}\ , \qquad E_{ik} u^k = 0\ , \qquad E_{ik} g^{ik} = 0$$

and $$H_{ik} = H_{ki}, \qquad H_{ik} u^k = 0\ , \qquad H_{ik} g^{ik} = 0\,. \qquad \ldots (5.5)$$

The Weyl tensor is said to be purely electric if $H_{hi} = 0$ and purely magnetic if $E_{hi} = 0$.

It is known that the Weyl tensor in terms of E and H can be decomposed as

$$C_{i\,k}^{\,j\,l} = 2u_{[i} E_{k]}{}^{[j} u^{l]} + \delta_{[i}^{[j} E_{k]}^{l]} - \eta_{ikmn} u^m H^{n[j} u^{l]} - \eta^{jlmn} u_m H_{n[i} u_{k]} \qquad \ldots (5.6)$$

which can be equivalently written as

$$C^{ijkl} = \left(\eta^{ijmn}\eta^{klrs} - g^{ijmn} g^{klrs}\right) u_m u_r E_{ns} + \left(\eta^{ijmn} g^{klrs} - g^{ijmn}\eta^{klrs}\right) u_m u_r H_{ns} \quad \ldots (5.7)$$

The time-like vector field u^i in terms of NP tetrad vector fields can be expressed as

$$u^i = \frac{1}{\sqrt{2}}(l^i + n^i) \qquad \ldots(5.8)$$

Consequently, the expressions (5.3) and (5.4) for electric and magnetic parts of the Weyl tensor become

$$E_{hj} = \frac{1}{2} C_{hijk} U^{ik}$$

and $$H_{hj} = \frac{1}{2} C^*_{hijk} U^{ik} \qquad \ldots(5.9)$$

where $U^{ik} = (l^i l^k + l^i n^k + n^i l^k + n^i n^k)$

Now Weyl tensor can be written as composition of electric and magnetic parts as follows

$$C_{hijk} = E_{ik}V_{hj} + E_{hj}V_{ik} - E_{ij}V_{hk} - E_{hk}V_{ij} + \frac{1}{2}\eta_{hi}^{pq}H_{qj}U_{kp} - \frac{1}{2}\eta_{hi}^{pq}H_{qk}U_{pj} + \frac{1}{2}\eta_{jk}^{pq}H_{hq}U_{ip} - -\frac{1}{2}\eta_{jk}^{pq}H_{iq}U_{hp} \quad \ldots(5.10)$$

where $V_{ij} = l_i l_j + 2m_{(i}\overline{m}_{j)} + n_i n_j$

Hasmani et al. (2008) have defined the tetrad components of electric part of Weyl tensor E_{hj} in to the four real and three complex scalars as

$$\begin{aligned} E_{11} &= E_{hj}l^h l^j, & E_{12} &= E_{hj}l^h n^j, \\ E_{13} &= E_{hj}l^h m^j, & E_{22} &= E_{hj}n^h n^j, \\ E_{23} &= E_{hj}n^h m^j, & E_{33} &= E_{hj}m^h m^j, \\ E_{34} &= E_{hj}m^h \overline{m}^j . \end{aligned} \quad \ldots(5.11)$$

Similarly, the tetrad components of the magnetic part of the Weyl tensor H_{hj} are obtained from equation (5.11) just by replacing E by H.

Using equations (5.9) and (5.11) one can determine the tetrad components of electric part of the Weyl tensor as

$$\begin{aligned} E_{11} &= E_{hj}l^h l^j \\ &= \frac{1}{2}C_{hijk}(l^i l^k + l^i n^k + n^i l^k + n^i n^k)l^h l^j \\ &= \frac{1}{2}C_{hijk}(l^h l^i l^j l^k + l^h l^i l^j n^k + l^h n^i l^j l^k + l^h n^i l^j n^k) \\ &= \frac{1}{2}(C_{1111} + C_{1112} + C_{1211} + C_{1212}) \\ &= \frac{1}{2}C_{1212} \end{aligned}$$

Similarly, we obtain

$$E_{11} = -E_{12} = E_{22} = \frac{1}{2}C_{1212},$$

$$E_{13} = -E_{23} = -\frac{1}{2}(C_{1213} + C_{1223}),$$

$$E_{33} = \frac{1}{2}(C_{1313} + C_{2323}),$$

$$E_{34} = \frac{1}{2}(C_{1324} + C_{2314}) \qquad \ldots(5.12)$$

Now we express the electric part of the Weyl tensor as a linear combinations of the basis of the null tetrad in the form

$$E_{ij} = E_{\alpha\beta} e_i^{(\alpha)} e_j^{(\beta)} \qquad \ldots(5.13)$$

Explicitly, we have

$$E_{ij} = E_{22} l_i l_j + E_{21} l_i n_j - E_{24} l_i m_j - E_{23} l_i \overline{m}_j + E_{12} n_i l_j + E_{11} n_i n_j - E_{14} n_i m_j - E_{13} n_i \overline{m}_j - $$
$$- E_{24} m_i l_j - E_{14} m_i n_j + E_{44} m_i m_j + E_{34} m_i \overline{m}_j - E_{23} \overline{m}_i l_j - E_{13} \overline{m}_i n_j + E_{34} \overline{m}_i m_j +$$
$$+ E_{33} \overline{m}_i \overline{m}_j$$

$$E_{ij} = E_{22} l_i l_j + E_{12}(l_i n_j + n_i l_j) + E_{11} n_i n_j - E_{24}(l_i m_j + m_i l_j) - E_{14}(m_i n_j + n_i m_j) -$$
$$- E_{23}(l_i \overline{m}_j + \overline{m}_j l_i) - E_{13}(\overline{m}_i n_j + n_i \overline{m}_j) + E_{44} m_i m_j + E_{33} \overline{m}_i \overline{m}_j +$$
$$+ E_{34}(m_i \overline{m}_j + \overline{m}_i m_j)$$

$$\ldots(5.14)$$

Using equation (5.12), above equation become

$$E_{ij} = \frac{1}{2}\{C_{1212}[l_i l_j - 2 l_{(i} n_{j)} + n_i n_j - 2(C_{1213} + C_{1223})[l_{(i} m_{j)} - m_{(i} n_{j)}] -$$
$$- 2(C_{1214} + C_{1224})[l_{(i} \overline{m}_{j)} - \overline{m}_{(i} n_{j)}] + (C_{1313} + C_{2323}) m_i m_j + (C_{1414} + C_{2424}) \overline{m}_i \overline{m}_j +$$
$$+ 2(C_{1324} + C_{2314}) m_{(i} \overline{m}_{j)}\}$$

$$\ldots(5.15)$$

Using the set of equations (4.9) in chapter I, equation (5.15) reduces to

$$E_{ij} = \frac{1}{2}\left\{C_{1212}[l_i l_j - 2 l_{(i} n_{j)} + n_i n_j] + 2(C_{1324} + C_{2314}) m_{(i} \overline{m}_{j)}\right\} \qquad \ldots(5.16)$$

Likewise the tetrad components of the magnetic part H_{ij} and its expression in terms of basis of the tetrad are obtained by simply replacing the tetrad components of the Weyl tensor by it's duals in the set of equations (5.12) and (5.16).

However, the tetrad components of the dual Weyl tensor are given by Hasmani et. al (2008)

$$C^{*}_{\alpha\beta\gamma\delta} = \frac{i}{2} \varepsilon_{\gamma\delta}{}^{\sigma\rho} C_{\alpha\beta\sigma\rho} \qquad \ldots(5.17)$$

This can also be written as

$$C^*_{\alpha\beta\gamma\delta} = \frac{i}{2}\varepsilon_{\gamma\delta\varepsilon\nu}\eta^{\varepsilon\sigma}\eta^{\nu\rho}C_{\alpha\beta\sigma\rho}$$

$$C^*_{\alpha\beta\gamma\delta} = \frac{i}{2}C_{\alpha\beta\sigma\rho}[\varepsilon_{\gamma\delta12}\eta^{1\sigma}\eta^{2\rho} + \varepsilon_{\gamma\delta13}\eta^{1\sigma}\eta^{3\rho} + \varepsilon_{\gamma\delta14}\eta^{1\sigma}\eta^{4\rho} + \varepsilon_{\gamma\delta23}\eta^{2\sigma}\eta^{3\rho} + \varepsilon_{\gamma\delta24}\eta^{2\sigma}\eta^{4\rho} + \\ + \varepsilon_{\gamma\delta21}\eta^{2\sigma}\eta^{1\rho} + \varepsilon_{\gamma\delta31}\eta^{3\sigma}\eta^{1\rho} + \varepsilon_{\gamma\delta32}\eta^{3\sigma}\eta^{2\rho} + \varepsilon_{\gamma\delta34}\eta^{3\sigma}\eta^{4\rho} + \varepsilon_{\gamma\delta41}\eta^{4\sigma}\eta^{1\rho} + \\ + \varepsilon_{\gamma\delta42}\eta^{4\sigma}\eta^{2\rho} + \varepsilon_{\gamma\delta43}\eta^{4\sigma}\eta^{3\rho}]$$

$$C^*_{\alpha\beta\gamma\delta} = \frac{i}{2}[C_{\alpha\beta\,21}\varepsilon_{\gamma\delta\,12} + C_{\alpha\beta\,12}\varepsilon_{\gamma\delta\,21} - C_{\alpha\beta\,24}\varepsilon_{\gamma\delta\,13} - C_{\alpha\beta\,42}\varepsilon_{\gamma\delta\,31} - \\ - C_{\alpha\beta\,23}\varepsilon_{\gamma\delta\,14} - C_{\alpha\beta\,32}\varepsilon_{\gamma\delta\,41} - C_{\alpha\beta\,14}\varepsilon_{\gamma\delta\,23} - C_{\alpha\beta\,41}\varepsilon_{\gamma\delta\,32} - \\ - C_{\alpha\beta\,13}\varepsilon_{\gamma\delta\,24} - C_{\alpha\beta\,31}\varepsilon_{\gamma\delta\,42} + C_{\alpha\beta\,43}\varepsilon_{\gamma\delta\,34} + C_{\alpha\beta\,34}\varepsilon_{\gamma\delta\,43}]$$

...(5.18)

By giving different values to the indices $\alpha, \beta, \gamma, \delta$ from 1 to 4 we can establish the following non-vanishing relations between the tetrad components of the Weyl tensor and it's duals as

$$C^*_{1212} = \frac{i}{2}[C_{1221}\varepsilon_{1212} + C_{1212}\varepsilon_{1221} - C_{1224}\varepsilon_{1213} - C_{1242}\varepsilon_{1231} - C_{1223}\varepsilon_{1214} - C_{1232}\varepsilon_{1241} - \\ - C_{1214}\varepsilon_{1223} - C_{1241}\varepsilon_{1232} - C_{1213}\varepsilon_{1224} - C_{1231}\varepsilon_{1242} + C_{1243}\varepsilon_{1234} + C_{1234}\varepsilon_{1243}]$$

...(5.19)

Using definition of Levi-Civita permutation symbol equation (5.19) gives

$$C^*_{1212} = \frac{i}{2}[C_{1243}\varepsilon_{1234} + C_{1234}\varepsilon_{1243}]$$

$$= \frac{i}{2}[-C_{1234} - C_{1234}]$$

$$C^*_{1212} = -iC_{1234}$$

Similarly, we obtain

$$C^*_{1234} = -iC_{1212} \ ; \qquad C^*_{1324} = iC_{1324} \ ; \qquad C^*_{3434} = -iC_{3412} \qquad \text{...(5.20)}$$

Using equations (4.6) and (5.20), we obtain the non-vanishing components of the dual of Weyl tensor as follows

$$C^*_{1212} = -iC_{1234} = -\frac{2a\cos\theta}{R^6}[2r(2mr - e^2) - mR^2],$$

$$C^*_{1234} = -iC_{1212} = \frac{4imr}{R^4} - \frac{i(3r^2 - a^2\cos^2\theta)}{R^6}(2mr - e^2) - \frac{ie^2}{R^4},$$

$$C^*_{1324} = iC_{1324} = \frac{im}{R^2\overline{R}^*} - \frac{i(2mr - e^2)}{R^2(\overline{R}^*)^2}, \qquad \ldots(5.21)$$

$$C^*_{3434} = -iC_{3412} = -\frac{2a\cos\theta}{R^6}[2r(2mr - e^2) - mR^2].$$

Now the expressions for E_{ij} and H_{ij} with reference to the Kerr-Newman metric become

$$E_{ij} = \frac{1}{2}R^{-6}\{[2mr(r^2 - 3a^2\cos^2\theta) - e^2(3r^2 - a^2\cos^2\theta)][l_il_j - 2l_{(i}n_{j)} + n_in_j] - 4[mr(r^2 - 3a^2\cos^2\theta) - e^2(r^2 - a^2\cos^2\theta)]\, m_{(i}\overline{m}_{j)}\} \qquad \ldots(5.22)$$

and

$$H_{ij} = (-a\cos\theta)R^{-6}[mr(3r^2 - a^2\cos^2\theta) - 2re^2][l_il_j - 2l_{(i}n_{j)} + n_in_j + 2m_{(i}\overline{m}_{j)}] \qquad \ldots(5.23)$$

respectively.

From equations (5.11) and (5.22) we obtain

$$E_{11} = -E_{12} = E_{22} = \frac{1}{2}R^{-6}[2mr(r^2 - 3a^2\cos^2\theta) - e^2(3r^2 - a^2\cos^2\theta)],$$

$$E_{34} = -R^{-6}[mr(r^2 - 3a^2\cos^2\theta) - e^2(r^2 - a^2\cos^2\theta)] \qquad \ldots(5.24)$$

and all complex tetrad components of electric part of Weyl tensor are zero.

Similarly, we find

$$H_{11} = -H_{12} = H_{22} = H_{34} = (-a\cos\theta)R^{-6}[mr(3r^2 - a^2\cos^2\theta) - 2re^2] \qquad \ldots(5.25)$$

and all other complex tetrad components of the magnetic part of the Weyl tensor vanish.

At very far distance from the gravitating object i.e. $r \to \infty$ one can observe that the electric and magnetic parts of the Weyl tensor vanish.

6. Conclusion: The expressions for the electric and magnetic parts of the Weyl tensor with reference to the Kerr-Newman space-time are obtained in terms of the basis of the tetrad. It has been observed that both the angular momentum per unit mass and the electric charge of the gravitating body are the sources of the electric part and magnetic part of the Weyl tensor. We see that if angular momentum per unit mass of a gravitating body is zero, the magnetic part of Weyl tensor ceases to be zero in the Kerr-Newman space-time while electric part E_{ij} still exists.