PREFACE

The dissertation entitled "ON THE PROPAGATION EQUATIONS OF ELECTRIC AND MAGNETIC PARTS OF WEYL TENSOR IN EINSTEIN-CARTAN THEORY OF GRAVITATION", is devoted mainly to the study of propagation equations of electric and magnetic parts of Weyl tensor in Einstein-Cartan theory of gravitation.

The Einstein's special theory of relativity only accounts for inertial systems, in the region of free space where gravitation effects are neglected. But the gravitational field is present everywhere and it cannot be switched ON and OFF at will. It is everlasting and omnipresent. The special theory of relativity fails to take into account the phenomenon of gravitation field and acceleration of particles.

Minkowskian space-time continuum holds in regions far from gravitational field. Thus the Minkowskian space-time continuum does not account the natural phenomenon of gravitation in non-inertial frames. Therefore in the modified Minkowskian space-time structure with gravitation, the line element is given by

$$ds^2 = g_{ij}dx^i dx^j \ ,$$

where g_{ij} are components of the fundamental metric tensor and are functions of coordinates. The above equation represents the curved geometry or the geometry of space in a gravitational field. Such a geometry is called as Riemannian geometry. Thus gravitation no longer to be regarded as a force but as a manifestation of curvature of space-time. The free motion in gravitational regions (which is curved) is not straight but curvilinear. This gravitational field effect is taken into consideration by Einstein and he put the generalized form of special theory of relativity which is known as general theory of relativity.

Einstein's general theory of relativity describes fundamental interaction of gravitation as a result of space-time being curved by matter and energy. Thus Einstein's field equations equate space-time curvature (expressed by Einstein tensor) with energy and momentum within that space-time (expressed by stress-energy tensor). These field equations are used to determine the space-time geometry resulting from mass, energy and momentum. That is they determine the metric tensor of space-time for a given arrangement of stress-energy in the space-time. The relationship between the metric tensor and Einstein's tensor allows field equations to be expressed as a set of ten nonlinear partial differential equations and the solution of these equations are nothing but the components of metric tensor. The Einstein's field equations in theory of gravitation are,

$$R_{ij} - \frac{1}{2} R g_{ij} = -K T_{ij} ,$$

where R_{ij} is the symmetric Ricci tensor ,

T_{ij} is stress energy momentum tensor .

These equations describe how space-time is influenced by the matter. The basis of this theory is Riemannian geometry in which the Ricci curvature tensor is symmetric hence the Einstein curvature tensor (G_{ij}) must be symmetric which is given by

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} .$$

In general theory of relativity the Einstein curvature tensor models local gravitational forces and it is equal (gravitational constant) to stress-energy tensor T_{ij}. The symmetry of Einstein curvature tensor forces the stress energy tensor (momentum tensor) T_{ij} to be symmetric.

Einstein's theory of gravitation has known flaws that it cannot describe intrinsic feature of spin of gravitating matter, it cannot prevent formation of singularities and it has not been able to account for Mach's principle. Cartan was the first man to introduce torsion in gravitation theory. In 1973 Trautman formulated Einstein-Cartan theory of gravitation by incorporating the spin of gravitating matter. This theory postulates that the spin of matter is the source of torsion of space-time geometry, with the hopes of preventing singularities through the spin of gravitating matter. However, when spin and angular momentum are being exchanged the momentum tensor is known to be non-symmetric as per the general equation of conservation of angular momentum. Hence to include the spin (affine torsion) the Ricci tensor is also to be non-symmetric, transferring the geometry to be non-Riemannian. Thus the geometry of space-time in Einstein-Cartan theory of gravitation is non-Riemannian, and the non-Riemannian part is described through the affine connections $\omega_{ij}{}^{l}$ and are defined as

$$\omega_{ij}{}^{l} = \Gamma_{ij}{}^{l} - K_{ij}{}^{l} ,$$

where $\Gamma_{ij}{}^{l} = \Gamma_{ji}{}^{l}$ and K_{ijk} is the contorsion tensor satisfying $K_{i(jk)} = 0$.

In the present thesis, the propagation equations of electric and magnetic parts of Weyl tensor with respect to Kerr-Newman metric are studied in Einstein-Cartan theory of gravitation. The techniques used for this study are:

- Newman-Penrose (NP) tetrad formalism (1962).
- The Extension of NP tetrad formalism called Jogia Griffiths formalism (1980).
- Differential forms and exterior calculus.

The work carried out in this dissertation is divided into three chapters and the study is exhibited sequentially chapter wise and section wise as follows:

Chapter 1 is preliminary. A brief history of development of general theory of gravitation is given in the Section1. In the Section 2, Einstein-Cartan theory of gravitation is introduced. Section 3 presents the differential forms, wedge product and exterior derivative. Cartan's equations of structure in Einstein theory of gravitation and in Einstein-Cartan theory of gravitation are described in Section 4 and Section 5 respectively. In the Section 6, a cursory account of Newman-Penrose tetrad formalism in Einstein-Cartan theory of gravitation is given. In the last section of this chapter the tetrad components of the Weyl tensor are enumerated. No originality is claimed in this chapter.

Chapter 2 deals with study of Kerr-Newman metric in Einstein-Cartan theory of gravitation. In Section 2, spin coefficients are obtained with respect to Kerr-Newman space-time by using Cartan's first equations of structure given by Katkar (2009). Tetrad components of connection 1-forms and curvature 2- forms in Einstein-Cartan theory of gravitation are derived in the Section 3. Further Riemannian curvature tensor, Ricci tensor and Ricci scalar are also obtained in the Section 4.

The study of Electric and magnetic parts of Weyl tensor and their propagation equations in Einstein-Cartan theory of gravitation is presented in Chapter 3. In Section 2, the tetrad components of Weyl tensor are obtained and subsequently those are used to get the electric and magnetic parts of Weyl tensor with respect to Kerr-Newman metric in Section 3. The Section 4 presents the propagation equations of electric and magnetic parts of Weyl tensor in Einstein-Cartan theory of gravitation. The necessary and sufficient conditions for the electric and magnetic parts to be propagated along the null vector l^i are obtained.