CHAPTER - II

THEORY OF FILTER

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2.1 DISCUSSION OF LOWER AND HIGHER ORDER FILTER CIRCUITS :-

Active - filter representations of passive filters can be accomplished in many ways. Before discussing the lower and higher order filter, we will discuss the order of active RC filter .

In the Laplace transform terminology the 'order" is related to the "number of poles". The actual mathematical significance of these designations is related to the transfer function of the filter. For a given filter type, the performance generally becomes closer to the ideal characteristic as the number of poles (i.e. the order) increases.

The gain roll-off in the stop-band is determined by the order (n = 1,2,3,......., etc) of the filter. The order of a filter equals the number of RC pairs in the circuit. Each increase in order increases the roll-off by 20 db/decade as shown for an LP filter in fig 2.1 Active filters with order of six or more are practical. The larger the order, the more closely the response approaches the ideal one. However, each 20 db increment in the roll-off is accompanied by a phase angle change of -45^{o} at W_{o} .

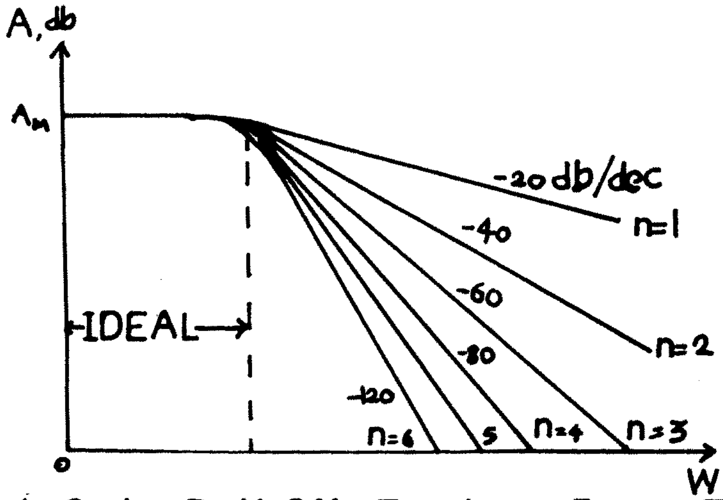

Fig. 2·1: Gain Roll-Off For Low Pass Filter With Increasing Order.

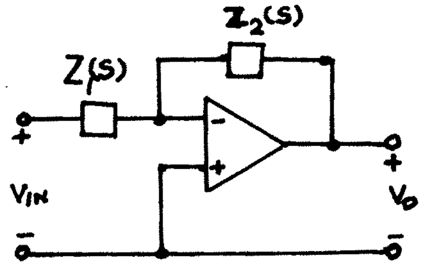

Fig. 2·2: First Order Low Pass Filter.

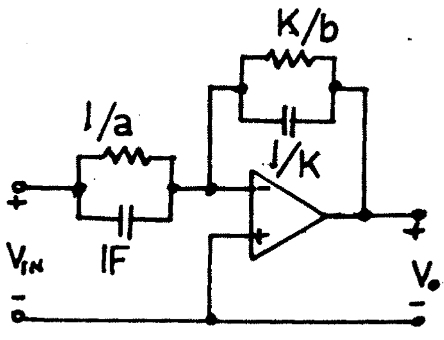

Fig. 2·3: Realization For First Order Low Pass Filter.

In general, the higher order filter is realized by the "direct method" where a single circuit is used to realize the entire transfer function, as was done for the passive filter design or the technique known as "cascade method". The transfer function to be realized is first factored into a product of first-order and / or second order terms. Each term is then individually realized by an active RC circuit. The cascade connection of individual circuit realizes the overall transfer function.

2.1.1 THE FIRST - ORDER FILTER :- First order low pass filters are often used to perform a running average of a signal having high frequency fluctuations superimposed upon a relatively slow mean variation[1]; for this purpose it is simply necessary to make the filter time constant CR much greater than the period of the high frequency fluctuations.

All operational amplifier active high pass filters show a band pass characteristic, for their response eventually falls off at frequencies beyond the closed loop bandwidth limit.

The first order low pass filter is, in inverting amplifier structure, shown in fig 2.2. It realizes a first

$$H(S) = K \frac{S + a}{S + b} \qquad \text{..... (2.1)}$$

where a and b are real

The voltage- ratio transfer function is

$$H(S) = \frac{Vo(S)}{V_{in}(S)} = - \frac{Z_2(S)}{Z_1(S)} \qquad \ldots\ldots \quad (2.2)$$

Apart from the minus sign in equation (2.2), our objective is to identify the impedances Z_1 and Z_2 from the right hand side of equation (2.1) so they represent the input impedances of the RC one-port networks. An RC impedance requires that all of its zeros and poles be simple, lie on the negative real axis, and alternate with each other, the first critical frequency being a pole. For negative K and nonnegative a and b, we can make the following identifications :

$$Z_1(S) = \frac{1}{S + a} \quad \text{and} \quad Z_2(S) = \frac{-K}{S + b} \qquad \ldots\ldots \quad (2.3)$$

The realization is shown in fig 2.3

2.1.2 The Second Order Filter :- A second order filter has a response whose magnitude falls at 40 db/decade in the stop band. The sharpness of the transition between the pass and stopbands depends upon the choice of filter constants which are fixed by circuit parameters.

A band pass filter characteristic can be obtained by cascading a high and a low pass filter, but when a highly selective (high Q) band pass characteristic is required a different approach is necessary. Many examples of active

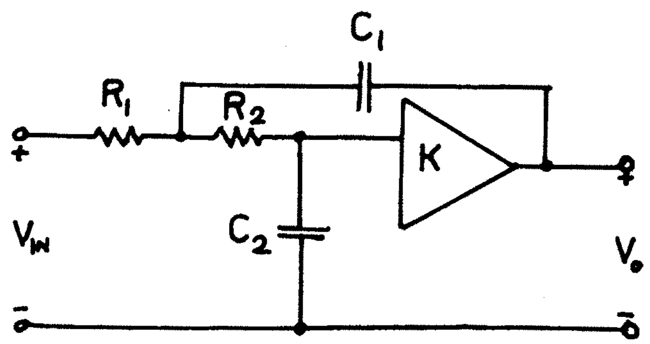

Fig.2.4: Second Order Low Pass Filter.

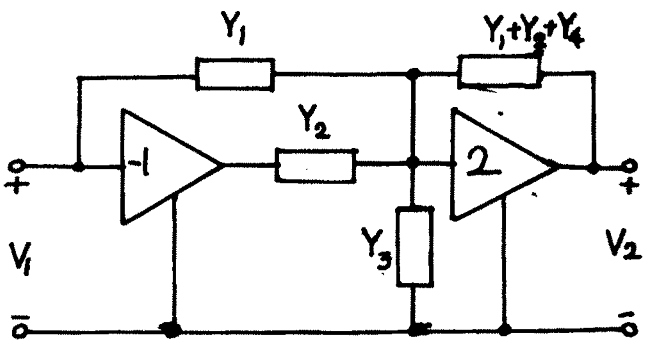

Fig.2.5: General Biquadratic Filter.

bandpass filters will be found in the literature and in manufacturer's notes. High Q bandpass filters, based upon a single operational amplifier, have a Q value which is very sensitive to component variation. The so called state variable filter approach, we will discuss in latter section.

The fig 2.4 shows the second order low pass filter realization[3].

2.1.3 BIQUAD SECOND ORDER FILTER :- For certain specialized filtering applications, especially those with critical phase requirements, a biquadratic network function in which the position of the complex conjugate poles and zeros can be independently specified may be required. Such a function has the form...

$$\frac{V_o}{V_1(S)} = \frac{m_2 S^2 + m_1 S + m_o}{S^2 + (Wo/Q)S + Wo^2} \qquad \ldots\ldots (2.4)$$

$$\text{or} \quad H(S) = K \frac{S^2 + cS + d}{S^2 + aS + b}$$

$$= K \frac{S^2 + (W_z/Q_z)S + W_z^2}{S^2 + (W_p/Q_p)S + W_p^2}$$

The frequency Wz is known as the zero frequency and Qz the zero Q. Likewise the frequency Wp is pole frequency & Qp the pole Q.

The numerator coefficients determine the filter bandwidth and the denominator coefficients the filter response.

The fig. 2.5 shows the general biquadratic filter which uses two VCVSs, the first an inverting one with a gain of -1, the second a noninverting one with a gain of 2. The voltage transfer function for this filter is

$$\frac{V_2(S)}{V_1(S)} = \frac{2(Y_1 - Y_4)}{Y_3 - Y_4} \qquad \ldots\ldots (2.5)$$

The values of the admittances Yi are found by dividing the numerator and denominator of equation (2.4) by the factor S+C,C>0 and making partial fraction expansions of the resulting functions. As an example of the use of this network, consider the realization of normalized all-pass (constant magnitude) transfer function.

$$\frac{V_2(S)}{V_1(S)} = \frac{S^2 - 2S + 1}{S^2 + 2S + 1} \qquad \ldots\ldots (2.6)$$

It is shown that the biquadratic function can be characterized by its zero and pole frequencies and zero and pole Q's. Depending on these parameters, the biquadratic functions can be classified into low-pass, high-pass, band-elimination and all-pass functions.

2.1.4 HIGHER ORDER FILTERS :- Higher order filters having better gain roll-off characteristics can be obtained by cascading, coupling or direct method. In direct method, a single circuit is used to realize the entire transfer function.

A] CASCADE APPROACH :- Many filtering applications, however, require filters of higher than second order, either to provide greater stop-band attenuation and sharper cut off at the edge of the passband in the low-pass or high-pass case or to provide a broad passband with same special transmission characteristic in the bandpass case .

A standard method of realizing a higher order transfer function is to express it as a product of second-order transfer functions and to realize each second-order transfer function as a single amplifier or state variable filter. The final filter is obtained by cascading the individual second order filter blocks. For example, a 3rd order LP filter may be obtained by cascading 1 st order and 2nd order LP filters. A 4th order filter may be obtained by cascading two second order filters. A fourth order bandpass filter response can be obtained by cascading LP filter with an HP filter with cut off frequencies properly adjusted.

Since these circuits all have an operational amplifier as an output element, their output impedance is low and thus a simple cascade of such second-order realizations may be made without interaction occurring between the individual stages. As a result, the overall voltage transfer function is simply the product of the individual transfer functions. may be tuned separately, a point of considerable practical importance when high-order network functions are to be realized. The success of the cascade method, depends on the use of operational amplifiers which have as low an output impedance as possible. The use of high impedance normalization levels for the passive elements, which minimizes loading an the interior operational amplifier output stages, is also significant in reducing interaction.

The general transfer function for cascade approach is [4]

$$T(S) = \prod_{i=1}^{N} T_i(S) \qquad \text{..... (2.6)}$$

where Ti(S) is of the form

$$TI(S) = K_i \frac{m_i S^2 + c_i S + d_i}{n_i S^2 + a_i S + b_i} \qquad \text{..... (2.7)}$$

The general cascade topology is shown in figh. 2.6

The output voltage at block T1 is

$V_{o1} = T_1 . V_{in}$

The output voltage at block T2 is

$V_{o2} = T_2 V_{o1} = T1.T2. V_{in}$

Extending the argument to cascade of N sections, the output voltage V_o is

$V_o = T_1 . T_2 . T_3 ...... T_n V_{in}$

$$\frac{V_o}{V_{in}} = T_1 . T_2 . T_3 .... T_n$$

$$= \prod_{i=1}^{N} T_i \qquad ..... (2.8)$$

Thus the transfer function of a cascade of networks is the product of the individual transfer functions, provided that the input impedance of each network is very large compared with the output impedance of the preceding network.

In the N_{th} order case;

The general low pass voltage transfer function is

$$T(S) = \frac{H}{S^n + a_{n-1} S^{n-1} + ... + a_1 S + a_o} \qquad ..... (2.9)$$

if n is even, equation (2.9) can be written as

$$T(S) = \prod_{i=1}^{n/2} \frac{Hi}{S^2 + ai_1 S + aio} \qquad \ldots. (2.10)$$

if n is odd equation (2.9) can be written as

$$T(S) = \frac{1}{S - \sigma_o} \prod_{i=1}^{(n-1)/2} \frac{Hi}{S^2 + ai_1 S + aio} \qquad \ldots.. (2.11)$$

The general high pass voltage transfer function is

$$T(S) = \frac{HS^n}{S^N + a_{n-1} S^{n-1} + \ldots.. + a_1 S + a_o} \qquad \ldots.. (2.12)$$

if n is even, this may be put in the form

$$T(S) = \prod_{i=1}^{n/2} \frac{Hi\ S^2}{S^2 + ai_1 S + a_{io}} \qquad \ldots.. (2.13)$$

if n is odd, this may be written as

$$T(S) = \frac{S}{S - \sigma_o} \prod_{i=1}^{(n-1)/2} \frac{Hi\ S^2}{S^2 + a_i S + a_{io}} \qquad \ldots.. (2.14)$$

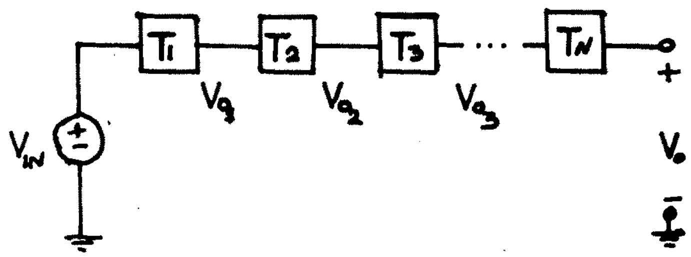

Fig.2.6: The Cascaded Topology.

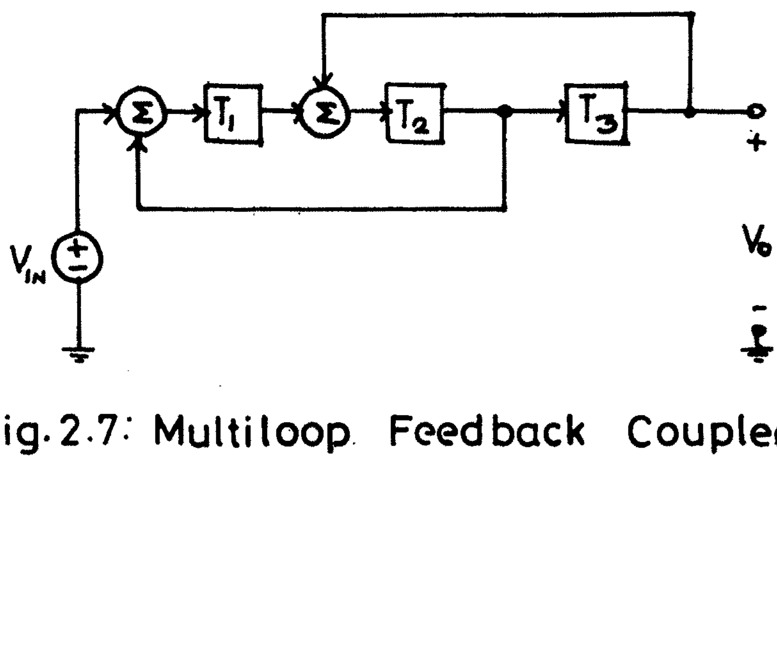

Fig.2.7: Multiloop Feedback Coupled System.

The general nth order bandpass filter voltage transfer function has th form

$$T(S) = \frac{H\,S^{n/2}}{S^{n} + a_{n-1}S^{n-1} + \dots + a_1 S + a_o} \qquad \dots\dots (2.15)$$

where n can only be even. The factored form of this function is

$$T(S) = \prod_{i=1}^{n/2} \frac{H_i\,S}{S^{2} + a_{i1}S + a_{io}} \qquad \dots\dots (2.16)$$

B] COUPLED STRUCTURE OR LEAP FROG FILTER :- In a second family of structures the individual biquadratic blocks are coupled to each other via feedback path. This method of active synthesis relies on simulation of the interactive effects in a ladder structure by feedback loops as in a control system. The passive reactances may then be replaced on a one to one basis by Miller integrators[5]. The importance of this early procedure is only now being appreciated in the context of the manifest advantages of multi-loop cascade techniques. An example of these so called coupled structure is shown in fig. 2.7

These structures are more complex than for the cascaded structures, since a change in one biquad affects the currents and voltages in all the biquads. Moreover, this lack of isolation between the blocks makes their tuning more difficult. On the other hand, one distinct advantage of using coupled structures is that the sensitivity is usually lower than for the equivalent cascaded realization.

The biquad is the basic building block used in both cascaded and coupled realizations. Therefore, biquad circuits are the fundamental importance in the design of active filters.

2.2 SENSITIVITY :- Active filters are designed to perform certain functions such as wave shaping or signal processing. Given perfect components, there would be little difference among the many possible designs. In practice, however, all components deviates from their nominal values because of manufacturing tolerances, changes in environmental conditions such as temperature and humidity or chemical changes due to the aging of the components.As a consequence, the performance of a practical filter differs from the nominal design. This causes the network transfer function to drift away from its nominal value. The cause and effect relationship between the network element variation and the resulting changes in the network transfer function is known as the "Sensitivity". One way to minimize this change or to reduce the sensitivity is to chose components with small tolerances, low temperature, again and humidity coefficients. However, this approach will usually result in more expensive networks than necessary. A practical solution is to design a network that has a low sensitivity to element changes. This is especially important in active filter design where active element such as op. amp. is much more sensitive to environmental changes.A good understanding of sensitivity is essential to the design of practical active filter.

Defination :-One of the earlier definations of sensitivity was made by Bode H.W.[6]

Sensitivity function is defined as the ratio of the fractional change in network function to the fractional change in an element for the situation when all changes concerned are differentially small[2].

The symbol S is used to denote sensitivity. In addition, a superscript character is used to indicate the performance characteristic that changes and subscript character is used to indicate the specific network element that is causing the change.

classically the sensitivity of a network function F(S) w.r.t. a parameter x is defined as

$$S_X^{F(X)} = \frac{\partial \ln F(S)}{\partial \ln x} = \frac{X}{F} \cdot \frac{\partial F}{\partial x} \qquad \text{..........} \quad (2.17)$$

Let the transfer function (open - circuit voltage transfer ratio) of a network N be T(S) = P(S) / q(S). Then the sensitivity of T(S) with respect to the parameter x can be shown to be

$$S^T_X = S_X^P - S^q_X$$

$$= X \left[\frac{1}{P} \cdot \frac{\partial P}{\partial X} - \frac{1}{q} \cdot \frac{\partial q}{\partial X} \right] \quad \ldots\ldots\ldots\ldots (2.18)$$

2.2.1 W AND Q SENSITIBITY :- In a qualitative sense, the sensitivity of a network is a measure of the degree of variation of its performance from nominal, due to change in the elements constituting the network. The biquadratic filter function can be expressed in terms of the parameter W_p, W_z, Q_p, Q_z and k as

$$T(S) = K \frac{S^2 + \frac{W_z}{Q_z} S + W_z^2}{S_2^2 + \frac{W_p}{Q_P} S + W_P^2} \quad \ldots\ldots\ldots\ldots (2.19)$$

Let us first consider the sensitivity of the pole frequency W_p to a change in a resistor R. Pole sensitivity is defined as the per unit change in the pole frequency, $\Delta W_P/W_P$, caused by a per - unit change in the resistor, $\Delta R/R$. Mathematically.

$$S_R^{\omega_P} = \lim_{\Delta R \to o} \frac{\frac{\Delta \omega_P}{\omega_P}}{\frac{\Delta R}{R}} = \frac{R}{W_P} \frac{\partial \omega_P}{\partial R}$$

This is equivalent to

$$S_R^{W_P} = \frac{\partial(\ln W)}{\partial(\ln R)} \quad \ldots\ldots (2.20)$$

Note that the cost of manufacturing a component is a function of the percentage change (100 x Δ R/R) rather than the absolute change (Δ R) of the component. For this reason it is desirable to measure sensitivity in terms of the relative changes in components

Similarly,

$$S_R^{Q_P} = \frac{R}{Q_P} \cdot \frac{\partial Q_P}{\partial R} \qquad \ldots\ldots\ldots (2.21)$$

2.2.2 MAGNITUDE AND PHASE SENSITIVITY :- The computation of the sensitivity functions for the magnitude and phase functions are given below.

We express a transfer function in polar form and substitute S by jw to give

$$H(jw) = | H(jw) | e^{J\phi(w)} \qquad \ldots\ldots\ldots (2.22)$$

Then the sensitivity function becomes

$$S_X^{H(jw)} = \frac{X}{H(jw)} \cdot \frac{\partial}{\partial X} [| H(jw) | e^{j\phi(w)}] \qquad \ldots\ldots (2.23)$$

Which can be expanded by making use of the product rule for differentiation of a product to give

$$S_X^{H(jw)} = S_X^{|H(jw)|} + j \phi(w) S_X^{\phi(w)} \qquad \ldots\ldots\ldots (2.24)$$

They state that the magnitude and phase sensitivity of a transfer function with respect to an element are simply related to the real and imaginary parts of the transfer function sensitivity with respect to the same element.

2.2.3 **THE MULTIPARAMETER SENSITIVITY :-** So for we have considered the situation where a network function is changed due to a change in a particular network element. In this section, we extend this concept by considering the change of a network function due to the simultaneous variation of many elements in the network.

Let H be the network function and let X_j (j = 1, 2, ... m) be the network elements such as resistores, capacitors, inductors or the parameters describing the active devices that are subject to change in values. Then the change ΔH in H due to the simultaneous variations of all the elements X_j may be obtained by the multivariable Taylor series expansion of H with respect to X_j

$$\Delta H = \sum_{j=1}^{m} \frac{\partial H}{\partial X_j} \Delta X_j + \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial^2 H}{\partial X_i \, \partial X_j} \cdot \frac{\Delta X_i \, \Delta X_j}{2\,!} + \ldots$$

$$+ \ \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.25)$$

If the changes in the elements ΔX are small, the second and higher order terms can be ignored and the first order approximation is given by

$$\Delta H = \sum_{j=1}^{m} \frac{\partial H}{\partial X_j} \Delta X_j \qquad \text{..............(2.26)}$$

To bring the sensitivity function in to evidence,the equation (2.26) can be written as

$$\Delta H = \sum_{j=1}^{m} \left(\frac{X_j}{H_j} \cdot \frac{\partial H}{\partial X_j} \right) \left(\frac{\Delta X_j}{X} H \right)$$

$$= H \sum_{j=1}^{m} S_{X_j}^{H} V_{X_j} \qquad \text{................(2.27)}$$

$$\text{Where } V_{X_j} = \frac{\Delta X_j}{X_j}$$

denotes the fractional change in the element X_j and is can be approximated by expression

$$\frac{\Delta H}{H} = \sum_{j=1}^{m} S_{X_J}^{H} \cdot V_{X_J} \qquad \text{........ (2.28)}$$

The multiparameter sensitivity defination above is somewhat simplisitic and does not take into account the random element variations. A more realistic and accurate measure is known as "Statistical multi parameter sensitivity "[7].

2.2.4 GAIN SENSITIVITY :-In the filter design, requirements are frequently stated in terms of the maximum allowable deviation in gain over specified band of frequencies. In such situation, it is convenient to consider the logarithm of the transfer function of a networkoperating under the sinusoidal steady state. Thus the magnitude of the transfer function in db can be written as.

$$G(W) = 20 \log \left| T(JW) \right| \quad \ldots\ldots\ldots\ldots\ldots(2.28)$$

Where T(JW) is given by equation (2.19).

$$G(W) = \sum_{i=1}^{N} 20 \log \left| S^2 + \frac{W_{zi}}{Q_{zi}} S + W_{zi}^2 \right|_{S=jW}$$

$$- \sum_{i=1}^{N} 20 \log \left| S^2 + \frac{W_{pi}}{Q_{pi}} S + W_{pi}^2 \right|_{S=jw}$$

$$+ 20 \log \left| K \right| \quad \ldots\ldots\ldots\ldots(2.29)$$

"Gain sensitivity " is defined as the gain in dB due to a per-unit change in an element (or parameter)X :[4]

$$S_x^{G(w)} = \frac{\partial\, G(W)}{\partial\, X/X}$$

$$= X \frac{\partial\, G(W)}{\partial\, X} \text{ dB} \quad \ldots\ldots\ldots\ldots(2.30)$$

From this equation

$$\Delta\, G(w) = \lim_{\Delta x \to 0} S_X^{G(w)} \frac{\Delta\, X}{X}$$

and for small changes in X

$$\Delta\, G(w) = S_X^{G(w)} \frac{\Delta\, X}{X} \quad \ldots\ldots\ldots(2.31)$$

Gain variation is affected by

A. The approximation function.

B. The choice of circuit topology .

C. The types of components used in realization.

2.2.5 ROOT SENSITIVITY :- The functions and the filter transfer functions are often specified by their poles and zeros. The location of the poles of an active filter determines the stability of the network. The poles and zeros themselves, being functions of network parameter get perturbed due to variation in these parameters. The change in pole-zero location changes the frequency response characteristics of a filter and may indeed indicate potential instability of the filter[8]. In order to study this aspect the sensitivity of a root (a pole or a zero) is found to be useful.

Let S_i be a root of either the numerator or denominator. Then the root sensitivity S_x^{si} is defined as

$$S_x^{si} = X \frac{dS_i}{dX} \Big|_{S=S_i} \qquad \text{..........(2.32)}$$

The root sensitivity in contrast to the transfer - function sensitivity is a complex constant. When S_i is the neumerator polynomial of the network function, equation (2.32) defines "the zero sensitivity". Likewise if S_i is the denominator polynomial equation (2.32) is referred to as the "pole sensitivity".

We have defined several sensitivity functions and interrelated them. However, in the final anlysis we are interested in minimizing the deviation of the filter response due to increamental variation of some network parameters. In highly selective networks (i.e. high Q networks) the pole sensitivity is an important factor to consider because of the stability of the network. In the design of the active filters the sensitivity of the filter transfer function to the variation of the active parameters is a major considerations.

2.3 VARIOUS APPROXIMATIONS IN FILTERS IN FILTER CIRCUIT THEORY AND DESIGN CONSIDERATION :- The modern filter design is based on the selection of the filter transfer function to satisfy the specification and then the ralization of this function by synthesis techniques[9]. The step involved in the modern filter design can be summarized as[8].

1. Selection of the filter specification.
2. Selection of a realizable rational function which satisfies this specication.
3. Ralization of the transfer function and calculation of the component values of the chosen filter structure.

4. Construction and testing of the filter.

Filter performance is prescribed in the frequency or time domain. Since electrical-network reactances have continuous frequency characteristics (except at a resonance), the adrupt cut off inherent in the ideal response of fig. (2.8) cannot actually be attained by any finite connection of elements. Hence arises the so called 'approximation problem' that is, the determination of system functions which approximate the given curve within specified tolerances[5] which are at the same time realizable as physical networks.

For LP filter, The filter is required to pass all frequencies below W_c with no attenuation and frequencies above W_c with infinite attenuation. The phase of the filter network is to be a linear function of W.

These specifications cannot be achieved by a physical network. If we consider the magnitude as the Fourior transform, the corresponding time function is of the form sin α_t / α_t and exits for all t. This makes the empulse response of the filter network non-causal and hence non-realizable. It can be shown that any band limited frequency response results in a non causal time response.

In order to overcome this problem the filter specificaions are

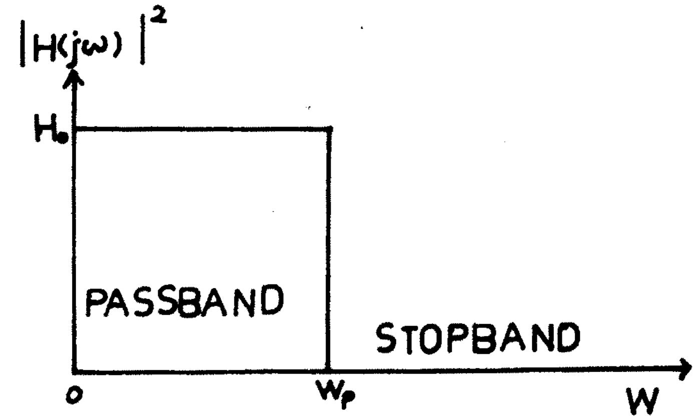

Fig. 2.8: An Ideal Low Pass Filter.

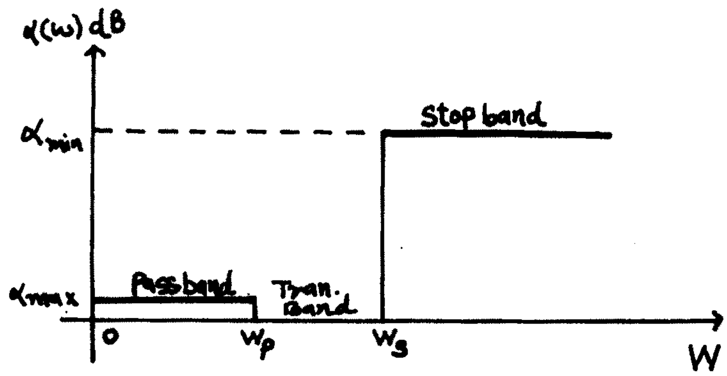

Fig. 2.9: The Specification For A Realizable Low Pass Filter.

modified. Instead of seeking an idealistic performance criterion, we specify the maximum permissible loss or attenuation over a given frequency band of interest called the 'pass band' the minimum allowable loss over another frequency band called the "stop band" and a statement about the selectivity or the tolerable interval between these two bands called the "transitinal band".

A method of approximation is based on the Bode plots[4]. This method is suitable for low order simple filter designs. More complex filter characteristics are approximated by using some well-described rational functions whose roots have been tabulated. The most popular among these approximations are the Butterworth, Chebyshev, Bessel and the elliptic (or cauer) types. These approximations are directly applicable to low-pass filters. However, they can also be used to design high pass filters, and symmetrical band pass and band reject filters by employing the frequency transformation functions.

2.3.1 BUTTERWORTH APPROXIMATION :- Butterworth approximation[8] (1936) is a special form of Taylor series approximation in which the approximating function and specified function are identical at $W = 0$. Butterworth polynomials can be used to approximate more practical characteristic function as shown in fig. (2.9).From fig. (2.9), the requirement are characterized by pass band from dc to

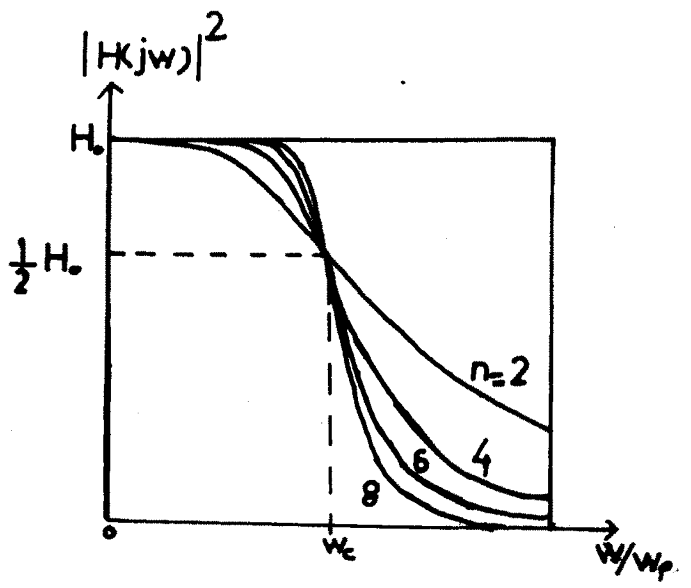

Fig. 2.10: Butterworth Filter For Various Values Of n.

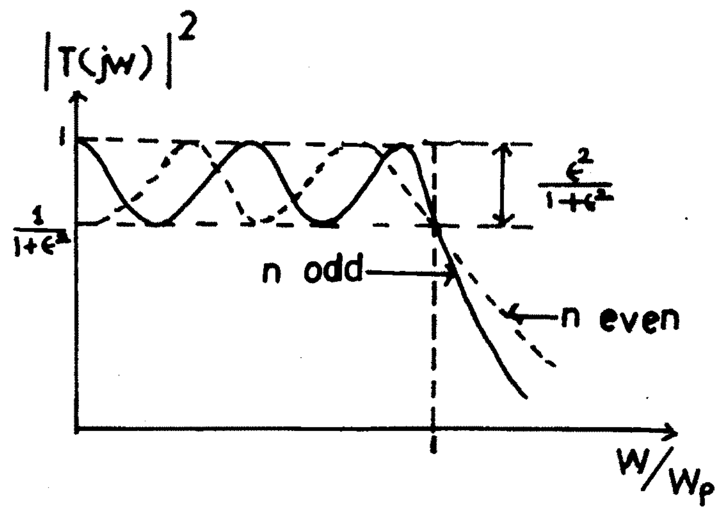

Fig. 2.11: Chebyshev Equiripple Low Pass Filter

w_p, the stopband from W_s to infinity, the maximum passband loss A_{max}, and the minimus stop band loss A_{min}.

The rational function Lp approximations, which have geneal form.

$$|H(jw)|^2 = 1 + |K(jw)|^2 = 1 + \left|\frac{N(jw)}{D(jw)}\right|^2 \quad ......(2.33)$$

where H(s) is desired loss function and K(S) = N(S)/D(S) is a rational function in S. The function K(S) must be chosen so that its magnitude is small in pass band, to make the magnitude of H(jw) close to unity. In the stop band the magnitude of K(S) must be large in order to satisfy the stopband loss requirements. For this approximation K(S) is selected.

$$K(S) = P_n(S) = a_o + a_1 s + a_2 s^2 + ... + a_n s^n \quad ....(2.34)$$

where the coefficients of the n^{th} order polynomial $P_n(s)$ are chosen so that the corresponding loss function H(S) satisfies the given filter requirements.

For the Taylor series approximation the function K(S) must be "maximally flat" at the origin (w = 0). Hence as many derivatives of K(S) as possible must vanish at w = 0. Hence for butterworth approximation [4] :

$$K(S) = P_n(S) = \varepsilon \left(\frac{S}{W_p}\right)^n \qquad \ldots\ldots\ldots\ldots(2.35)$$

where ε is constant, n is the order of the polynomial and Wp is desired passband edge frequency. The corresponding loss function is

$$\left| H(jw) \right| = \left| \frac{V_{IN}(jw)}{V_o(jw)} \right|$$

$$= \left(1 + \varepsilon^2 \left(\frac{W}{W_p}\right)^{2n}\right)^{1/2} \ldots\ldots\ldots(2.36)$$

This expression shows that the first (2n - 1) derivatives are zero at w= 0. Since K(S) was chosen to be an n th order polynomial, this is the maximum number of derivatives that can be mode zero. Thus the slope is as "flat as possible" at dc. For this reason the Butterworth approximation is also known as " maximally flat" approximation.

So the n th order Butterworth polynomial and approximation K(S) satisfies the following conditions.[10]

1. $K_n(S)$ is an n^{th} order polynomial.

2. $K_n(o) = 0$

3. $K_n(S) = 1$ is maximally flat at the origin

4. $K_n(1) = 1$

5. As n (order of filter) is increased, the passband is flat over a wider interval.

6. As n is increased, the stopband loss is increased.

The frequency response of a Butterworth filter for various values of n is shown in fig(2.10). All the curves pass through the same point at w = Wp and this point is determined by α_p.

The price that is paid for this simplicity is a very slow transition between the pass band and stop band. However Bitterworth filter does provide a convenient foundation on which other more practical maximumally flat filters are realisable.

2.3.2 CHEBYSHEV APPROXIMATION :- Butterworth approximation concentrates on the polonomial at $w = 0$ instead of distributing it over range $0 < w < 1$. This yields maximally flat lowpass filters. These networks had the disadvantage of requiring a very high order polynomial for a sharp transition region.

A better result in this regard may be obtained if we look for a rational function that approximates the constant value unity throughout this range in an oscillatory manner, rather than in a monotonic manner[11]. One such approximation is " Chebyshev approximation".

Chebyshev approximation can be defined as follows[10] :

"A function C(w) is a Chebyshev approximation of F(w) if the available parameters are adjusted so that The magnitude of the largest error is minimized".

In definition, the " availabe parameters" refer to the quatities that determine the function C(w) (for example, they meight be resitors in a specific network). Since the Chebyshev approximation minimizes the maximum error, it is often called a "min- max" approximation.

The increased stopband attenuation is achieved by changing the approximation conditions in the pass band. The criterion used is to minimize the maximum deviation from the ideal flat characteristic. We get the equiripple characteristic shwon in fig. (2.11).

$$|H(jw)|^2 = \frac{H_0}{1 + \varepsilon^2 C_n^2 (w/w_c)} \qquad \text{.......... (2.37)}$$

Where $C_n(w)$ is the n^{th} order chebyshev polynomial of the first kind and $\varepsilon^2 < 1$ and H_o is a constant. The response of equation (2.37) is called " n th order Chebyshev or equiripple response ".

Chebyshev polynomials are defined as linearly independent solutions of the differential equation.

$$(1 - W^2) y'' - Wy' - n^2 y = 0 \qquad ..... (2.38).$$

One of the solution is, a n^{th} order chebyshev polynomial is

$$Y = C_n (w) = \cos (n \cos^{-1} w) \quad , 0 < w < 1 \qquad (2.39)$$

$$= \cosh (n \cosh^{-1} w) \quad , \quad w > 1 \qquad (2.40)$$

In fact, these two expressions are completely equivalent, each being valid for all W.

The properties of the Chebyshev polynomials are given below [2].

1. $C_n(w)$ is either an even or an odd functions depending on whether n is even or odd. Thus,we can write.

$$Cn (- w) = C_n(w) \ , \ n \text{ even} \quad .... (2.41 A)$$

$$C_n (- w) = -Cn(w), \ n \text{ odd} \quad .... (2.41 B)$$

2. Every coefficient of $C_n(w)$ is an integer, and the One associated with w_n is 2^{n-1}

Thus, in the limit as w approaches infinity

$$C_n(w) \longrightarrow 2^{n-1} w^n \quad .... (2.42)$$

3. In the range $-1< w < 1$, all of the Chebyshev polynomials have the equal ripple property, varying between a miximum of 1 and a minimum of -1. Outside of this interval, their magnitude increases monotonically as w is increased and approaches infinity in accordance with equation (2.42).

4. The Chebyshev polynomials posses special values at w = 0,1 or -1.

$$C_n(0) = (-1)^{n/2} , \quad n \text{ even} \quad ...... (2.43\ A)$$

$$= 0 , \quad n \text{ odd} \quad ...... (2.43\ B)$$

$$C_n(\pm 1) = 1 , \quad n \text{ even} \quad ...... (2.44\ A)$$

$$= \pm 1 , \quad n \text{ odd} \quad ...... (2.44\ B)$$

There is a practical problem that arises if one attempts to realize an even - order Chebyshev low- pass filter with a passive network. Even order Chebyshev low pass filters have a zero - frequency loss which is equal to the pass band ripple maximum gain. However, this implies that the source resistance cannot be equal to the load impedance. One way around this restriction is to use a frequency transformation which changes the loss at dc. However, it should also be pointed out here that active and digital filters can easily realize lowpass characteristics that have nonzero loss at dc.

2.3.3 THE INVERSE CHEBYSHEV APPROXIMATION :- The Butterworth lowpass filter is maximally flat at origin. The input/output transfer function H(S) is a polynomials; that is, it has no poles of attenuation. The inverse Chebyshev low pass filter is also maximally flat at the origin, but it does have poles of attenuation. The attenuation poles give the inverse chebyshev filter a transition region much steeper than that of the Butterworth approximation.

The Chebyshev lowpass filter has an equiripple passband and its input/output transfer function H(S) is a polynomial. Even though it has no attenuation poles, its transition region is just as steep as that of the inverse Chebyshev. Inverse Chebyshev filters are sometimes better than Chebyshev filters because of the maximally flat passband which results in better delay performance.

Delay is a frequency domain parameter that indicates how much a pulse will be distorted.

Fig. (2.12) demonstrates that the inverse Chebyshev filter

For Chebyshev filter

$$|T(jw)|^2 = \frac{\varepsilon^2 T_n^2(w)}{1+\varepsilon^2 T_n^2(w)} \quad \ldots\ldots\ldots\ldots\ldots(2.45)$$

Where $T_n = C_n$ for equation (2.37)

By replacing w by (1/w), we have

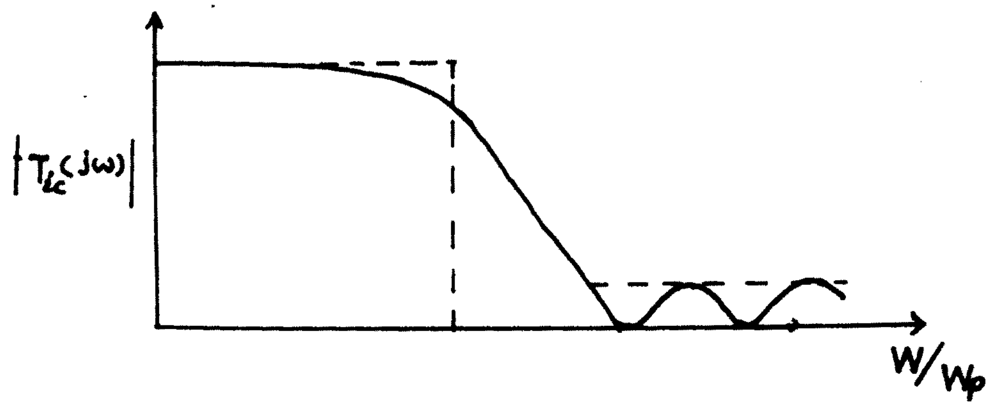

Fig. 2.12: Inverse Chebyshev Low Pass Filter.

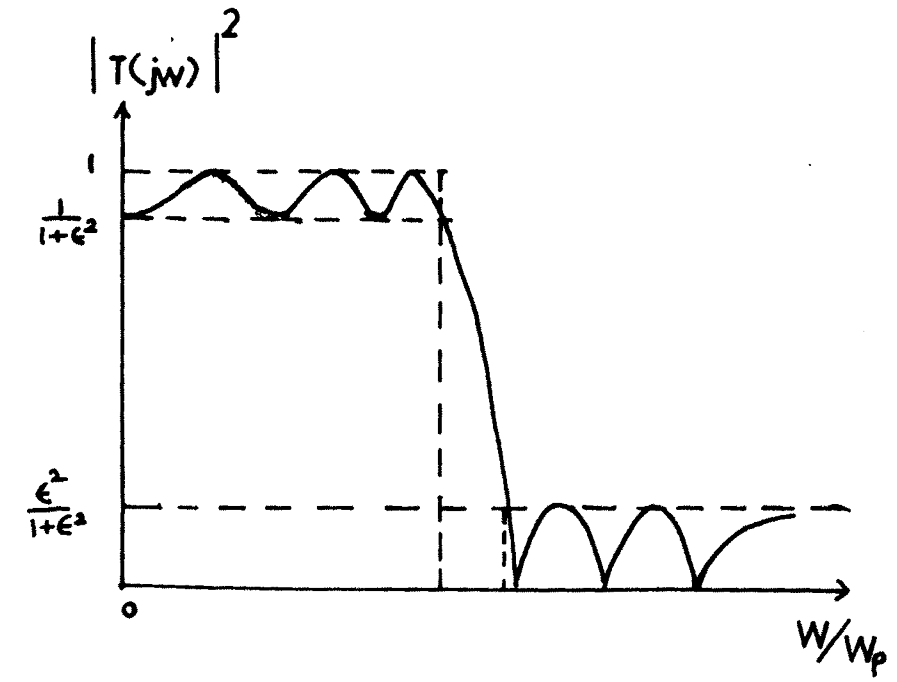

Fig. 2.13: Elliplical Low Pass Filter.

$$| T_{Ic} (jw) |^2 = \frac{\varepsilon^2 T_n^2 (1/w)}{1+\varepsilon^2 T_n^2 (1/w)} \quad ....(2.46)$$

or

$$| H (jw) |^2 = \frac{H_0 \varepsilon^2 C_n^2 (w_c / w)}{1+\varepsilon^2 C_n^2 (w_c / w)} \quad ....(2.47)$$

$| T_{Ic}(jw)|^2$ is a low pass function with monotonic pass band and equiripple stop band (stop band edge being w = 1).This is called "Inverse Chebyshev" approximation.

2.3.4 ELLIPTIC APPROXIMATION :- The Chebyshev approximation has an equiripple pass band. It yields a greater stop band loss than the maximally flat Butterworth appoximation. In both approximations the stopband loss keeps increasing at the maximum possible rate of 6n dB/octave for an n^{th} order function. Therefore these approximations provide increasingly more loss than the flat A_{min} needed above the edge of the stopband.

If we are to improve the performance of a filter beyond that which is achived by the chebyshev filter, we have to allow equirepple response in both pass and stop band. This leads to narrower transition band. These filters are found by using elliptic function and referred to as " elliptic filters". The approximation is called "elliptical approximation. They are also known as "Cauer" or "Zolotarev" approximation. These filter are also called " Darlington filters", as S. Darlington did much original work.[10].

A typical elliptic approximation fuction is sketched in fig. (2.13).

The distinguishing feature of elliptic approximation is that it has poles of attenuation in the stopband. Thus the elliptic approximation is a rational function with finite poles and zeroes, while the Butterworth and Chebyshev are polynomials and such have all thier loss poles at infinity. In particular, in the elliptic approximation the location of the poles must be chosen to provide the equiripple stopband characteristic shown. The pole closest to the stop band edge (wp_1) significantly increases the slop in the transition band. The further poles (wp_2 and infinity) are needed to maintain the required level of stopband attenuation. By using finite poles, the elliptic approximation is able to provide a considerably higher flat level of stopband loss than the Butterworth and Chebyshev approximations. Thus for a given requirement the elliptic approximation will, in general, require a lower order than the Butterworth or Chebyshev[12]. Since a lower order corresponds to less components in the filter circuit, the elliptic approximation will lead to the least expensive filter realization.

2.3.5. BESSEL APPROXIMATION :- In all approximation techniques discussed so far, we have concentrated on approximating the magnitude of the transfer function. In many signal processing requirements linearity of the phase or constant phase delay is an important factor. The phase distortion is more in the Chebyshev filter than in the Batterworth filter. It can be shwon that a sudden change in the amplitude is accompanied by a similar change in the phase. An equiripple filter has greater amount of phase distortion than the maximally flat filter. Bessel's approximation deals with phase and delay charactristics.

A constant delay response can be approximated by a maximally flat delay at w = 0 by using Bessel polynomials. It turns out that the coefficients of the polynomials used in the transfer function H(S) are closely related to Bessel polynomials and Thomson was the one of the first to use these polynomials in the approximation for this response"[2].

The lose function for the indeal delay characteristic is given by [4].

$$H(S) = e^{ST_o} \qquad \ldots\ldots\ldots\ldots\ (2.48)$$

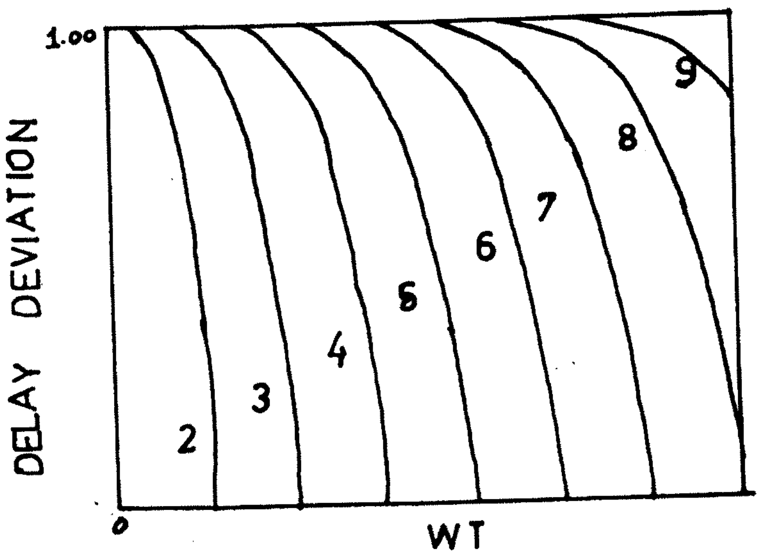

Fig.2.14: Delay Error Of Bessel Approximation.

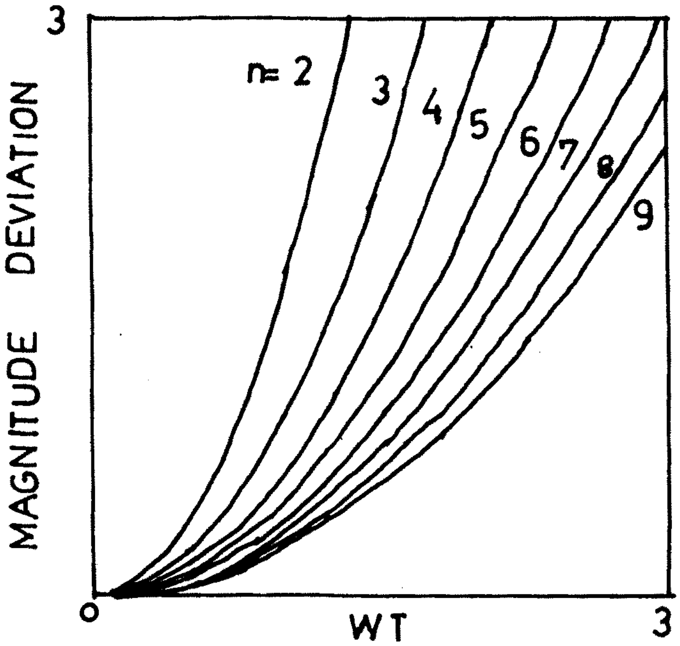

Fig.2.15: Magnitude Error Of Bessel Approximation.

The Bessel approximation is a polynomial that approximates this ideal characteristic. In this approximation the delay at the origin is maximally flat, that is, as many derivatives as possible are zero at the origin. It is convenient to consider the approximation of the normalized function, with the dc delay $T_o=1$.

$$H(S) = e^{S} \qquad \text{.......... (2.49)}$$

The Bessel approximation to this normalized function is

$$H(S) = \frac{B_n(s)}{B_n(o)} \qquad \text{........(2.50).}$$

Where $B_n(s)$ is the n^{th} order Bessels polynomial which is defined by following equation

$$B_O(S) = 1$$

$$B_n(S) = S + 1$$

and

$$B_1(S) = (2n-1)\, B_{n-1}^{(s)} + S^2\, B_{n-1}^{(s)} \quad \text{.....(2.51)}$$

The polynomial B (S) is called Bessel polynomal of order n.

The delay and magnitude of Bessel approximation are sketched in fig.(2.14) and fig.(2.15) respectively.

The delay characteristic of the Bessel approximation are far superior to thsose of the Buttreworth and Chebyshev. It concentrated on the requirement of flatness of the time delay. As a result ,the step responce is also superior having no overshoot.However ,the flat delay is achived at the expance of the stopband attenuation which, is even lower than that for the Butterworth.

The poor stopband characteristics of the Bessel approximation makes it an imprατical approximation for most filtering applications .An alternate solution to the problem of attaining a flat delay characteristic is by the use of delay equalizers.

2.4 FREQUENCY AND IMPEDANCE SCALING :- In most of examples, the values of the elements R,L,C have been of the order of unity. It is very difficult, to build a capacitor of 1 Farad. The circuits considered so far have normalized element values. There are mainly two reasons for resorting to normalized designs. The first reason is simplicity in numerical computation. It is easier to manipulate numbers of the order of unity. The round-off errors that occur in normalized designs is less sever. The second reason is that if we have a normalized design of, a band - pass filter, then it is easy to generate band -pass filters of similar characteristics, of varying center frequencies and impedance levels without redesigning the whole circuit. To obtain the element value of the required band pass filter we amplitude (impedance) and frequency scale the normalized design.

After obtaining the nominal design, " impedance scaling" is used to change the element values of the circuit in order to make the circuit practically realizable. The empedance normalizing factor is given by [8].

$$Y_n = \frac{\text{desired impedance level}}{\text{normalized impedance level}}$$

Frequency scaling is used to shift the frequency response of a filter to a different part of the frequency axis. This is useful in designing filters using normalized frequency requirement, such as those given in standerd tables.

One example of frequency scaling is in denormalization of an LP transfer function which has a cut off frequency of 1 rad/sec, to realize a LP function with cut off frequency at W_p rad/sec.

In general, the frequency response of a given active filter can be scaled up by a factor α by decreasing all capacitors (or resistors) by the factor α.

The frequency normalizing factor is given by[8].

$$\Omega_n = \frac{\text{desired frequency}}{\text{normalized frequency}}$$

2.5 STATE VARIABLE ANALYSIS :- The network may be divided into three basic aspects[13],

a] Linear system :- A system containing linear componants e.g.

resistor, capacitor and inductance, etc.

b] Non - linear system :- A system containing non linear elements e.g. diode, transistors, FET, Tunnel diode, etc.

c] Time varying system :- The value of the element changes with time e.g. Capacitor microphone, mass of a rocket.

To analyse the system, we have two usual circuit analysis methods.

1] Mesh analysis KVL.

2] Nodal analysis KCL

These analysis methods are not convienient for the higher order system, non-linear systems and time varying systems. So all these difficulties are removed by " State variable technique ".

A " State Variable " is the term used to define the effect of an evergy-storing element in a physical system. There is one state variable for each energy storage element representad by the capacitor voltages and inductor currents[14].

$$E_C = \frac{1}{2} C V^2 \quad \text{and} \quad E_L = \frac{1}{2} L I^2.$$

These currents and voltages inform us about energy stored in system.

Response to a given input depends on the zero - input response. The zero - input response in an RLC network is completely determined once the initial inductor currents and capacitor voltages are known. Hence, we call the initial capacitor voltages and inductors currents (initial conditions) as the initial states of the system. The knowledge of capacotor voltages and inductor current, at a given time is sufficient to calculate any of the network variables (Current and Voltages) at that particular time. Hence, we call the capacitor voltages and the inductor currents at a specified time, as the " state Variables " of the network.

The state of a network as a set of real or complex quatities that satisfy the following conditions[8].

a] The state at any time t_1 and the inputs from t_1 to t $(t > t_1)$ uiquely determine the state at time t.

b] The state at time t and the inputs at time t determines uniquely the value at time t of any network varible.

The network equations are written in the form of a set of first order differential equations. Such equations are called "State Equations". The concept of state and of state equations form an important part of the study of optimal control system.

The state equations of a linear time-invariant network can be written as.

$$\frac{dX(t)}{dt} = A X(t) + B U(t) \qquad \text{..........} \quad (2.52)$$

and the output equations

$$Y(t) = C\,X\,(t) + D\,U\,(t) \qquad \text{..........} \quad (2.53)$$

where X is the state vector, U (t) the input vector and Y(t) the output vector. A,B,C and D are matrices of appropriate dimensions. If the network is time varying, then the elements of these matrices are functions of time. An advantage of the state equations is that similar equations can be written even for a nonlinear network where the conventional network function technique is, in general, not applicable.In writing equation (2.52) and (2.53) we have made a tacit assumption that the network does not have any cicuit (cut-off) of capacitor (inductors), or capacitor (inductor) voltage (current) sources. If it were not the case, the state and output equations take the general form (here the tiem t isnot written to simplify notation).

$$\frac{d\,X}{d\,t} = A\,X + B\,U + E\,\frac{d\,U}{d\,t} \qquad \text{.......} \quad (2.54)$$

$$Y = C\,X + D\,U + F\,\frac{d\,U}{d\,t} \qquad \text{.......} \quad (2.55)$$

2.5.1. STATE VARIABLE FILTER :- A state variable is one way of representing the effect of the an evergy storing element in any physical system. The electrical method is to implement an analog cumputer simulation cicuit, where the integrator outpot voltages represents the state variables. It requires one integrator to represent each energy - storing element. Thus one can count the integrators and know how many energy storing elements are in the system being represented. The analog computer having a biquadratic transfer function of fig. (2.16), is called " state - variable biquad"

Several circuits are available for implementing the biquadratic equations. The first of these is the KHN state - variable circuit, named after Kerwin, Huelsman and New-Comb, which is illustread in fig. (2.16). This cicuit can simultaneoulsy represent the regular low - pass, high pass and band pass filter at three different output points[14].

An infinite - gain state variable network configuration is illustrated in fig. (2.17). This configuration makes use of op. amps. in the same way they would be used in an analog computer realization of transfer functions (i.e. using integrators and summers).

The voltage transfer function has the form.7

$$\frac{V_o}{V_1} = \frac{a_0+a_1S+.......+a_n^{n-1}S+ a_nS^n}{b_0+b_1S +........+b_{n-1}S^{n-1}+ b_nS^n} \quad ....(2.56)$$

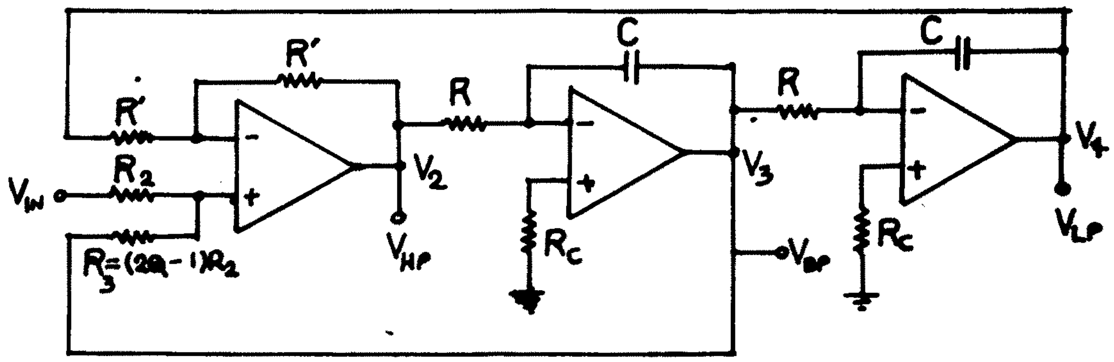

Fig. 2.16: KHN State Variable Biquad Circuit.

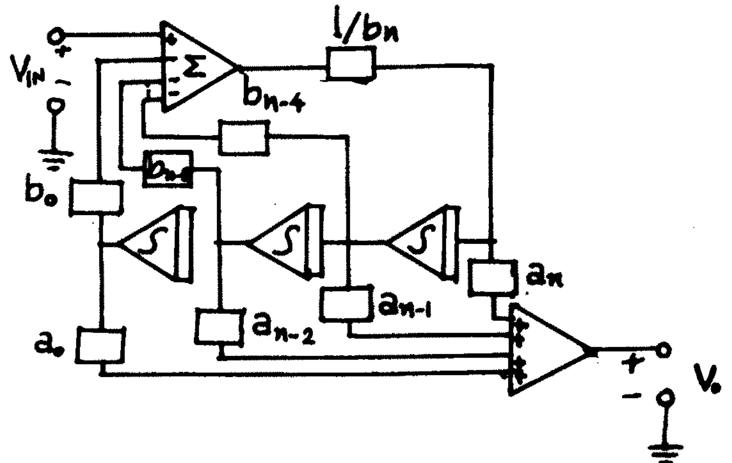

Fig. 2.17: State Variable Infinite Gain Network Configuration.

So the state variable technique is the only viable direct approach presented. The state-variable realization in general provides low Q sensitivity to element variation than a single- amplifier realization and for this reason is sometimes used for high -Q band pass applications (Q > 50). A single method for tunning a state - variable bandpass filter over more than octave without serious loss in the Q - value is presented by Antonio L. Eguizabal - Rivas[15]. One of the advantages is the implementation of the higher order filter with few op.amps.Although op. amps. are cheap and physically small, they still consume power and are the predomianant sources of noise.

One of the important drawbacks of state variable approach is that, it is applicable to low pass and high pass applications. It is a rather expensive circuit to use. The most widely used method in industry for highr order filter is the cascade approach because the systhesis procedure needed for determining the element values of a biquiad is relatively simple and minimization of sensitivity is usually ease to attain.

2.6 FILTER TOPOLOGY :- Topology formalizes the formulation of the network equilibrium equations (loop equations,node equations). Most of the computer aided analysis and design methods utilize topological formulation. The deviation of the state equations of a network inherently depends on the topological matrices of the network.Topology or geometry of the network, is concerned with the interconnections of the element in the network.The network is represented by a linear graph.So the study of topology proves helpfull in solving complex problem.

In this section we discuss the commonly used single amplifier biquad topologies.These structures require an RC-network in conjuction with one op.amp.and can be used to realize a complex pole-zero pair.The transfer function of an RC-network will have poles on the negative real axis while the zeros can be anywhere in the S plane. The general biquad circuit must realize complex poles as well as complex zeros. The op.amp. must somehow be used to realize complex poles instead of the RC-Network poles are real.There are many circuits that can accomplish this. The majority of circuits can be classified into two basic categories, namely,the negative feedback topology and the positive feedback topology.This classification is based on to which input terminals of the op.amp.,the RC-Network is connected.

2.6.1 NEGATIVE FEEDBACK TOPOLOGY :- Consider the RC-Network associated with amplifier fo fig.(2.18),where the RC-Network provides a feedback path to the negative input terminal of op.amp.Such a structure is capable of realizing a biquadratic function and is termed the "Negative feedback topology "[2].The transfer function of this general structure can be expressed in

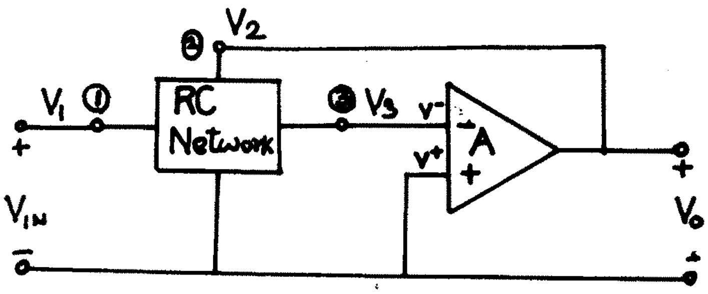

Fig.2.18: The Negative Feedback Topology.

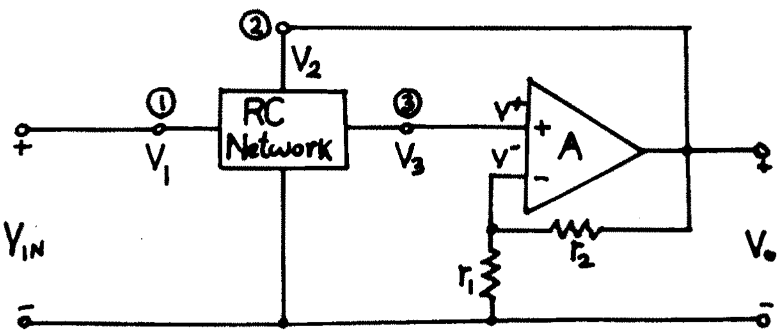

Fig.2.19: Positive Feedback Topology.

terms of the feedforward and feedback transfer functions of the RC network defined by the equations.

The transfer function of negative feedback topology is given by

$$T_v = - \frac{N_{FF}}{N_{FB}} \qquad \text{..............(2.57)}$$

Where N_{FF} and N_{FB} represents the zeros of RC network at feedforward and feedback ports .

Summerizing the negative feedback topology[4] we have

1.The zeros of the feedback network determine the poles of the transfer function .

2.The zeros of the feedforward network determine the zeros of transfer function .

3.The poles and zeros can be complex ; however for a stable network the poles cannot lie in the right half S plane .

4.The poles of the RC network donot contribute to the transfer function (assumming the op.amp.to be ideal).

2.6.2 POSITIVE FEEDBACK TOPOLOGY :- The positive feedback topology is shown in fig.(2.19), where a feedback provided by the RC network is connected to the positive terminal of the op.amp. In addition to this feedback, a part of output voltage is also fedback to the negative terminal via the resistor $r_1$and r_2.This is realy a mixed feedback topology containing both positive as well as negative feedback. The negative feedback is used to define a positive gain for the VCVS.

As in the case for the negative feedback topology , the transfer function of the network of fig.(2.19) can be expressed in term of the feedforward and feedback transfer function of the RC network . The transfer function is given by

$$T_v = \frac{K\ N_{FF}}{D - K\ N_{FB}} \qquad \text{...........(2.58).}$$

where NFF aan NFB represents the zeros of an RC network which can be complex .

D represents the poles of an RC network which must be real .

$$K = 1 + \frac{r_2}{r_1}$$

The zeros of T_v are determined by N_{FF} while the poles of
Summerizing the positive feedback topology :

1. The zeros of the transfer function are the zeros of the feedforward RC network which can be complex .

2. The poles of the transfer function can be located anywhere in the left half S plane, being determined by the poles of the RC network and factor K.

27. CONCLUDING REMARKS :- The selection of active filter structure depends mainly on sensitivity of transfer function to the variation in the passive elements and active element parameters. A standard method of designing a higher order filter is to realize it a cascade of second order filters. However, a coupled biquad structure has lower sensitivity than the cascade structures. Here we have discussed various types of sensitivities e.g. w and Q sensitivity,magnitude and phase sensitivity, the multiparameter sensitivity,gain and root sensitivity. Since the ideal response cannot be attainable by many finite connection of element, the determination of system functions which approximate the given curve within a specified tolerance is called "approximation problem". We discussed several types of low pass filter approximations . They are Butterworth (maximally flat) response , the Chebyshev (equiripple) response ,the inverse Chebyshev response and Bessel-Thomson (maximally flat delay) response.The frequency transformation techniques were applicable to design for high pass filter and symmetrical bandpass and band reject filter

State variable approach has found application in network analysis and synthesis .A state variable is one way of representing the effect of an energy storing element in any physical system. The state variable realization provides less Q sensitivity to element variable than a single amplifier realization . For this reason, it is used for high Q bandpass applications .

Finally we discussed the negative feedback and positive feedback topologies for the realization of the basic building block of active filters , the biquad.

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