PREFACE

This dissertation entitled "*Some Results in Multiplicative Lattices*" is based on abstract commutative ideal theory. Around 1938, M. Ward and R. P. Dilworth began a study in abstract form of the ideal theory of commutative rings. Their absolute aim was to extend results of commutative ring theory to general lattice theory. For such generalisation, it was then obvious to introduce new binary operation called multiplication on lattices. Since then there has been a lot of development in multiplicative lattices. Meanwhile the development was ceased due to weak concept of principal element, for the theorems corresponding to the deeper results of the ideal structure were not being obtained. R. P. Dilworth introduced the satisfactory notion of principal element in 1961. It was an absolute breakthrough. Beside him, most of the pioneer work has been done by M. Ward, K. P. Bogart, E. W. Johnson, D. D. Anderson, N. F. Jonowitz, etc.

This dissertation is an exposition of abstract commutative ideal theory, wherein we give simplified proofs of the work of D. D. Anderson, K. P. Bogart, E. W. Johnson, C. Jayaram, P. J. McCarthy. All the works have been sorted in many chapters as per types of multiplicative lattices. There are seven chapters. The elementary aspects of multiplicative lattices utilised frequently in this dissertation are listed in chapter 0.

Chapter I provides further introduction to multiplicative lattices mainly to principal elements. The results presented here are used constantly in the study. Chapter II is restricted to finite Boolean algebras. Some equivalent condition are developed for multiplicative lattices to be finite Boolean algebras. For a couple of these, we use the latest versions of

Nakayama's lemma and Krull intersection theorem which are given in first chapter. Chapter III focuses on local Noetherian lattices. A satisfactory concept of Noether lattices as an abstraction of lattices of ideals of a Noetherian ring was first introduced by M. Ward & R. P. Dilworth in 1961.

Chapter IV introduces regular lattices and develops some equivalent conditions for lattices being regular.

Gilmer did pioneer work in ' Rings in which semiprimary ideals are primary'. We study abstraction of these rings in chapter V. The lattices in which semiprimary elements are primary are called as the lattices satisfying the condition (*). For characterization of these lattices, the lattice of filters of a bounded commutative multiplicative semilattice is recalled. We study also investigation of prime elements of the lattices satisfying the condition (*). Further investigation of lattices of filters and using this, further characterization of regular lattices will be studied in chapter VI.

Chapter VII strengthens concept of weak r-lattices. In this chapter, mainly we study prime and primary elements of weak r-lattices. Also we study some equivalent conditions for weak r-lattice to be regular.

All the chapters are restricted to multiplicative lattice with the greatest element 1 compact. Many of the simple results are separated from the proofs of some lengthy theorems, since we desire to maintain the continuity of the proofs. During the study of the course, we observe that many of the so far published results of multiplicative lattices can be improved well. Some of them relative to the main theme of the theory are given in this book. Also new simple proofs to some of the results are given. Thus we have tried our best to make the exposition as self-contained as possible.

It is important to note that, whenever brackets are not introduced the operations viz., residuation & multiplication are to be performed first and then the operations viz., join & meet will be performed. The list of references is attached at the end of this book. The list contains a very incomplete bibliography. The utilised material obtained from other sources has been acknowledged.

Beside deserving the degree of M. Phil., our purpose of this dissertation was two fold: first to study the depth and beauty of the multiplicative lattice theory and second, to make deeper the knowledge of the theory for pursuing the line of thoughts. It is our pleasure to say that, we are all set now to continue the line of thoughts.

- *Chavan nitin sadashiv.*