CHAPTER – II

The Mixed Stieltjes Transform

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THE MIXED STIELTJES TRANSFORM

2. 1 Introduction:

The transform theory introduce various types of integral transform. Such as Laplace, Fourier, Mellin , Stieltjes, Hankel, Hilbert transform and similar types of transforms, their properties and corresponding application .

Our objective in this chapter is to study the mixed Stieltjes transform defined by M. Giona and O. Patierno [1] . We shall study some classical properties and application of the mixed Stieltjes transform. We shall unify some important properties of both the Laplace and Stieltjes transformations.

Definition:-

The mixed Stieltjes transformation [3] is defined by the equation,

$$m[f(x)] = \int_0^\infty \frac{e^{-zx}}{1+tx} f(x)dx \quad \text{.......(2.1.1)}$$

For the function f(x) which is piecewise continuos and locally integrable over the interval $0 < x < \infty$. Where 'z' is complex variable and 't' is real variable on interval $0 < t < \infty$. It is denoted by m[f(x)] or m(t,z).

Remarks:

For $z = 0$, the mixed Stieltjes transform reduces to the Stieltjes transform $S[f(x)] = m(t, 0)$. For $t = 0$, it reduces the Laplace transform $L[f(x)] = m(0, z)$.

2.2 Existance of the Mixed Stieltjes Transform.

Theorem : If f(x) is a function which is piecewise continuous on every finite interval in the range $x > 0$ and satisfies

$$\left|\frac{f(x)}{x}\right| \leq Me^{ax}$$

and for some constants a & M, then the mixed Stieltjes transform of function f(x) exists for all Re $z > a$ and for $0 < t < \infty$.

Proof: We have,

$$m[f(x)] = \int_0^\infty \frac{e^{-zx}}{1+tx} f(x)dx$$

$$= \int_0^{x_0} \frac{e^{-zx}}{1+tx} f(x)dx + \int_{x_0}^{\infty} \frac{e^{-zx}}{1+tx} f(x)dx$$

$$\text{the integral} \int_0^{x_0} \frac{e^{-zx}}{1+tx} f(x)dx - \text{exists,}$$

since f(x) is piecewise continuous on every finite interval $0 < x \leq x_0$ and for $0 < t < \infty$.

Now,

$$\left| \int_{x_0}^{\infty} \frac{e^{-zx}}{1+tx} f(x)dx \right| \leq \int_{x_0}^{\infty} \left| \frac{e^{-zx}}{1+tx} f(x) \right| dx$$

$$\leq \int_{x_0}^{\infty} e^{-zx} \left| \frac{f(x)}{1+tx} \right| dx$$

$$\leq \int_{x_0}^{\infty} e^{-zx} M e^{ax} dx$$

$$\leq M \int_{x_0}^{\infty} e^{-(z-a)x} dx$$

$$= \frac{Me^{-(z-a)x_0}}{(z-a)}$$

$$\therefore \left| \int_{x_0}^{\infty} \frac{e^{-zx}}{1+tx} f(x)dx \right| \leq \frac{Me^{-(z-a)x_0}}{(z-a)}, z > a$$

But $\frac{M.e^{-(z-a)x_0}}{(z-a)}$

can made as small as we have taking x_0 sufficiently

large.

The mixed Stiltejes transform exists for all $z > a$ and $0 < t < \infty$.

Classical Properties of the Mixed Stieltjes Transformation.

2.3 Linearity Property.

The mixed Stielties transformation m is linear for every pair of function $f_1(x)$ and $f_2(x)$ and for every pair of constants a_1 and a_2

$$\therefore \quad m[\, a_1 f_1(x) + a_2 f_2(x)\,] = a_1\, m[\, f_1(x)\,] + a_2\, m[\, f_2(x)\,] \quad ......(2.3.1)$$

2.4 Translation Property or Shifting Theorem.

If $m[\, f(x)\,] = m(t, z)$, when $\text{Re } z > \alpha$, α is positive number then,

$$m[\, e^{-ax} f(x)\,] = m(t,\ z + a) \qquad ....(2.4.1)$$

$\text{Re } z > \alpha + a$, a is positive number.

Proof : By definition, we have,

$$m[f(x)] = \int_0^\infty \frac{e^{-zx}}{1+tx} \cdot f(x)\,dx$$

$$m[e^{-ax} f(x)] = \int_0^\infty \frac{e^{-zx}}{1+tx} \cdot e^{-ax} f(x)\,dx$$

$$= \int_0^\infty \frac{e^{-(z+a)x}}{1+tx} f(x)\,dx$$

$$= m(t,\ z+a)\ .$$

2.5 Change of Scale Property.

If $m[f(x)] = m(t, z)$, then

$$m[f(ax)] = \frac{1}{a} m\left(\frac{t}{a}, \frac{z}{a}\right) \ldots\ldots (2.5.1)$$

where a is non-zero constant.

Proof : By definition we have,

$$m[f(x)] = \int_0^\infty \frac{e^{-zx}}{1+tx} f(x)\,dx$$

$$\therefore\ m[f(ax)] = \int_0^\infty \frac{e^{-zx}}{1+tx} f(ax)\,dx$$

Putting ax = y so that dx = 1/a . dy

$$= \int_0^\infty \frac{e^{-z\left(\frac{y}{a}\right)}}{1+t\left(\frac{y}{a}\right)} f(y)\,\frac{1}{a}\,dy$$

$$= \frac{1}{a}\int_0^\infty \frac{e^{-\left(\frac{z}{a}\right)y}}{1+\left(\frac{t}{a}\right)y} f(y)\,dy$$

$$= \frac{1}{a}\, m\left(\frac{t}{a}, \frac{z}{a}\right)$$

$$\therefore \quad m[f(ax)] = \frac{1}{a}\, m\left(\frac{t}{a}, \frac{z}{a}\right)$$

2.6 Multiplication by x^n.

If f(x) is piecewise continuous and locally integeable in interval $0 < x < \infty$ and if $m[f(x)] = m(t,z)$

then

$$m[x^n f(x)] = (-1)^n \frac{d^n}{dz^n} m(t,z) \quad \ldots\ldots(2.6.1)$$

where n = 1,2,3,..........

Proof : By definition we have,

$$m\,[\,f(x)\,] = \int_0^{\infty} \frac{e^{-zx}}{1+tx} f(x)\,dx$$

$$\text{i.e.. } m\,(t,z) = \int_0^{\infty} \frac{e^{-zx}}{1+tx} f(x)\,dx$$

differentiating with respect to z on both sides , we have

$$\frac{d}{dz} m\,(t,z) = \frac{d}{dz} \int_0^{\infty} \frac{e^{-zx}}{1+tx} f(x)\,dx$$

By using Lebnitz rule for differentiating under the integral

sign given by (0.2.10)

$$L.H.S = \int_0^{\infty} \frac{\partial}{\partial z}\left(\frac{e^{-zx}}{1+tx}\right) dx$$

$$= \int_0^{\infty} (-x)\,\frac{e^{-zx}}{1+tx} f(x)\,dx$$

$$= -\int_0^{\infty} \frac{e^{-zx}}{1+tx}[\,x f(x)\,]\,dx$$

$$= -\,m\,[\,x f(x)\,]$$

$$\text{Thus,. } m\,[\,x f(x)\,] = -\frac{d}{dz} m\,(t,z)$$

$$m\,[\,x f(x)\,] = (-1)^{1} \frac{d}{dz} m\,(t,z)$$

i.e. the theorem is true for n = 1

Now assume that the theorem is true for a particular value of n, say s

Then we have,

$$m[x^s f(x)] = (-1)^s \frac{d^s}{dz^s} m(t, z)$$

$$\therefore \int_0^\infty \frac{e^{-zx}}{1+tx} x^s f(x)\,dx = (-1)^s \frac{d^s}{dz^s} m(t, z)$$

Now differentiating both sides with respect to z,

we have

$$\frac{d}{dz} \int_0^\infty \frac{e^{-zx}}{1+tx} x^s f(x)\,dx = (-1)^s \frac{d^{s+1}}{dz^{s+1}} m(t, z)$$

By using Leibnitz rule for differentiating under the sign of integral, we have

$$\Rightarrow \int_0^{\infty} \frac{\partial}{\partial z}\left(\frac{e^{-zx}}{1+tx} x^{s} f(x)\right) dx = (-1)^{s} \frac{d^{s+1}}{dz^{s+1}} m(t,z)$$

$$\Rightarrow \int_0^{\infty} (-x) \frac{e^{-zx}}{1+tx} x^{s} f(x) dx = (-1)^{s} \frac{d^{s+1}}{dz^{s+1}} m(t,z)$$

$$\Rightarrow \int_0^{\infty} \frac{e^{-zx}}{1+tx} [x^{s+1} f(x)] dx = (-1)^{s+1} \frac{d^{s+1}}{dz^{s+1}} m(t,z)$$

$$\Rightarrow m[x^{s+1} f(x)] = (-1)^{s+1} \frac{d^{s+1}}{dz^{s+1}} m(t,z)$$

Hence the property is true for $n = s + 1$

Therefore, by mathematical induction the property is true for every positive integral value of n.

2.7 Division by x^n

If function f(x) is piecewise continuous and locally integrable in the interval $0 < x < \infty$ **and if** $m[\,f(x)\,] = m(t, z)$, then

$$\therefore\ m\left[\frac{f(x)}{x^n}\right] = \int_z^\infty \int_z^\infty \cdots\cdots\cdots \int_z^\infty m(t, u)(du)^n \quad \ldots.(2.7.1)$$

$$(\text{where\_ } n = 1, 2, 3\ldots..)$$

$$\text{provided\_ that\_ } \lim_{x \to o} \int \frac{f(x)}{x}\, dx\text{\_ exists.}$$

Proof : Let $G(x) = f(x)/x$

i.e. $f(x) = x\,G(x)$

$\therefore\ m[f(x)]\,(t,z) = m[xG(x)]\,(t,z)$

$\therefore$ by property 2.6 we have,

$$m(t, z) = -\frac{d}{dz} m[G(x)](t, z)$$

Now integrating both sides w.r.t. u from z to ∞,

$$\int_z^{\infty} m(t,u)du = -\{ \ m[G(x)](t,u) \ \}|_z^{\infty}$$

$$\therefore -\lim_{u\to\infty} m[G(x)](t,u) + m[G(x)](t,z) = \int_z^{\infty} m(t,u)du$$

$$\left[\because \lim_{u\to\infty} m[G(x)] = \lim_{u\to\infty} \int_0^{\infty} \frac{e^{-ux}}{1+tx} G(x)dx = 0\right]$$

$$\therefore 0 + m[G(x)] = \int_z^{\infty} m(t,u)du$$

$$\therefore m[\frac{f(x)}{x^1}] = \int_z^{\infty} m(t,u)du$$

i.e. theorem is true for n=1.

Now assume that the theorem is true for a particular value of 'n' say s.

Then we have,

$$m\left[\frac{f(x)}{x^s}\right] = \int_z^{\infty} \ldots\ldots\ldots\ldots \int_z^{\infty} m(t,u)\cdot(du)^s$$

Now integrating both sides w. r. t. u from z to ∞

We have,

$$\int_z^{\infty} m\left[\frac{f(x)}{x^s}\right]\cdot du = \int_z^{\infty}\left\{\int_z^{\infty}\ldots\ldots\int_z^{\infty} m(t,u)\cdot(du)^s\right\}\cdot du$$

$$\therefore \int_z^{\infty}\left\{\int_0^{\infty}\frac{e^{-ux}}{1+tx}\frac{f(x)}{x^s}dx\right\}du = \int_z^{\infty}\left\{\int_z^{\infty}\ldots\ldots\int_z^{\infty} m(t,u)(du)^s\right\}.du$$

By change of order of integration given by [0.2.11] , we have

$$\int_z^{\infty}\int_0^{\infty}\frac{e^{-ux}}{1+tx}\frac{f(x)}{x^s}dx.du = \int_z^{\infty}\ldots\ldots\ldots\int_z^{\infty} m(t,u)\cdot(du)^{s+1}$$

$$\int_0^{\infty}\frac{f(x)}{x^s(1+tx)}\left\{\int_z^{\infty} e^{-ux}du\right\}\cdot dx = \int_z^{\infty}\ldots\ldots\int_z^{\infty} m(t,u)\cdot(du)^{s+1}$$

$$\int_0^{\infty}\frac{f(x)}{x^s(1+tx)}\left[\frac{e^{-ux}}{-x}\right]_z^{\infty}dx = \int_z^{\infty}\ldots\ldots\int_z^{\infty} m(t,u)\cdot(du)^{s+1}$$

$$\int_0^{\infty}\frac{f(x)}{x^s(1+tx)}\left[\frac{e^{-zx}}{x}\right]\cdot dx = \int_z^{\infty}\ldots\ldots\int_z^{\infty} m(t,u)\cdot(du)^{s+1}$$

$$\int_0^{\infty} \frac{e^{-zx}}{1+tx} \cdot \left[\frac{f(x)}{x^{s+1}} \right] \cdot dx = \int_z^{\infty} \ldots\ldots \int_z^{\infty} m(t,u) \cdot (du)^{s+1}$$

$$m\left[\frac{f(x)}{x^{s+1}} \right] = \int_z^{\infty} \ldots\ldots\ldots \int_z^{\infty} m(t,u) \cdot (du)^{s+1}$$

$(s+1)^{th}$ repeated integrals.

Hence the property is true for $n = s + 1$,

Therefore, by mathematical induction the property is true for every positive integral value of n.

2.8 Mixed Stieltjes Transform of the derivative of f(x).

If functions f(x), f '(x) are continuous and locally integrable in the interval $0 < x < \infty$ and

if $\quad m[\, f(x)\,] = m\,(t,z),\quad$ then

$$m\,[f'(x)\,] = -\,f(0) + (z+t).\, m(t, z) + t^2 \, d/dt \; m(t, z)$$

where f '(x) denotes derivatives of f(x).

Proof: By definition, we have

$$m\left[f(x)\right] = \int_0^\infty \frac{e^{-zx}}{1+tx} \cdot f(x) \cdot dx \ldots (2.8.1)$$

$0 < t < \infty$, and z is complex variable.

$$\therefore \ m\left[f'(x)\right] = \int_0^\infty \frac{e^{-zx}}{1+tx} \cdot f'(x) \cdot dx$$

By integration by parts we have

$$= \frac{e^{-zx}}{1+tx} f(x)\Big|_0^\infty - \int_0^\infty \frac{\partial}{\partial x} \cdot \left(\frac{e^{-zx}}{1+tx}\right) \cdot f(x) \cdot dx$$

$$= -f(o) - \int_0^\infty \left\{ \frac{(1+tx)(-z)e^{-zx} - e^{-zx} \cdot t}{(1+tx)^2} \right\} \cdot f(x) \cdot dx$$

$$\left[\because \lim_{x \to \infty} \frac{e^{-zx}}{1+tx} f(x) = 0, z > o \right]$$

$$= -f(0) - \int_0^\infty \left\{ \frac{(-z)e^{-zx}}{1+tx} - \frac{t.e^{-zx}}{(1+tx)^2} \right\} \cdot f(x) \cdot dx$$

$$= -f(0) - \int_0^{\infty} (-z) \cdot \frac{e^{-zx}}{1+tx} \cdot f(x) \cdot dx + \int_0^{\infty} t \cdot \frac{e^{-zx}}{(1+tx)^2} \cdot f(x) \cdot dx$$

$$= -f(0) + z \int_0^{\infty} \frac{e^{-zx}}{1+tx} \cdot f(x) \cdot dx + t \int_0^{\infty} \left\{ \frac{e^{-zx}}{t^2 (\frac{1}{t}+x)^2} \right\} \cdot f(x) \cdot dx$$

$$= -f(0) + z.m(t,z) + t \int_0^{\infty} \left\{ \frac{e^{-zx}}{t^2 (\frac{1}{t}+x)^2} \right\} \cdot f(x) \cdot dx$$

$$\because \frac{d}{dt} \left[\frac{1}{\frac{1}{t}+x} \right] = -\left[\frac{1}{(\frac{1}{t}+x)^2} \right] \cdot \left[-\frac{1}{t^2} \right]$$

$$= \frac{1}{t^2 (\frac{1}{t}+x)^2}$$

$$= -f(0) + z.m(t,z) + t \int_0^{\infty} \frac{\partial}{\partial t} \left\{ \frac{1}{(\frac{1}{t}+x)} \right\} \cdot e^{-zx} f(x) \cdot dx$$

By using Lebnitz's rule for differentiating under the sign of integral, we have

$$= -f(0) + z \cdot m(t,z) + t \frac{d}{dt} \left\{ \int_0^{\infty} \frac{e^{-zx}}{\frac{1}{t}+x} \cdot f(x) \cdot dx \right\}$$

$$= -f(0) + z \cdot m(t,z) + t\frac{d}{dt}\left\{\int_0^\infty \frac{t \cdot e^{-zx}}{1+tx} \cdot f(x) \cdot dx\right\}$$

$$= -f(0) + z \cdot m(t,z) + t\frac{d}{dt}\left\{t\int_0^\infty \frac{e^{-zx}}{1+tx} \cdot f(x) \cdot dx\right\}$$

$$= -f(0) + z \cdot m(t,z) + t\frac{d}{dt}\{t \cdot m(t,z)\}$$

$$= -f(0) + z \cdot m(t,z) + t\{t\frac{d}{dt}m(t,z) + m(t,z)\}$$

$$= -f(0) + z \cdot m(t,z) + t^2\frac{d}{dt}m(t,z) + t \cdot m(t,z)$$

$$= -f(0) + z \cdot m(t,z) + t \cdot m(t,z) + t^2\frac{d}{dt}m(t,z)$$

$$\therefore\ m[f'(x)] = -f(o) + (z+t) \cdot m(t,z) + t^2\frac{d}{dt}m(t,z).$$

Hence proved.

Remark 1:

In case $f'(x)$ is piecewise continuous, the integral (2.8.1) may be broken as the sum of integrals in different ranges from 0 to : and that $f'(x)$ is continuous in each of such parts, Then preceding as above (2.8), we have,

$$m[f'(x)] = -f(0) + (z+t)\,m(t,z) + t^2\, d/dt\,[m(t,z)]$$

Remark 2 :

For $t = 0$ some properties of the mixed Stieltjes transform reduces to classical properties of the Laplace transform and for $z = 0$ the properties of the mixed Stieltjes transform reduces to classical properties of the Stieltjes transform.

2.9 Result:

If function f(x) is piecewise continuous and locally integrable over the interval $0 < x < \infty$ and

If $m[f(x)] = m(t, z)$, then

$$m\left[e^{-ax} f(bx)\right] = \frac{1}{b} \cdot m\left[\frac{t}{a}, \frac{z+a}{b}\right]$$

Where a, b are non-zero constants.

Proof: By definition , we have

$$m[f(x)] = \int_0^\infty \frac{e^{-zx}}{1+tx} \cdot f(x) \cdot dx$$

$$\therefore\ m\left[e^{-ax} f(bx)\right] = \int_0^\infty \frac{e^{-zx}}{1+tx}\left[e^{-ax} f(bx)\right] \cdot dx$$

$$= \int_0^\infty \frac{e^{-(z+a)x}}{1+tx} \cdot f(bx) \cdot dx$$

put $bx = y$, $dx = 1/b\, dy$

$$= \int_0^\infty \frac{e^{-(z+a)(y/b)}}{1+t(y/b)} \cdot f(y) \cdot \frac{1}{b} dy$$

$$= \frac{1}{b} \int_0^\infty \frac{e^{-\left(\frac{z+a}{b}\right)y}}{1+(t/b)y} \cdot f(y) \cdot dy$$

$$= \frac{1}{b} m\left[\frac{t}{b}, \frac{z+a}{b}\right]$$

$$\therefore \quad m\left[e^{-ax} f(bx)\right] = \frac{1}{b} m\left[\frac{1}{a}, \frac{z+a}{b}\right]$$

2.10 Application of the Mixed Stieltjes Transform:

The mixed Stieltjes transform is used in all the problems involving distribution function characterised by an highly singular (extraordinary) structure.

The chemical reaction, kinetics in continuous mixtures is a suitable example of such type of structures. Continuous thermodynamics and kinetics are connected with complex chemical mixtures, for which different chemical species can be labeled with respect with to a continuous parameter that is a reaction rate coefficient or boiling point temperature [3] such a explanation is very useful to describe complex fuel mixtures, polymeric solutions, waste water solutions etc. For example, Moore and Anthony [4] have been studied experimentally the chromatographic methods that diesel fuel mixtures are characterized by a singular composition. It can be described by a smooth continuous distributions.

Consider a first order reaction developing gradually in a Completely mixed Stirred Tank Reactor, CSTR (the concentration inside the reactor is uniform and equal to the outlet concentration) under the isothermal conditions.

Let $c(x, t)$ = concentration inside the reactor

[Volume V, Inlet volumetric flow Q]

x = dimensionless reaction rate coefficient

$= k/k_{max} \in [0, 1]$

where k is rate coefficient of & k_{max} is maximum value attained by k.

Let $c_0(x) = c_o\,\rho(x)$ the inlet (innert) concentration

defined on support c of [0, 1] with

$$\int_c \rho(x).dx = \int_c d\mu(x) = 1$$

$\rho(x) =$ density function of reactant vanish

outside the interval [0, 1]

The balance equation inside the reactor is given by

$$\frac{\partial(x,\theta)}{\partial\theta} = \frac{\rho(x)-u(x,\theta)}{T} - x.u(x,\theta)$$

Where

$\theta = t\ k_{max},$ the dimensionless time,

$u(x, \theta\ ) = c\,(\,x, \theta / k_{max}\,) / c_o$ the concentration inside the reactor.

$T = V\ k_{max} / Q$, dimensionless residence time of the reactor.

Then the total reactant concentration $U(\theta)$ is given by

$$U(\theta) = \int_0^1 u(x,\theta).\,dx$$

By using given condition we can write,

$$U(\theta) = \int_0^\infty u(x,\theta)dx$$

Since $$\frac{\partial u(x,\theta)}{\partial\theta} = \frac{\rho(x) - u(x,\theta)}{T} - xu(x,\theta)$$

$$= \frac{\rho(x)}{T} - \frac{(1+tx)u(x,\theta)}{T}$$

$$\frac{(1 + tx)u(x,\theta)}{T} = \frac{\rho(x)}{T} - \frac{\partial u(x,\theta)}{\partial\theta}$$

$$u(x,\theta) = \frac{\rho(x)}{1 + Tx} - \left(\frac{T}{1 + Tx}\right)\frac{\partial u(x,\theta)}{\partial\theta}$$

$$= \frac{\rho(x)}{1+Tx} - \left(\frac{T}{1+Tx}\right)\frac{\partial}{\partial\theta}\left[\frac{c(x,\frac{\theta}{k_{max}})}{c_0}\right]$$

$$= \frac{\rho(x)}{1+Tx} - \left(\frac{T}{1+Tx}\right)\frac{\partial}{\partial\theta}\left[\frac{c(x,\frac{\theta}{k_{max}})}{c_{o(x)}/\rho(x)}\right]$$

$$= \frac{\rho(x)}{1+Tx} - \left(\frac{T\rho(x)}{1+Tx}\right)\frac{\partial}{\partial\theta}\left[\frac{c(x,\frac{\theta}{k_{max}})}{c_{o(x)}}\right]$$

$$\text{Let} - c(x,\theta/k_{max}) = -\frac{c_0(x)\cdot e^{-\theta(\frac{1+Tx}{T})}}{1+Tx}$$

$$\text{L.H.S.} = \frac{\rho(x)}{1+Tx} - \left(\frac{T\rho(x)}{1+Tx}\right)\frac{\partial}{\partial\theta}\left[-\frac{e^{-\theta(\frac{1+Tx}{T})}}{1+Tx}\right]$$

$$= \frac{\rho(x)}{1+Tx} - \left(\frac{T\rho(x)}{1+Tx}\right)\left[\frac{e^{-\theta(\frac{1+Tx}{T})}}{T}\right]$$

$$= \frac{\rho(x)}{1+Tx} - \frac{e^{-\theta(\frac{1+Tx}{T})}}{1+Tx}\cdot\rho(x)$$

Integrating on both sides with respect to x from 0 to ∞.

$$\int_0^\infty u(x,\theta)dx = \int_0^\infty \frac{\rho(x)}{1+Tx}dx - \int_0^\infty \frac{e^{-\theta(\frac{1+Tx}{T})}}{1+Tx}\rho(x)dx$$

$$U(\theta) = \int_0^\infty \frac{\rho(x)}{1+Tx}dx - \int_0^\infty \frac{e^{-(\theta/T)-x\theta}}{1+Tx}\rho(x)dx$$

Since the density function $\rho(x)$ is generally indicated as

$$d\mu(x) = \rho(x)dx$$

$$U(\theta) = \int_0^\infty \frac{d\mu(x)}{1+Tx} - e^{\theta/T}\int_0^\infty \frac{e^{-\theta x}}{1+Tx}d\mu(x)$$

$$U(\theta) = S(T) - e^{-\theta/T}m(T,\theta)$$

S(T) : The Stieltjes transformation.

m(T, θ) : the mixed Stieltjes transformation.

At steady state ($\theta \longrightarrow \infty$), the total reactant concentration is equal to the Stieltjes transform of the inlet distribution.

Therefore if we know the time of reaction θ and residence time of reactor T for a particular reaction rate x ,then we can calculate total reactant cocentration of continuous mixture by using the Stieltjes and mixed Stieltjes transforms.

The mixed Stieltjes transform can also be used in chromatographic and absorption experiments.

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