PREFACE

This dissertation entitled "*Principally Generated Multiplicative Lattices*" is based on abstract commutative ideal theory. Around 1938, M. Ward & R. P. Dilworth began study in abstract form of ideal theory of commutative rings. Their absolute aim was to extend results of commutative ring theory to general lattice theory. For such generalization, it was then obvious to introduce new binary operation called multiplication on lattices.

For such multiplicative lattices, theorems analogues of the Noether decomposition theorems for commutative rings were formulated and proved. However the theorems corresponding to the deeper results on the ideal structure of ring were not obtained, because of the lack of a proper abstraction of *principal ideals*. In [16], a weak concept of 'principal element' was introduced as follows:

" An element a of a multiplicative lattice L is *principal*, if $x \leq a$ implies $\exists$ an element $y \in L$ such that $x = ay$."

Though this concept was sufficed for the proof of the decomposition theorem into primaries, it had serious defects and it was immediately obvious that this concept of principal element is not adequate for the further development.

It was R. P. Dilworth [8] who was able to give a stronger formulation for the notion of principal elements in two identities, as follows :

" In a multiplicative lattice L, an element $a \in L$ is said to be *join principal*, if $x \vee y:a = (xa \vee y):a$ and *meet principal*, if $x \wedge ay = (x:a \wedge y)a$, for all $x, y \in L$. An element is *principal*, if it is both join and meet principal."

Principal elements are the cornerstones on which the theory of multiplicative lattices and abstract ideal theory now largely rests.

Using this new version of principal elements, R. P. Dilworth proceeded to demonstrate the richness of principally generated multiplicative lattices by giving a purely lattice theoretic development of the most basic constructions and results of classical ideal theory.

The resultant setting not only admits natural formulations of the fundamental definitions but is simultaneously rich enough to yield results of classical ideal theory. The key richness of the setting lies, of course, in the definition of a principal element which is expressed into two identities. Almost all basic concepts are studied in the chapter one.

Anderson and Jayaram [4] introduced the concept of a regular lattice as an abstraction of a lattice of ideals of regular rings. A compactly generated multiplicative lattice with 1 compact is regular, if each compact element is complemented. Further regular lattices are investigated and several conditions equivalent to a lattice being regular are given.

Further characterizations of regular lattices are given in the chapter second wherein the concepts of Baer elements, closed elements, $*$-elements, Baer lattices and quasiregular lattices, defined by Anderson and others [4], are studied. Some of the concepts are as follows.

Let L be a compactly generated lattice with 1 compact and every finite product of compact elements is compact in L and let L_* be set of all compact elements.

A nonempty subset F of L_* is called a filter, if (i) a, $b \in F$ implies $ab \in F$ and (ii) a, $b \in L_*$ with $a \in F$ and $a \leq b$ implies that $b \in F$. Note that, an element x of a multiplicative lattice is called a radical element, if $\sqrt{x} = x$. An element $a \in L$ is called a Baer element, if for any compact element $x \leq a$, $0:(0:x) \leq a$. An element $a \in L$ is called a closed element, if $0:(0:a) = a$. An element $a \in L$ is called a $*$-element, if $a = 0_F$ or equivalently $a = \vee_{r \in F} (0:r)$, for some filter F of L_* such that $F \cap \{0\} = \varnothing$. Next we study, the set of elements of these lattices.

Further in the same chapter, some equivalent conditions for a multiplicative lattice to be an M-lattice are obtained and using them Baer lattices with respect to radical elements are characterized.