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PRELIMINARIES AND NOTATIONS

0.1) Notations

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CHAPTER - 0

PRELIMINARIES AND NOTATIONS

(0.1) Notations:

∞	:	Infinity
$\lvert \cdot \rvert$	:	Modulus
$\in$	:	Belongs to
$=$	:	equal
$>$	:	Greater than
$<$	:	Less than
$\geq$	:	Greater than or equal to
$\leq$	:	Less than or equal to
$\neq$	:	Not equal to
$\longrightarrow$	:	Tends to
$\sqrt{}$	:	Square root
$\cap$	:	Intersection
$\cup$	:	Union
Σ	:	Summation
$\oplus$ or $\dot{\Sigma}$	:	Direct sum
Id	:	Identity
$\mathcal{F}$	:	Fourier operator
$\mathbb{R}$	:	Set of real numbers
$\mathbb{Z}$	:	Set of integers

ε : Epsilon

$L^p(\mathbb{R})$: The class of measurable functions f on $\mathbb{R}$ such that the (Lebesgue) integral

$$\left\{ \int_{-\infty}^{\infty} | f(x)|^p \, dx \right\}^{1/p} \text{ is finite.}$$

$L^\infty(\mathbb{R})$: The collection of almost everywhere (a.e.) bounded functions.

$L^p(0,2\pi)$: The Banach space of functions f satisfying $f(x + 2\pi) = f(x)$ a.e. on $\mathbb{R}$ and

$$\left\{ \frac{1}{2\pi} \int_{0}^{2\pi} | f(x)|^p \, dx \right\}^{1/p} \text{ is finite.}$$

$l^p(\mathbb{Z})$: The space of square summable complex sequences indexed by $\mathbb{Z}$.

(0.2) Definitions:

1) The $L^p(\mathbb{R})$ norm of f is defined as,

$$\| f \|_p = \left\{ \int_{-\infty}^{\infty} | f(x)|^p \, dx \right\}^{1/p} \quad \text{for} \quad 1 \leq x < \infty$$

$$\| f \|_\infty = \underset{0 \leq x < \infty}{\text{ess. sup}} | f(x) |.$$

2) Inner product in $L^p(\mathbb{R})$ is defined as,

$$< f , g > = \int_{-\infty}^{\infty} f(x) \overline{g(x)} \, dx \qquad \text{for } f , g \in L^p(\mathbb{R}).$$

3) Minkowski Inequality for $L^p(\mathbb{R})$

$$\| f + g \|_p = \| f \|_p + \| g \|_p$$

4) Holder Inequality for $L^p(\mathbb{R})$

$$\| f g \|_p = \| f \|_p \| g \|_{p(p-1)^{-1}}$$

5) Schwarz Inequality for $L^p(\mathbb{R})$

$$\| f g \|_1 = \| f \|_2 \| g \|_2$$

6) The $L^p(0,2\pi)$ norm of f is defined as,

$$\| f \|_{L^p(0,2\pi)} = \left\{ \frac{1}{2\pi} \int_0^{2\pi} | f(x) |^p \, dx \right\}^{1/p}$$

$$\text{for } 1 \leq x < \infty$$

$$\| f \|_{L^\infty(0,2\pi)} = \operatorname*{ess.\,sup}_{0 \leq x < 2\pi} | f(x) | .$$

7) Inner product in $L^p(0,2\pi)$ is defined as,

$$< f , g > = \frac{1}{2\pi} \int_0^{2\pi} f(x) \overline{g(x)} \, dx$$

$$\text{for } f , g \in L^p(0,2\pi).$$

The inequalities of Minkowski, Holder and Schwarz for $L^p(\mathbb{R})$ are also valid for $L^p(0,2\pi)$.

8) The $l^p(\mathbb{Z})$ norm of f is defined as,

$$\| \{ a_k \} \|_{l^p} = \left\{ \sum_{k\in\mathbb{Z}} |a_k|^p \right\}^{1/p} \quad \text{for} \quad 1 \le x < \infty$$

$$\| \{ a_k \} \|_{l^\infty} = \sup_k |a_k| .$$

9) Inner product in $l^p(\mathbb{Z})$ is defined as,

$$<\{ a_k \} , \{ b_k \}> = \sum_{k\in\mathbb{Z}} a_k \bar{b}_k$$

Again, the inequalities of Minkowski, Holder and Schwarz for $L^p(\mathbb{R})$ are also valid for $l^p(\mathbb{Z})$.

10) Riesz Basis

A function $\psi \in L^2(\mathbb{R})$ is said to generate a Riesz basis (or unconditional basis) $\{ \psi_{b_o;j,k} \}$ with sampling rate b_o if both of the following two properties are satisfied,

(i) the linear span

$$< \psi_{b_o;j,k} \; ; \; j, k \in \mathbb{Z} >$$

is dense in $L^2(\mathbb{R})$;and

(ii) there exists a positive constants A and B, with $0 < A \le B < \infty$ such that

$$A \, \|\{c_{j,k}\}\|^2_{l^2} \le \left\| \sum_{j,k\in\mathbb{Z}} c_{j,k} \psi_{b_o;j,k} \right\|^2_2 \le B \, \|\{c_{j,k}\}\|^2_{l^2}$$

for all $\{c_{j,k}\} \in l^2(\mathbb{Z}^2)$. Here A and B are called Riesz bounds of $\{ \psi_{b_o;j,k} \}$.

(0.3) Results:

Result(1): For any $a > o$

$$\int_{-\infty}^{\infty} e^{-ax^2}\, dx = \sqrt{\frac{\pi}{a}}\,.$$

Result(2): If G is Hermitian operator on $\mathbb{H}$ such that $\langle G^* G f , f \rangle \geq 0$ for all $f \in \mathbb{H}$, then all the eigenvalues of G are necessarily nonnegative. We then say that the operator G itself is nonnegative and write this as an operator inequality $G \geq 0$.

Result(3): If a positive bounded linear operator T on $\mathbb{H}$ is bounded below by a strictly positive constant α, then T is inversible and its inverse T^{-1} is bounded by α^{-1}.

Result(4):

$$\cot(x) = \lim_{n \longrightarrow \infty} \sum_{k=-n}^{n} \frac{1}{(x + \pi k)}$$