CHAPTER 0

PRELIMINARIES

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This chapter is devoted to the summary of known concepts and results which will be used in subsequent chapters.

§ 0.1 DEFINITION :-

0.1.1 Right near-ring : ([7] , Page- 7)

A right near-ring is a triplet < N, +, . > consisting a set N with two binary operations + and . such that

(1) < N, + > is a group .

(2) < N, . > is a semigroup.

(3) $(a+b).c = a.c + b.c$, $\forall$ a, b, $c \in N$.

0.1.2 Abelian near -ring : ([7], Page - 11)

Let < N, +, . > be a near-ring . If < N, + > is abelian group we call N an abelian near-ring.

0.1.3 Commutative near-ring : ([7], Page - 11)

Let < N, +, . > be a near-ring. If < N , . > is commutative we call N itself a commutative near-ring.

0.1.4 Near - field : ([7], Page-11)

Let N be a near-ring. If < $N^* = N \setminus \{0\}$, . > is a group , then N is called a near-field.

0.1.5 Zero-symmetric part of near-ring : ([7]-Page-10)

Let < N, + , . > be a near-ring. The set

$N_0 = \{ n \in N / n.0 = 0\}$ is called the zero - symmetric part of N.

0.1.6 Zero-symmetric near-ring : ([7],Page-10)

A near-ring < N, +, . > is called a zero- symmetric near-ring if n.0=0, ∀ n∈N.

0.1.7 Subnear-ring :([7],Page -10)

A subset M of a near-ring < N,+,. > is a subnear-ring of N if < M,+,. > is also a near-ring.

0.1.8 Left ideal in a near-ring :([7],Page-15)

Let < N, +, . > be a near-ring . A normal subgroup < I , + > of < N, +> is called a left ideal in N if n.(n′+ i) - n.n′∈I , ∀ i ∈ I and ∀ n, n′ ∈ N.

0.1.9 Right ideal in a near-ring : ([7],Page-15)

Let < N, +, . > be a near-ring . A normal subgroup < I , + > of < N, + > is called a right ideal in N if i.n ∈ I , ∀ i ∈ I and ∀ n ∈ N.

0.1.10 Ideal in a near-ring : ([7],Page-16)

Let < N, +, . > be a near-ring . A normal subgroup < I, + > of < N, + > is called an ideal in N if

(1) n.(n′+i)-n.n′∈ I , ∀ i ∈ I and ∀n , n′ ∈ N.

(2) i.n ∈ I , ∀ i ∈ I and n ∈ N.

0.1.11 Pseudo-left ideal in a near-ring : ([8],Page-)

Let < N, +, . > be a near-ring . A subnear-ring < I , +, . > is called a pseudo-left ideal in N if

(1) < I, + > is a normal subgroup of < N,+ >

(2) n.i - n.0 ∈ I , ∀ i ∈ I and ∀ n∈N.

0.1.12 Booleannear-ring : ([7],Page -300)

A near-ring < N, +, . > is called a Boolean near-ring iff $n^2 = n$, ∀ n∈N.

<u>0.1.13 Bi-ideal in a near-ring</u> : ([3],Page- 1002)

Let $< N, +, . >$ be a near-ring . A subgroup $< B, + >$ of $< N, + >$ is called a bi-ideal in N if $BNB \cap (BN) * B \subseteq B$.

Note :-- operation $*$ is defined as

$A * B = \{a(a'+b) - a.a' / a, a' \in A , b \in B\}$.

<u>0.1.14 Quasi-ideal in a near-ring</u> : ([3],Page- 1002)

Let $< N, +, .>$ be a near-ring . A subgroup $< Q, + >$ of $< N, + >$ is called a quasi-ideal in N if $QN \cap NQ \cap N*Q \subseteq Q$.

<u>0.1.15 Moore family</u> : ([7],Page- 2)

$\mu \subseteq 2^A$ is called a Moore -system on A if

(1) $A \in \mu$

(2) μ is closed w.r.t. arbitrary intersection.

<u>0.1.16 Regular near-ring</u> :([7],Page-345)

A near-ring $< N, +, . >$ is called a regular near-ring if $\forall$ $n \in N$ $\exists$ $x \in N$: $n.x.n = n$.

<u>0.1.17 Nilpotent</u> : ([7],Page -69)

Let $< N, +, . >$ be a near-ring. An element $n \in N$ is called nilpotent if $k \in IN : n^k = 0$. where IN is set of all natural numbers.

<u>0.1.18 Idempotent</u> : ([7],Page-11)

Let $< N,+, . >$ be a near-ring . An element $n \in N$ is called an idempotent element if $n^2 = n$.

<u>0.1.19 Centre</u> : ([7],Page- 253)

Let $< N, +, \cdot >$ be a near-ring. Let $c(N) = \{n \in N / n.n' = n'.n, \forall n' \in N\}$. c(N) is called the centre of $< N, . >$

<u>0.1.20 Central idempotent</u> : ([7],Page -33)

An idempotent $e \in N$ is called central idempotent if it is in the centre of $< N,. >$ i.e. if $\forall n \in N$: $e.n = n.e$.

§ 0.2 RESULT

<u>0.2.1 Result</u> :([7],Page -9)

If $< N,+,. >$ is any near-ring , then $0.n = 0$, $\forall$ $n \in N$.

<u>0.2.2 Result</u> :([6],Page - 45)

Intersection of any collection of normal subgroups in N is a normal subgroup.

<u>0.2.3 Result</u> : ([6],Page - 45)

If $< A,+ >$ and $< B,+ >$ are two normal subgroups of $< N,+ >$ then $< A+B,+ >$ is a normal subgroup of $< N,+ >$.

<u>0.2.4 Result</u> : ([7] , Page -346)

Let $N \neq \{0\}$ be a regular near-ring with identity . Equivalent are:

(a) $N = N_o$. has no non -zero nilpotent element.

(b) All idempotents of N are central.

(c) N is a subdirect product of near-field.

<u>0.2.5 Result</u> : ([7],Page -249)

If N is a near-field then either $N \cong M_c(z_2)$ or N is zero symmetric.

<u>0.2.6 Result</u> :([7],Page -14)

A near - ring with three or more elements is a near-domain.

<u>0.2.7 Result</u> : ([7],Page-345)

[A near-ring N is called regular near-ring if $\forall$ $n \in N$ $\exists x \in N$: $n.x.n = n$] $n.x$ and $x.n$ are idempotents in a regular near-ring.

0.2.8 Result : ([7],Page-249)

Equivalent are for $N \in \eta_0$ (the set of all zero - symmetric near-ring)

(a) N is a near-field.

(b) $N_d \neq \{0\}$ and $\forall\, n \in N^* = N \setminus \{0\}$: Nn=N. where N_d is the set of all distributive element in N.

(c) N has a left identity and N^N is N-simple.

(d) N has a left identity and N is 2- primitive on N^N.

(e) N has a left identity and N is a 1- primitive on N^N.

0.2.9 Result : ([7], Page-)

If N is commutative near-ring then N is zero- symmetric near-ring.

0.2.10 Result : ([3], Page -1002)

Every quasi-ideal in a near-ring is a bi-ideal.

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