

CHAPTER - IV

IONOSPHERIC  
PROPAGATION  
OF RADIO WAVE

#### 4.1 Derivation of the Appleton formula

The formula for the complex refractive index of a medium such as the ionosphere were derived by a number of people during the 1920's but the name most commonly associated with the theory is that of Sir. Edward Appleton. In 1931, D.R.Hartree suggested that it was appropriate to include the Lorentz polarization term in the theory. Although the bulk of both theoretical and experimental evidences indicate that the inclusion of the term is unjustified, the formula for the complex refractive index is often referred to as Appleton - Hartree formula.

First we will apply Maxwell's equations to the wave and, secondly, we shall impose the properties of the medium, the so called "Constitutive relations".

Let us express the refractive index in terms of polarization term  $P$  and electric field  $E$ . Fig. 4.1 shows system of coordinate axes.

Consider characteristic wave to be travelling along the 1 - axis of a right-handed orthogonal system.

Components of the wave vary in time as  $\exp(+i\omega t)$  and in space as  $\exp(-ikx_1)$  and do not vary in the 2 and 3 directions.

Since we are considering the charge oscillations as a polarization of the medium then we have, from eq,3.19.

$$D = \epsilon_0 E + P \quad 4.1$$

Where  $D$  is displacement vector.

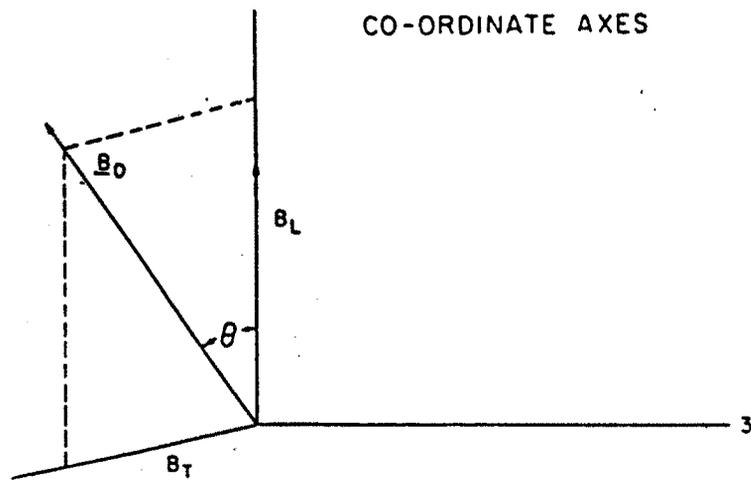


Fig.4.1 System of axes.

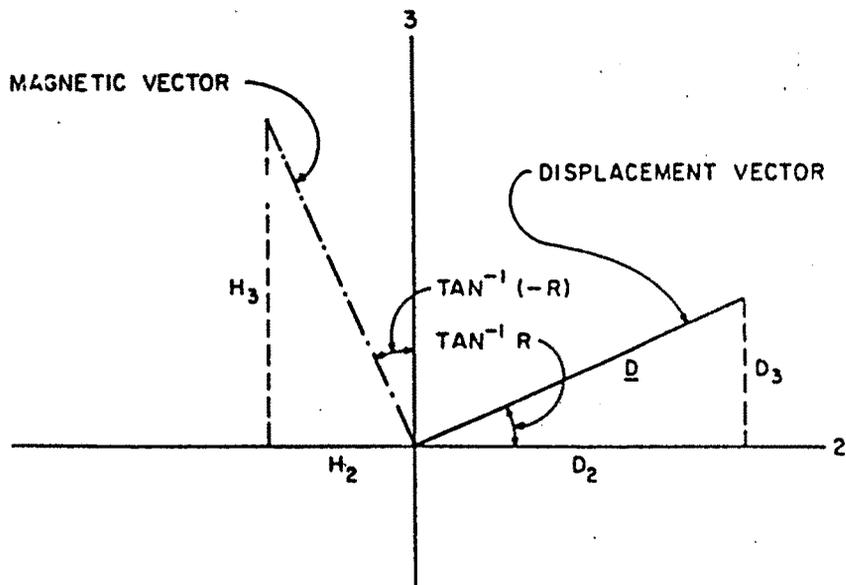


Fig.4.2 Relationship between the instantaneous D and H vectors in the plane of the wavefront.

Relationship between the instantaneous D and H (magnetic field vector) of the wavefront.

R is called the wave polarization and is real. R is given by

$$R = i/2Y_L \{ Y_T^2 / 1 - X - iZ \pm \sqrt{Y_T^4 / (1-x-iz)^2 + 4Y_L^2} \} \quad 4.2$$

Where

$$X = Ne^2 / \epsilon_0 m \omega^2, \quad Y_L = e B_L / m \omega \quad 4.3$$

$$Y_T = e B_T / m \omega, \quad Z = \nu / \omega$$

For the propagation of high frequency radio waves through the E and F regions of the ionosphere, Z is usually vary small and can be neglected.

$$R = i/2Y_L \{ Y_T^2 / 1 - X \pm \sqrt{Y_T^4 / (1-x)^2 + 4Y_L^2} \} \quad 4.4$$

This formula tells us that, in general, there are two, and only two, characteristic waves capable of propagation so refractive index formula becomes,

$$\mu^2 = 1 - \left( X / \left( 1 - iZ - \left( Y_T^2 / 2(1-x-iz) \right) \pm \sqrt{Y_T^4 + 4(1-x-iz)^2 + Y_L^2} \right) \right) \quad 4.5$$

Which is Appleton formula.

In the upper regions of the ionosphere the collision frequency is sufficiently small so that, for frequencies greater than about 1 mc/s, we may put  $Z = 0$  and hence the real part of refractive index is given by

$$\mu^2 = 1 - \left( 2x(1-x) / \left( 2(1-x) - Y_T^2 \pm \sqrt{Y_T^4 + 4(1-x)^2 Y_L^2} \right) \right) \quad 4.6$$

in the absence of an imposed magnetic field  $Y_T = Y_L = 0$ , and of collisions ( $Z = 0$ ) the refractive index is given by

$$\mu^2 = 1 - X = 1 - (fN / f)^2 = 1 - (KN / f^2) \quad 4.7$$

Where,  $K = (e^2 / 4\pi^2 \epsilon_0 m) = 80.5 \text{ m}^3/\text{sec}^2$  and  $f$  in cycles per sec. and  $N$  is electrons / meter<sup>3</sup>

#### 4.2 The variation of collision - frequency with height

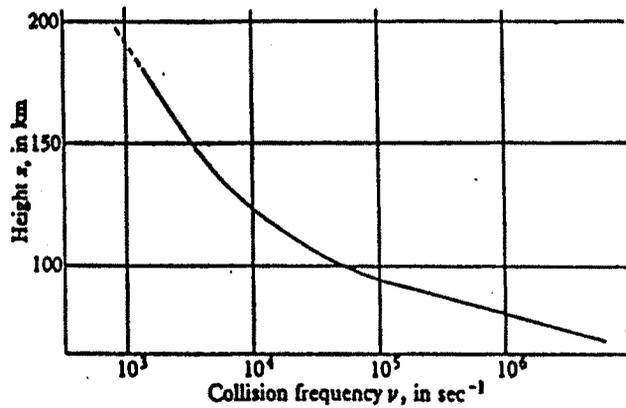
The average number of collisions  $\bar{n}$  which an electron makes per unit time with the air molecules depends upon the number density of the molecules, and therefore on the density and composition of the air. It also depends on the velocity of the electron, but for many purposes it is permissible to neglect this effect. Then in an atmosphere which is constant in composition and temperature.

$$\bar{\nu} = \nu_0 \exp(-Z / H) \quad 4.8$$

Where  $H$  is scale height and  $\nu_0$  is a constant.

In practice  $H$  takes different values at different levels and the law can only be expected to hold over ranges of  $Z$  so small that  $H$  may be treated as constant. Fig. 4.3 shows how  $\bar{\nu}$  depends on the height  $Z$  according to the best estimates at present available.

It is found that changes of the value  $\bar{\nu}$  affect the propagation of radio waves for less than changes of the electron number density  $N$ . For many purposes it is therefore permissible to



The dependence of electron collision-frequency  $\nu$  upon height  $z$ . This curve is based partly on the work of Crompton, Huxley and Sutton (1953). The author is greatly indebted to Dr K. Weekes for supplying the data from which it was plotted.

Fig.4.3

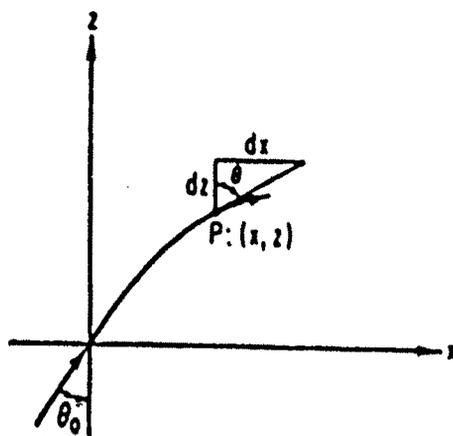


Fig.4.4

Ray path co-ordinates in a plane

treat  $n$  as constant over a small range of height  $Z$ . This is especially true at high frequencies (greater than about 1 Mc/s), where the wavelength is small compared with the scale height  $H$ , which is about 10 Km. The dependence of  $n$  on electron velocity gives rise the phenomenon of wave interaction. Here  $n$  is treated as a constant at each level.

#### 4.3 Ray Paths :

It is aim of the present section to describe the methods for determining the path or trajectory of a ray propagated in the ionosphere and to discuss the results obtained when these methods are applied to special form of ionospheric layers. We first assume that  $\mu$  varies with height only, and that for negative values of the height  $Z$  the electron density is zero and consequently the index of refraction equals unity. The geometry of the ray is shown in Fig. 4.4.

Where ray is incident at an angle  $\theta_0$  from the vertical, and the point of incidence is taken to be the point  $X = 0, Z = 0$ . At an arbitrary point  $P : (X, Z)$  on the ray, the ray makes an angle  $\theta$  with upward directed vertical.

For the normal ionosphere, where the ionosphere is assumed to vary with height only, we can consider geometry for Snell's law in plane slabs.

Here three parallel slabs in each of which index of refraction  $\mu_1, \mu_2, \mu_3$  is constant. Refer Fig (4.5). Let the ray in medium 1 be incident at an angle  $i_1$ , from the interface between the first

two slabs. By Snell's law the ray emerges in region 2 at an angle  $r_1$  given by,

$$\sin r_1 / \sin i_1 = \mu_1 / \mu_2$$

Now the ray is incident on the second interface at an angle  $r_2$ . Applying Snell's law at this interface, we find that,

$$\sin r_3 / \sin r_2 = \mu_3 / \mu_2$$

Eliminating  $\sin r_2$ , we get,

$$\sin r_3 / \sin r_1 = \mu_3 / \mu_1$$

Now let us assume horizontal strata (Fig 4.4)

$$\mu(z) \sin \theta = \sin \theta_0 \quad 4.9$$

At point P we consider a small variation  $ds$  along the ray tangent. This corresponds to increments  $dx$  and  $dz$  along the horizontal and vertical directions. We see that

$$dx / dz = \tan \theta = \pm \sin \theta / \sqrt{1 - \sin^2 \theta} \quad 4.10$$

Where we introduce an explicit  $\pm \sin \theta$  to permit the use of a positive value for the square root since eq.4.9 determines only the  $\sin \theta$ , it does not determine whether  $\theta$  lies between  $0$  and  $90^\circ$  or whether it lies between  $90^\circ$  and  $180^\circ$ .

Thus eq 4.9 is satisfied by an upgoing ray or downcoming ray. For downcoming ray  $dz$  is negative and  $dx$  is positive; this necessitates the use of the negative sign in eq.4.10. Introducing eq.4.9 in to 4.10 we find,

$$dx / dz = \pm \sin \theta_0 / \sqrt{u^2 \sin^2 \theta_0} \quad 4.11$$

The quantity  $x$  is therefore obtained by the following integration

$$x = \pm \sin \theta_0 \int^z d\xi / \mu^2(\xi) - \sin^2 \theta_0 \quad \text{eq. 4.12}$$

To establish some of the general properties of the ray, we introduce the concept of the curvature  $\Gamma$  of the ray. the curvature may be defined as follows :

Consider two points  $p_1$  and  $p_2$  located on the ray at a distance  $s$  apart, and let the respective tangent vectors to the ray makes angles  $\theta_1$  and  $\theta_2$  with the vertical. Let  $\Delta\theta = \theta_2 - \theta_1$ . Then  $\Gamma$  is the limit of  $\Delta\theta / \Delta s$  As  $\Delta s$  approaches zero, The radius of curvature  $R_c$  is given by  $R_c = 1 / \Gamma$  carrying out the limiting process. We see that,

$$\Gamma = (d^2 z / dx^2) / ([1 + (dz/dx)^2]^{3/2}) \quad \text{eq. 4.13}$$

Introducing eq 4.10, and noting that

$$d\mu / dx = (d\mu / dz) (dz / dx)$$

$$\Gamma = (1/\mu^2) (d\mu / dz) \sin \theta_0 \quad \text{eq 4.14}$$

When the curvature is positive, the ray is concave upwards, and when the curvature is negative, the ray is concave downward. In the special case  $\Gamma = 0$ , the ray is straight and its radius of curvature is infinite. For infinite  $\Gamma$  the ray suffers a discontinuous change of direction.

We can consider  $\mu^2$  to be non-negative, since evanescent wave can exists in a region where  $\mu^2$  is negative. The angle  $\theta_0$  can always be defined so as to yield a positive value of  $\sin \theta_0$ . Hence  $\Gamma$  always has the sign of  $d\mu / dz$ .

For the present problems, we neglect collisions and the effects of the earth's magnetic field. In this case, we can obtain an

adequate expression for  $\mu^2$ .

$$\mu^2 = 1 - (N e^2 / m \epsilon_0 \omega^2) = 1 - (K N / f^2)$$

given by (4.7)

Noting that  $\mu^2$  is non-negative, we see immediately that  $d\mu / dz$  is opposite in sign to  $dN / dz$ . Thus curvature  $\Gamma$  is always opposite in sign to  $dN / dz$ . This means that a ray always tends to curve away from the direction of increasing  $N$ .

Let us suppose initially that the electron density is zero at height  $Z = 0$  and then increases monotonically with height above this level.

From preceding paragraphs, it can be seen that, as the ray penetrates more and more deeply into the layer, it makes a larger and larger angle with the vertical, until the ray becomes horizontal. Putting  $\theta = 90^\circ$  in eq.4.9, we see that the ray still possesses negative curvature at the point where it becomes horizontal. Then it bends downwards and ultimately goes back, out of ionosphere. In this case, the wave is said to have been reflected by the ionosphere.

Eq. 4.9 permits us to find the value of  $\mu$  for which the ray becomes horizontal, i.e. for which  $\theta = 90^\circ$ . Denoting this value of the refractive index by the symbol  $\mu_r$ , we have

$$\mu_r = \sin \theta_0 \tag{4.15}$$

If the increase of  $N$  with height continues to sufficiently great heights, every value of  $\mu$  from  $\mu = 1$  to  $\mu = 0$  and into negative values, will exist in the layer. Thus there will always be

some level at which the value  $\mu = \sin \theta_0$  is reached and the wave will be turned downwards without reaching the level where  $\mu = 0$  (an exception occurs in the case of a vertically incident ray, for which  $\theta_0 = 0$ ).

For completeness of the present discussion, it is necessary to consider briefly a more realistic layer than the unterminated layer introduced above. Neglecting discontinuous changes in the layer, there must be at least one point at which  $dN / dZ = 0$ . Therefore, a ray which reaches the level of maximum ionization has zero curvature at that height. In general, such a ray passes through the level of maximum ionization at an angle  $\theta \neq 0$  and then begins to curve upwards, since  $dN / dZ$  becomes negative above the layer maximum.

However, for a given layer and for a sufficiently high frequency  $f$ , there is an angle  $\theta_0$  for each frequency such that the level of reflection, where  $\mu(z) = \mu_r = \sin \theta_0$ , occurs at the layer maximum. Excepting the case of vertical incidence, we see that  $\sin \theta_0 = \mu_r > 0$ . Therefore, the ray is both straight and horizontal. In the ideal case, this ray would continue horizontally to infinity; in practice, some disturbance would eventually deviate it upwards or downward.

The preceding paragraphs have shown that the ray is horizontal at its maximum height  $Z_r$  and has a curvature given by eq.4.14. Since Snell's law for a horizontally stratified layer shows that the angle  $\theta$  of a ray at any point is function only of the incid-

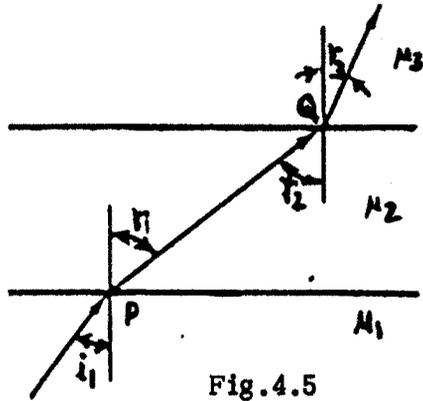


Fig.4.5  
Snell's Law

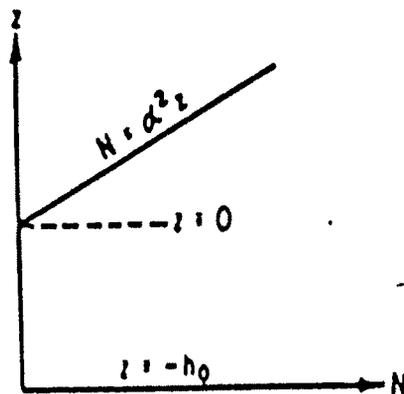


Fig.4.6  
Electron density varies linearly with height

ent angle  $\theta_0$  and of the index of refraction (and thereby the height) of the point, we see that if the highest point of the trajectory is considered to be the initial point, the ray will see the same index of refraction variation, whether it travels to positive or to negative value of  $x$ . We therefore conclude that the ray is symmetrical about its point of maximum height.

a) Linear layer

Let us now return to the case of an undetermined layer and assume also that the electron density varies linearly with height as shown in Fig. 4.6. Then we write  $N = \alpha^2 Z$ , so that the refractive index becomes

$$\mu^2 = 1 - (\alpha^2 K Z / f^2) \quad 4.15$$

Using this value of  $\mu^2$ , we see that eq.4.12 becomes

$$x = \pm (f \sin \theta_0 / k \alpha) \int_0^Z d\xi / \left( \sqrt{f^2 \cos^2 \theta_0 / \alpha^2 K} - \xi \right) \quad 4.16$$

The maximum value  $Z_r$  of  $Z$  is obtained by solving for  $Z$  in  $\mu(z) = \sin \theta_0$ . Then

$$Z_r = f^2 \cos^2 \theta_0 / \alpha^2 K \quad 4.17$$

This value  $Z = Z_r$  is called the reflection height of the ray. Eq. 4.16 can be simplified by the introduction of  $Z_r$  and yields

$$x = \pm \sqrt{Z_r} \tan \theta_0 \left( \int_0^Z d\xi / \sqrt{Z_r - \xi} \right) \quad 4.18$$

integrating, we get

$$x = (2 Z_r \pm 2 \sqrt{Z_r} (\sqrt{Z_r} - Z)) \tan \theta_0 \quad 4.19$$

The trajectory is a parabolic curve. The horizontal distance traveled by the ray from the point of incidence to the point of emergence is found by putting  $Z = 0$  and using the positive sign

in eq 4.18, since the emerging wave is downcoming. This horizontal distance or range in the ionosphere, is given by  $X_m$ , where

$$X_m = 4Z_r \tan \theta_0 = (4f^2 / \alpha^2 K) \sin \theta_0 \cos \theta_0 \quad 4.20$$

Because of the way in which the reflection height  $Z_r$  appears in these equations, it is sometimes convenient to use it as a normalizing factor by the introduction of the new variables  $G$  and  $n$ ,

$$G = X / Z_r = (\alpha^2 K / f^2 \cos^2 \theta_0) X \quad 4.21$$

$$n' = Z / Z_r = (\alpha^2 K / f^2 \cos^2 \theta_0) Z$$

The equations of the ray then can be written as

$$G = 2 (1 \pm \sqrt{1 - n'}) \tan \theta_0 \quad 4.22$$

and the range in the ionosphere is

$$G_m = 4 \tan \theta_0 \quad 4.23$$

The normalized curves as expressed in 4.22 would be useful in cases where extensive calculations of ray paths in a linear layer might be desired.

However, nonnormalized curves in eq.4.19 more clearly exhibit the behaviour of the rays as the angle of incidence is varied.

We plot a series of such rays in Fig 4.6(b)

In drawing this set of curves, we choose arbitrarily the value  $f^2 / (\alpha^2 K) = 100$ . since this value gives ranges and heights which are reasonable for ionosphere problems. If either the frequency  $f$  or the variation  $\alpha^2 Z$  of the electron density with height is changed, a new set of ray path curves will result,

unless the above relation among,  $f$ ,  $\alpha$  and  $K$  is retained.

The preceding paragraphs have treated the part of the path contained in the ionosphere itself. For moderate ranges of propagation, it is reasonable to assume a flat earth and a plane ionosphere starting at some height  $h_0$  above the earth and varying in the vertical direction only. The overall geometry is shown in Fig. 4.7. The total horizontal distance covered by the ray between the transmitter and the receiver is given by

$D = \overline{X_1 X_4}$  and is made up of three segments :

- (1) the distance  $\overline{X_1 X_2} = h_0 \tan \theta_0$  which the ray covers as it travels from the transmitter to the ionosphere ;
- (2) the range  $\overline{X_2 X_3}$  of the ray in the ionosphere.
- (3) the distance  $\overline{X_3 X_4} = h_0 \tan \theta_0$  as the ray descends from the ionosphere.

For the linear ionosphere, where  $N = 0$  at  $Z = h_0$ , we have

$$D = 4 (f^2 / \alpha^2 K) \sin \theta_0 \cos \theta_0 + 2 h_0 \tan \theta_0 \quad \dots 4.23$$

Using this equation, we plot in Fig. 4.8 (a) the range  $D$  as a function of the angle of incidence  $\theta_0$  for various parametric values of  $f^2 / (\alpha^2 K)$ . If we assume layer shape to be fixed then  $\alpha K = \text{constant}$ , and the parametric values correspond to various frequencies. Thus the curves indicate that at low frequencies the range increases monotonically with frequency. However, for high enough frequencies, the range curves have a maximum around  $\theta_0 = 50^\circ$  and a minimum around  $\theta_0 = 70^\circ$ . At all frequencies the range becomes infinite as  $\theta_0$  approaches  $90^\circ$ , since under this

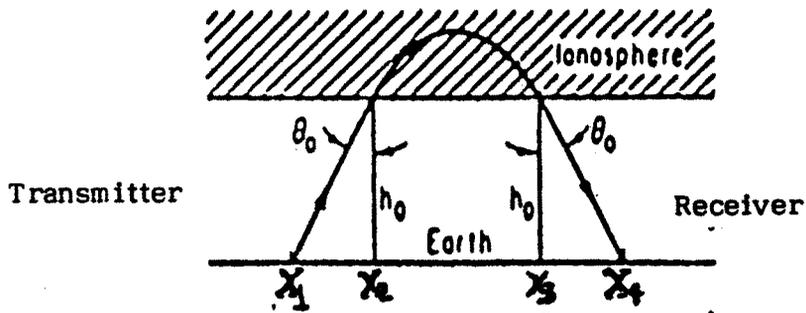


Fig.4.7 Geometry of ray between Sender and Receiver.

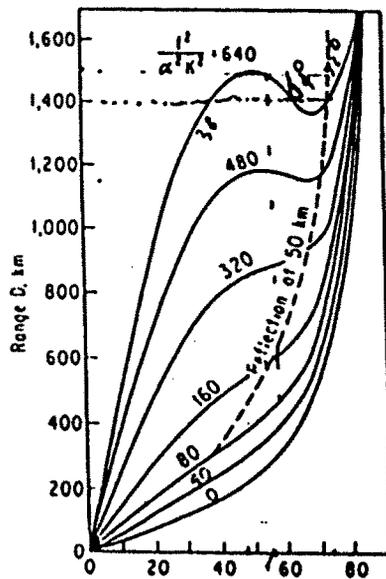


Fig.4.8(a) [Angle of incidence  $\theta_0$ -variation w.r.t.D],  $\theta_0$ , deg.

$$\frac{f^2}{2k^2} = 640.$$

condition the problem approaches that of a tangential reflection from a perfect mirror. For the high frequency cases there are three rays. Which yield same range.

Thus for  $f^2 / (\alpha^2 K) = 640$  the three cases are for the angles  $\theta_0 = 38^\circ, 64^\circ$  and  $73^\circ$ , which all lead to range of 1,400 Km.

The preceding results are unrealistic to the extent that they are based on the assumption that the layer keeps increasing indefinitely with increasing height. In fact, we know that the ionosphere layers have maximum values of electron density at some height above which the electron density decreases with height. If we assume that the linear layer is terminated at some height  $Z_m$ , the results will be more realistic. For the present problem, it is sufficient to require that electron density should not increase above  $Z_m$ . Then a ray which does not become horizontal by a height  $Z = Z_m$  will continue upward and will not return. If the layer is constant in electron density, above  $Z = Z_m$ , the ray will continue in a straight line at the angle at which it crossed  $Z = Z_m$ . The frequency which is just reflected at vertical incidence is called the critical frequency of the layer.

Now if we put  $Z_r = Z_m$  on eq.4.17 and solve for  $\theta_0$ , we can determine the angle at which a given frequency is reflected at the height  $Z_m$ . For this frequency, a ray at a smaller  $\theta_0$  (more nearly vertical) will not be reflected. For the given conditions, when  $f^2 / (\alpha^2 K) = 50$ , the ray does not exceed  $Z = Z_m$  for

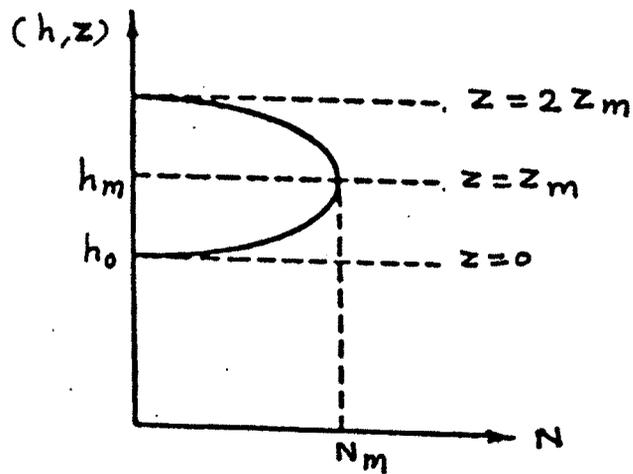


Fig.4.8(b) geometry of Parabola

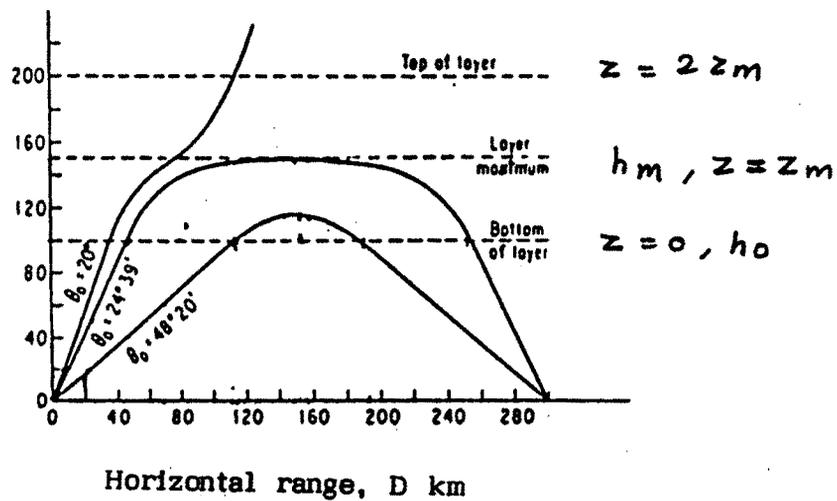


Fig.4.9 : Ray paths in Parabolic layer

$h_0 = 100 \text{ km.}$

$Z_m = 50 \text{ km.}$

$h_m = 150 \text{ km.}$

$f/f_c = 1.1.$

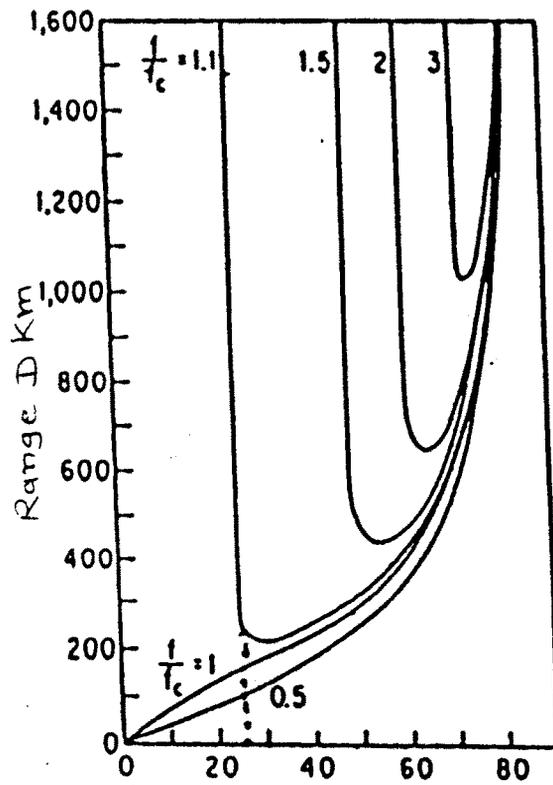


Fig.4.10 Angle of Incidence  $\Theta_0$  degree variation w.r.t. range D ; for different  $f/f_c$  ratio.

any angle of incidence; thus the corresponding frequencies are less than the critical frequency. For larger values of  $f^2 / (\alpha^2 K)$ , the dotted curve on Fig. 4.8(a) indicates that values of  $\theta_0$  less than that of dotted curve correspond to rays which are not reflected in the layer. Therefore, only the parts of the  $D V_s \theta_0$  region to the right of the dotted curve have meaning if the layer ceases to increase in electron density above the height  $Z_m = 50$  Km.

We now see that, for the assumed terminated layer, there is only a single value of  $\theta_0$  at any one frequency which leads to a given range. However, if we suppose that the layer termination occurs at various heights in excess of 50 Km, then the dotted curve in Fig. 4.8(a) moves to the left, and we introduce two and even three, values of  $\theta_0$  for a given  $D$  and  $f$ . It will be shown later that for more realistic layers, only two values of  $\theta_0$  at most correspond to a given  $D$  and  $f$ .

(b) Parabolic layers :

We now wish to consider the equation for the ray path in the parabolic layer shown in Fig. 4.8(b) which more nearly approximates the real ionosphere than does a linear layer. A Chapman layer may be approximated by parabola. Consider eq 3.32 (b) Substituting  $q = \alpha N^2$ , assuming a recombination process with coefficient  $\alpha$ , and for a layer following attachment process of electron loss, with  $\beta$  independent of height, we have at equilibrium when  $dN / dt = 0$ ,  $q = \beta N$ .

Then,  $N = N_m \exp \{1 - Z - \exp(-Z)\}$

Where  $N_m = (q_m / \beta) = (q_0 \cos X_1 / \beta)$  and  $Z = (h - h_m) / H$  4.25

Chapman layer may be approximated as,

$$N = N_m [1 - ((h - h_m)^2 / 4H^2)] \quad 4.26$$

Where  $N$  = electron density,  $N_m$  = electron density at the height of maximum ionization, and  $H$  = scale height =  $KT / (mg)$ .

The electron density becomes zero when  $(h - h_m)^2 = 4H^2$ .

So that vertical thickness of layer is  $4H$ .

Now let  $h = h_0 + Z$

$$h_m = h_0 + Z_m$$

$Z_m = 2H$  is thickness, we obtain

$$N = N_m [(2Z / Z_m) - (Z / Z_m)^2]$$

Then the index of refraction for this layer is given by

$$\mu^2 = 1 - (N e^2 / 4 \pi^2 m \epsilon_0 f^2)$$

or

$$\mu^2 = \{1 - ((e^2 / 4\pi^2 m \epsilon_0) \times (N_m / f^2) (2Z/Z_m - Z^2/Z_m^2))\} \quad 4.27$$

The critical frequency  $f_c$  of a layer was defined as frequency at which vertical signal is reflected at the layer maximum, i.e., for which  $\mu = 0$  when  $Z = Z_m$ . In the present case, we have for the critical frequency.

$$f_c = \sqrt{N_m e^2 / 4\pi^2 m \epsilon_0} \quad 4.28$$

and  $\mu^2$  becomes

$$\mu^2 = 1 - (f_c^2 / f^2 (2Z / Z_m - Z^2 / Z_m^2)) \quad 4.29$$

Introducing  $\mu^2$  from 4.29 into the general expression for a ray path given by (4.12), we see that

$$x = \sin\theta_0 \int_0^Z d\zeta / \left[ \sqrt{\cos^2\theta_0 - \zeta^2 (2f^2c/f^2 Z_m) + \zeta^4 (f^2c/f^2 Z^2m)} \right]$$

integrating,

$$x = Z_m (f/f_c) \sin\theta_0 \log \left[ \frac{1 - (Z/Z_m) - \sqrt{f^2/f^2c \cos^2\theta_0 - ((2Z/Z_m) - Z^2/Z_m^2)}}{1 - f/f_c \cos\theta_0} \right] \quad 4.30$$

Numerical Analysis with example :

The equation for the ray path is quickly adopted for determination of the transmission distance D. The range in the ionosphere is given by twice the value of x obtained from eq 4.30 by setting  $Z = Z_r$  the reflection height. To this range we must add the range of the upgoing and downcoming rays,  $2h_0 \tan\theta_0$ . The reflection height  $Z_r$  is the value of Z in eq.4.27 when  $\mu^2 = \sin^2\theta_0$ . Thus,

$$(f^2c/f^2) Z_m Z_r^2 - (2f^2c/f^2) Z_m Z_r + \cos^2\theta_0 = 0$$

or

$$Z_r = Z_m (1 \pm \sqrt{1 - (f^2/f^2c) \cos^2\theta_0}) \quad 4.31$$

The positive sign yields a point above the layer maximum (and thus gives the reflection height for a ray incident from above. For present problem involving incidence from below the layer maximum, we see the negative sign is  $Z_r$  and obtain

$$D = Z_m (f/f_c) \sin\theta_0 \left\{ \log \left[ \frac{1 + (f/f_c) \cos\theta_0}{1 - (f/f_c) \cos\theta_0} \right] + 2 h_0 \tan\theta_0 \right\} \quad 4.32$$

Curves of  $D$  vs  $\theta_0$  are plotted in Fig. 4.10 for a layer with  $h_0 = 100$  Km,  $Z_m = 50$  Km and for parametric values of  $f/f_c$ . For  $f/f_c \leq 1$ , the rays reflected from the layer regardless of the angle of incidence  $\theta_0$ . Further, this figure shows that, when  $f/f_c \leq 1$ , there is only one ray at a given angle  $\theta_0$  which corresponds to any specified transmission distance. On the other hand, when  $f/f_c > 1$ , rays striking the layer at sufficiently steep angles penetrate through the layer maximum and are therefore not reflected in the layer. The steepest angle which still leads to reflection is found by eq.4.31, by letting  $Z_r = Z_m$  (i.e. reflection at the level of maximum ionization), which occurs at  $(f/f_c) \cos\theta_0 = 1$ . The parameter  $(f/f_c) \cos\theta_0$  is an excellent index to the ray behavior under certain important conditions. At an angle  $\theta_0$  such that  $(f/f_c) \cos\theta_0 = 1$ , the ray travels parallel to the layer maximum, and the propagation distance becomes infinite. At larger  $\theta_0$ , the propagation distance decreases to minimum and then increases with increasing  $\theta_0$  until it becomes infinite at  $\theta_0 = 90^\circ$  (this corresponds to mirror reflection of tangential rays).

Thus for  $f/f_c > 1$ , there are two rays, within the range of  $\theta_0$  permitting reflection, which lead to each attainable distance for a given value of  $f/f_c$ . An exception occurs at the minimum distance, at which the two rays merge in to one :

An example of the two rays is shown in Fig. 4.9. The upper ray (at the smaller value of  $\theta_0$ ) is called "high ray" or often the

"Pedersen ray" after the Danish scientist Pedersen, whose book [PEDERSEN (1972)] is one of the classics of ionosphere work. The lower ray is usually called simply the "low ray". The Pedersen ray is always attenuated more than the "low ray"; so that the Pedersen ray is usually apparent only for distance near to the minimum distance.

The minimum distance for a given  $f/f_c$  is called the skip distance  $D_s$ . AS the  $D$  vs  $\theta_0$  curves shows, for frequencies such that  $(f/f_c) \cos\theta_0 > 1$ , there is a circle, of radius  $D_s$  around the transmitter, within which there are no received signals produced by reflection from the given layer. However, there can be ground wave signals or signals reflected from a higher and more dense layer. The region within this circle is called the skip zone for the given layer. At the skip distance both the low ray and the Pedersen ray can be received. The interference of these two rays can produce a fading phenomenon known as skip distance fading, resulting from the interference of the two signals.

If we consider a particular layer, the  $f_c$  must have some specific value. In this case, the quantity  $f/f_c$  must be considered to be a function of  $f$  alone. When  $f > f_c$ , the  $D$  vs  $\theta_0$  curves have a minimum value associated with them. This means that, for  $f > f_c$  and for a given distance  $D$ , there is some maximum frequency with which communication can be established by reflection from the given layer. For example Fig. 4.10 shows that the frequency can not exceed  $2f_c$  if communication is to be established over a

distance of 625 Km, for the layer assumed in the Fig. At a lower frequency, say  $f = 1.5 f_c$ , this range is possible one. Therefore, the limiting frequency is of great importance in communication problems. It is called maximum usable frequency and is abbreviated 'muf'. The estimation of muf is one of the basic problems in predicting ionospheric propagation, and many national governments have special services for preparing such predictions.

Thus far we have not mentioned the possibility of repeated or multiple reflections. From a given layer we have only mentioned the effects of several layers. The ray path and range calculations can be carried out by repeated applications of the above described procedures. For this reason, the following discussion of multiple reflections and multiple layer will be only qualitative.

The assumption of a mirror reflection of a downcoming ray when it strikes the ground is a fairly good approximation as far as range calculations in the high frequency band are concerned. A similar mirror reflection for the ionosphere forms a useful if somewhat crude, basis for qualitative discussions provided that the mirror contains a circular hole just above the transmitter, leaving an escape route for penetrating rays. Thus a simple picture of the parabolic layer just considered is shown in Fig. 4.11 In this picture, we do not attempt to exhibit the existence of two rays at the distance D. This neglect is justified by the

simplicity of the presentation and by the usual attenuation of the Pedersen ray. Similarly, a qualitative picture of multiple reflection from a simple layer (for which  $f$  is considered less than  $f_c$ , for simplicity the presentation is shown in Fig. 4.12

In this Fig. 4.12 we show that the transmitter and the receiver separated by a distance  $D = TR$  and the two rays arriving at the receiver - one ray being reflected from the ionosphere once, the other being reflected twice. These are called the 'one-hop' and 'two-hop' reflections respectively. If neither penetration nor attenuation occurs, three hop, four hop etc, rays can exist.

Detailed calculation for the two hop ray can be made by considering the transmission distance to be  $D/2$  and for the general  $n_1$  hop case  $D/n_1$ , if the ionosphere is uniform.

The complexity of the problem is increased by the presence of a higher layer of greater electron density above the layer originally considered. If the electron density of the higher layer is less than or equal to that of the lower layer then the problem on the ground is unchanged since rays which penetrate the lower layer will also penetrate the higher layer. However, both layers will affect the range if there is still higher layer capable of reflecting the signal. If there are two layers and upper layer has the greater electron density, then some rays will be reflected from the upper layer, as shown in Fig. 4.13. In this Fig. ray 1 penetrates both layers, and ray 2 is reflected from the lower layer, ray 3 however, penetrates the

lower layer is reflected from the upper layer at point P, and again penetrates the lower layer at point A.

A special type of signal, called an M echo, arises in cases where the lower layer varies horizontally in maximum electron density to the extent that, the downcoming signal is reflected upwards at the point A as indicated in the figure, but can penetrate when it comes down again. The same type of echo can also be produced by horizontal variations in the upper layer which change the angle of incidence of downcoming signal on the lower layer or by inhomogeneities in the lower layer which cause a scattered signal to be reflected upward at the point A. The preceding examples are sufficient to indicate the utility of simple argument in application to complicated cases of multiple reflections. The variety of paths where by a signal can reach a receiver from a transmitter can cause very serious problems in communication, since the signal requires varying times to travel the different paths, the received signal can, in suitable circumstances, be a garbled mixture of various samples of the signal delayed by different amounts. The problems of severals also appear in scatter communications and in cases of reflections from terrain features. The general situation in these cases is called multipath propagation, since downcoming wave is often a mixture of waves subject to different conditions, the transmitted signal consists of pulses whose duration is short compared with the round trip time between the earth and the

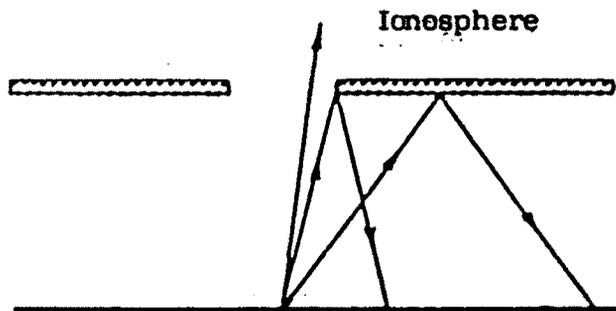


Fig.4.11 : Picture for parabolic layer

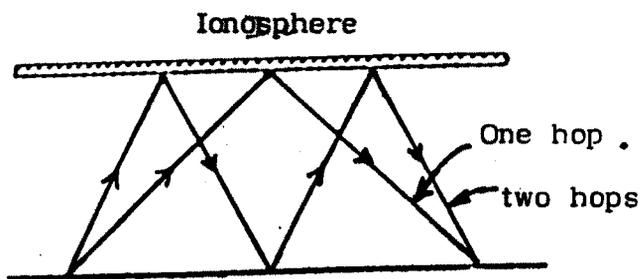


Fig.4.12 : Picture of multiple reflections from simple layer(  $f < f_c$  )

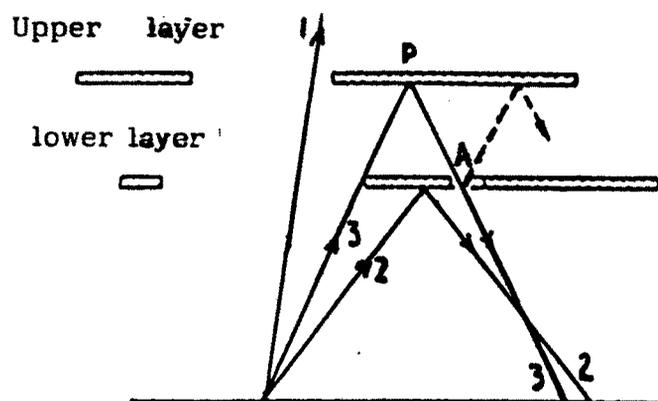


Fig.4.13 : Reflection of signal from upper layer and lower layer

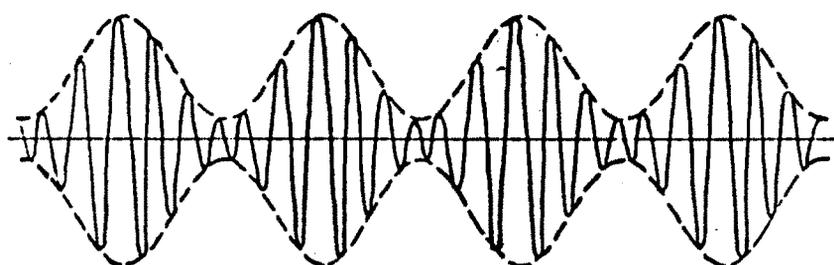


Fig.4.14 : Superposition of two harmonic waves

ionosphere. The various downcoming pulses are generally separated in time. Under the conditions of the experiment, the pulses themselves are propagated at the group velocity, while the planes of constant phase within the pulses are propagated at the phase velocity.

#### 4.4 Oblique propagation :

High frequency ( 3 to 30 Mc/s) radio waves have been still are one of the basic vehicles for long distance transmission of information. The reasons for this may be summerized as follows,

- (a) low cost of terminal equipment,
- (b) low power requirements
- (C) adequate signal strengths
- (d) adequate bandwidth

For propagation over long distances disadvantages in using high frequencies are,

- 1) The variability of propagation conditions. Which, for optimum results requires frequent changes in the operating frequency. Even on the optimum frequency, communications are often subject to interruption by ionospheric storms.
- 2) The large number of possible propagation paths and the resulting time dispersion of a single signal.
- 3) The large and rapid phase fluctuations.
- 4) The high interference.
- 5) Wide band signals suffer from frequency distortion.

In the lower part of the high frequency spectrum, refraction in and reflection from the E - region (particularly  $E_S$ ) contribute to complexity of the echo structure.

#### 4.5 Group Propagation :

Phase velocity of a wave from eq. (4.7) in plasma containing no imposed magnetic field and in which electronic collisions are negligible is given by

$$u = c / \mu = c [ 1 - Ne^2 / m \epsilon_0 \omega^2 ]^{1/2} \quad 4.33$$

This indicates that the phase velocity, in medium, is greater than that of light.

The relationship between phase velocity ( $u$ ) wavelength ( $\lambda$ ) and phase refractive index  $\mu$  are given by

$$u \mu = c \quad (a)$$

4.34

$$u / \lambda = c / \lambda_0 \quad (b)$$

Where  $\lambda_0$  is the free space wavelength.

Phase velocity  $u$  of a wave in a medium is a function of the wave frequency, the medium is said to be dispersive :

phase path :

We see that the electric field intensity  $E(\xi)$  at some point  $\xi$  on the ray can be expressed as,

$$E(\xi) = A(\xi) \exp [j (\omega t + k_0 \int \mu ds)] \quad 4.35$$

Where the integral is taken along the ray from some arbitrarily chosen origin  $r = 0$ , and  $A(r)$  is an attenuation function.

Solution (4.35) is solution to wave equation

$$\nabla^2 E + K_0^2 n^2 E = 0, \quad n = n(x, y, z)$$

Then, at any instant  $t$ , the difference in phase  $\Delta \phi$  between the signal at the point  $r$  and the signal at the origin is given by,

$$\Delta \phi = K_0 \int_0^r \mu ds = (2\pi / \lambda_0) \int_0^r \mu ds \quad 4.36$$

Where  $\Delta \phi$  is in radians,  $\lambda_0$  is the vacuum wavelength, and  $K_0$  is the propagation constant in free space. Now, since  $\Delta \phi / 2\pi$  is the number of vacuum wavelengths represented by the difference in phase,

We see that the integral,

$$P = \int_0^r \mu ds \quad 4.37$$

yields a distance equal to total length of the specified number of vacuum wavelengths. This quantity  $p$ , called the phase path, and is equal to the distance that would be travelled in free space by wave in the time required for a given wavefront to be propagated from  $O$  to  $r$  in the medium.

The group path is a quantity related to the phase path and is of great importance in ionospheric studies because it is relatively measured and it may be noted that the first direct observation of the ionosphere involved the group path

Group velocity is considered (somewhat loosely) to be the velocity with which variations in the signal amplitude are propagated in the ionosphere. We let the group velocity be  $\mu^1$ ,

and  $dt$  be the time of travel of a pulse, then the (actual) distance  $ds$  traveled by the pulse in the time  $dt$  is given by  $ds = u^1 dt$ . The time required for a pulse to travel over a given finite path is,

$$t = \int_{\text{path}} (ds/u^1) \quad 4.38$$

Where the integral is taken over the path. In ionospheric sounding by pulses, this value of  $t$  is measured. However it is convenient to suppose that the pulse travelled at the velocity of light in vacuum, and we therefore multiply  $t$  by  $C$  and, obtain an apparent distance of travel called the group path, the equivalent path, or the apparent path, and designated it by the symbol  $p^1$

From the definition above, we see that

$$p^1 = ct = C \int_{\text{path}} (ds/u^1) \quad 4.39$$

For our present propose it will suffice to consider the superposition of two harmonic waves  $\psi_1$  and  $\psi_2$  of equal amplitude which differ slightly in frequency and wave number

$$\psi_1 = \cos (kx_1 - \omega t) \quad (a)$$

$$\psi_2 = \cos \{ (k + \delta k) x_1 - (\omega + \delta \omega) t \} \quad (b)$$

$$\psi = \psi_1 + \psi_2 \quad (c)$$

} 4.40

This is shown diagrammatically in Fig. 4.14

Interference pattern produced by waves of equal amplitudes and slightly different frequencies.

The superpositions produces a "beat" signal, the envelope of

which is given by

$$A = 2 \cos \frac{1}{2} (x_1 \delta k - t \delta \omega) \quad \text{eq 4.41}$$

It follows from eq. 4.41 that the velocity of propagation of the envelope (i.e. the group velocity  $u^1$ ) is given by,

$$u^1 = \delta \omega / \delta k \quad \text{4.42}$$

This contrasts with phase velocity  $u$  which is given by

$$u = \omega / k$$

It is convenient to define a group refractive index  $\mu_1$  in a manner similar to a phase index.

$$\mu^1 = c / u = c \cdot dk / d\omega = c [d/d\omega (2\pi / \lambda)] \quad \text{4.43}$$

$$= d/d\omega (\mu \omega)$$

$$= \mu + \omega d\mu/d\omega = \mu + f d\mu/df \quad \text{eq 4.44}$$

It is important to remember that the group velocity is the component in the direction of phase propagation. Furthermore, under normal dispersion conditions its magnitude is always less than the velocity of light.

Substituting  $\mu = \sqrt{1 - (f_N/f)^2}$  we find.

$$\mu^1 = d/df (\mu f) = 1/\mu \quad \text{4.45}$$

When magnetic field and collisions are neglected in the presence of an imposed magnetic field the group refractive index  $\mu^1(\pm)$  is given by eq. (4.6)

$$\mu^1(\pm) = \mu(\pm) + \{1 - \mu^2(\pm)\}^2$$

$$\times \left[ \frac{f}{x\mu(\pm)} + \frac{Y_T^2}{2\mu(\pm)(1-x)^2} \pm \right.$$

$$\left. \frac{((2(1-x)^3 Y_L^2 - XY_T^4) / 2X\mu(\pm)(1-x)^2 \sqrt{Y_T^4 + 4Y_L^2(1-x)^2}) \right] \quad \text{4.46}$$

where the + and - signs refer to the ordinary and extraordinary

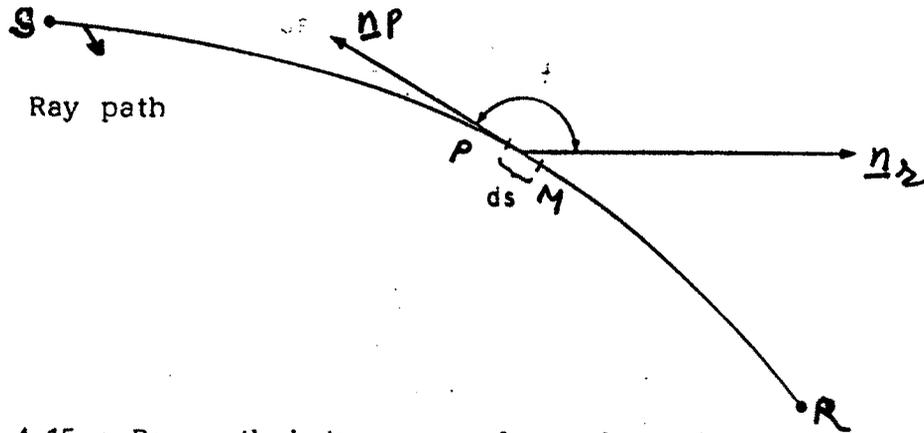


Fig.4.15 : Ray path between sender and receiver

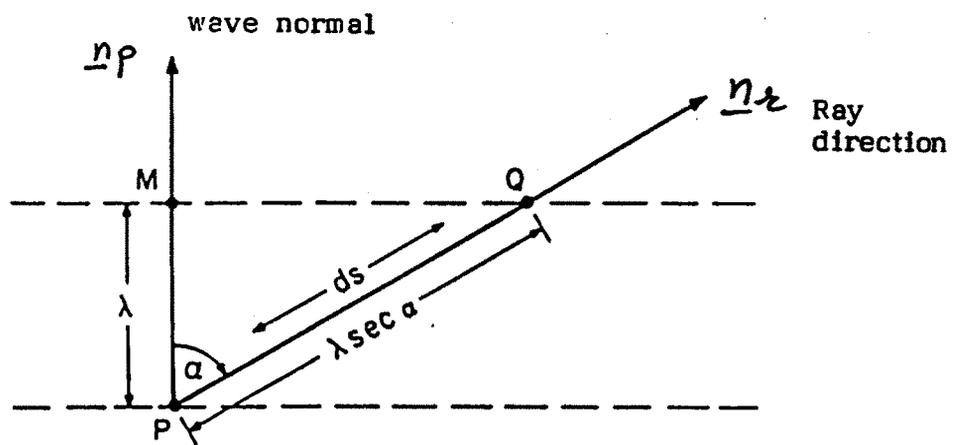


Fig.4.16 : Phase and group paths in anisotropic medium

waves respectively.

From eqs. 4.43 and 4.44 expression for the group path becomes.

$$P^1 = \int_{\text{path}} \mu^1 ds = \int_{\text{path}} \mu ds + f \int_{\text{path}} (\delta\mu + \delta f) ds$$

$$\text{From 4.41 } \mu^1 = f ds / \mu \quad (a)$$

4.47

$$\text{or } P^1 = P + f(\delta P / \delta f) \quad (b)$$

Where P is phase path.

The preceding results are applicable under suitable condition to problems in which effect of collisions and of earth's magnetic field are included.

For some purpose, it is convenient to use the concepts of phase and group paths which may be defined with reference to the transit times of a surface of constant phase and of a wave packet, respectively, between a sender and a receiver. It must be remembered that these are not paths in space but are distances which would have been covered if the wave (and wave packet) had traveled with the free space velocity.

Referring to the ray path between sender S and receiver R in Fig 4.15, the time  $dTP$  required for a surface of constant phase (see Fig 1.16) to travel from P to Q is given by

$$dTP = PM/u = PQ/V_r = ds \cos\alpha / u = \mu \cos\alpha ds / C \quad 4.48$$

The total transit time

$$T_p = 1/C \int_S^R \mu \cos\alpha ds = P/C \quad 4.49$$

$$\text{Where } P = \int_S^R \mu \cos\alpha ds \quad 4.50$$

is defined as the phase path.

The quantity  $K = (2\pi / \lambda_0)P$  is called angular phase path length. It is simply the number of radians of phase between sender and receiver. If  $K$  changes with time, either because of a change of real distance and/or a change in refractive index, the angular frequency of the received wave will differ from that transmitted by an amount  $\Delta W$ . If  $K$  increases then the received frequency decreases because more radians are needed in the medium between  $S$  and  $R$ . therefore the angular Doppler frequency  $\Delta W$  is

$$\Delta W = - dk/dt = -2\pi / \lambda_0 (dp/dt) \quad 4.51$$

In terms of linear frequency the Doppler shift  $\Delta f$  is given by

$$\Delta F = - 1/ \lambda_0 (dp/dt) = - f/c (dp/dt) \quad 4.52$$

The group path  $P^1$  may be defined in terms of flight of a pulse.

In an anisotropic medium the wave packet will, in general, have a velocity component parallel to the wavefront. the time  $dT_g$  for the packet to travel from  $P$  to  $Q$  is given by

$$dT_g = (ds \cos\alpha) / u \quad 4.53$$

The quantity  $u \cos\alpha$  is called ray group velocity and represents the velocity of the wavepacket along the ray path.

The total time of flight between sender and receiver is :

$$T_g = 1/c \int_S^R \mu^{-1} \cos\alpha ds = 1/c (P^1) \quad 4.54$$

Where

$$P^1 = \int_S^R \mu^{-1} \cos\alpha ds \quad 4.55$$

is called the group path. Another useful relationship between these quantities is

$$T_g = d/dw (wT_p), \quad (a)$$

4.56

$$\text{or } P^1 = d/dw (WP) \quad (b)$$

#### 4.6 Theorem of Breit and Tuve.

We wish now to discuss an important theorem obtained by BREIT and TUVE (1926), who also made the first pulse observation of the ionosphere.

Theorem states that the group (or equivalent) path  $P^1$  for transmission between a transmitter T and a receiver R Fig. 4.17 is given by the length of equivalent triangle TAR. That is,

$$P^1 = TA + Ar \quad (a) \quad 4.57$$

Consider the geometry shown in Fig, 4.14. The actual path of the ray is TABCR. However, if one observes only the angles  $\alpha$  of the ray above the horizontal at T and R, the ray appears to follow the path TAGCR. We may call this later path the apparent path. Then, for the specified conditions, the theorem of Breit and Tuve states : The length of the apparent path TAGCR is equal to the group path  $P^1$ . This theorem requires a proof since it cannot be considered self-evident that the apparent path based on angles of elevation is necessarily the same as the group path determined from time difference.

Below the ionosphere, the group path equals the actual paths TA and CR. Within the ionosphere, the symmetry of the ray path is sufficient to establish that the group paths along the upgoing

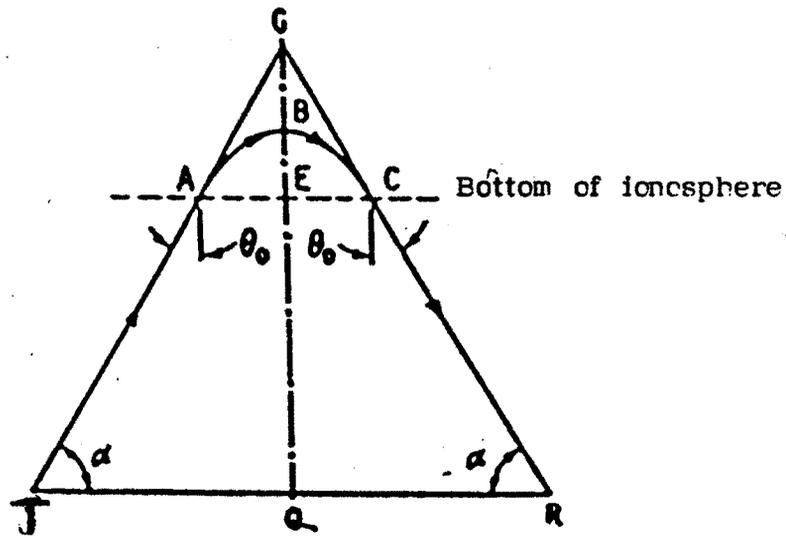


Fig.4.17 : Group path for transmission between sender T and Receiver R.

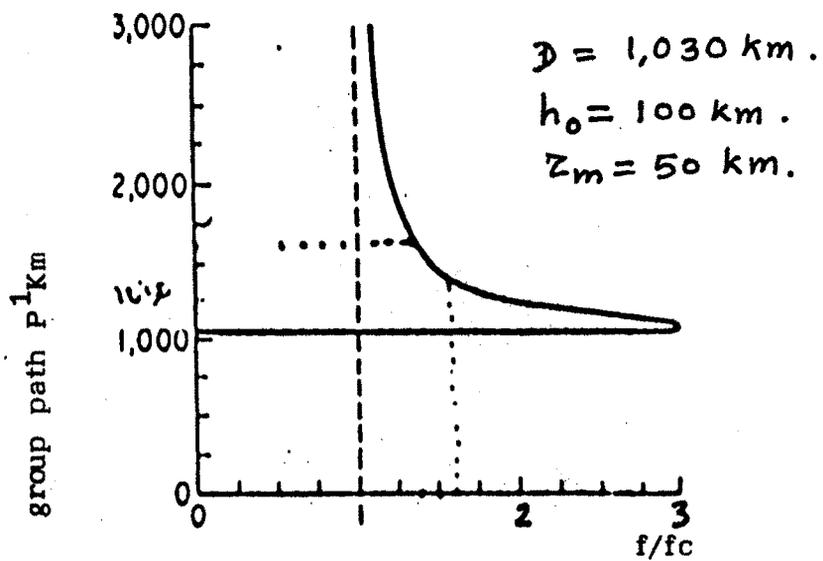


Fig.4.18 plot of  $P^1$  v/s  $f/fc$

path AB and along the downcoming path BC are equal. Thus from eqs. 4.45, 4.7

$$P^1 = \int_{\text{path}} (ds/\mu) \quad 4.58$$

From eq. 4.58;

$$P^1 = 2\overline{TA} + 2 \int_{A}^{T} ds/\mu$$

Since

$$ds = \mu dz / \sqrt{\mu^2 - \sin^2 \theta_0}, \quad P^1 \text{ becomes } P^1 = 2\overline{TA} + \int_0^{Z_r} dz / \sqrt{\mu^2 - \sin^2 \theta_0}$$

Where the height of reflection  $Z_r$  corresponds to the point B. The integral in this expression is the same as that appearing in eq. 4.12 for the ray path (with  $Z = Z_r$  and  $Z = AE$ ).

Hence,

$$P^1 = 2\overline{TA} + (\overline{AE} / \sin \theta_0) = 2(\overline{TA} + \overline{AD})$$

$$P^1 = D / \sin \theta_0 \quad 4.59$$

Where  $D = \overline{TR}$  is the horizontal range of the signal. Thus the theorem of Breit and Tuve is proved for a horizontally stratified ionosphere with an arbitrary height variation, provided only that the condition for reflection exists.

Since theorem of Breit and Tuve shows that we need to know only the ground range and the angle of incidence to determine the group path, we can make use of the previously obtained curves for the range in a parabolic layer.

Fig. 4.18 indicates that, at a frequency such that  $f/f_c = 3.0$ , the minimum range is 1030 Km. Let us now determine the group path as a function of frequency over a distance equal to this

minimum range of 1,030 Km. Thus the quantity  $D$  in eq.4.59 has the value 1030 Km and the angles  $\theta_0$  will be obtained from Fig 4.10. For example ,at  $F/F_c = 1.5$ , the range  $D=1,030$  Km occure at  $\theta_0 = 60^\circ$  and at  $\theta_0 = 78.5^\circ$  finding corresponding angles for different frequencies, and applying eq.4.59, We may plot  $P^1$  v/s  $f/f_c$ , as shown in figure 4.15. Just as the range vs. $\theta_0$  curves are double valued functions of range for  $f/f_c > 1$ , so also are the  $P^1$  vs. $f/f_c$  curves double valued functions of  $f/f_c$  for  $f/f_c > 1$ .

In theoretical curves ,  $P^1$  on the upper branch becomes infinite as  $f/f_c$  decreases to unity In expremental results, only a portion of the upper branch of the  $P^1$  curve is observed , since the upper value of  $P^1$  at a given  $f/f_c$  corresponds to the pedersen ray, which we have already noted to be highly absorbed, except at frequencies near to the  $muf$  . (In the present example, ' $muf$ ' = 3.0  $f_c$ ) In exprimental work, or in calculations for a specific ionosphere,  $f_c$  usually has some fixed value either measured or assumed - so that the curve of Figure 4.15 would be plotted as  $P^1$  vs. $f$ . Then the curve is commonly denoted as a  $P^1$ -  $f$  ("P-dash-f" or "P-prime -f") curve or sometimes, as a  $P'(f)$  curve. A number of exprimental  $P^1$ - $f$  curves have been presented by SULZER and FERGUSON (1952) and SULZER (1955).

#### 4.7 Ionospheric changes, Irregularities and Movements.

The results of ionospheric sounding show that the attitude, thickness and density of ionization in the layers depends on sush factors as :

- (a) Time of day.
- (b) Phase of moon.
- (c) Season of the year.
- (d) Latitude (and the lesser degree the longitude as well).
- (e) Phase of the sunspot cycle, and solar activity.
- (f) Meteor showers.

In addition it is found that both systematic and turbulent motions and also considerable irregularities occur.

#### 4.8 Ionospheric drifts.

The amplitude of a pulse of radio energy received on the ground after reflection from the ionosphere is not constant, but is found to vary from moment to moment in somewhat irregular manner with about 20 maxima and minima per second.

If the ionosphere had a somewhat granular structure or if the contours of constant ionization contained undulations like the ripples on a beach, then movement of the grains or ripples or their appearance or disappearance would give rise to variation in the strength of received signal.

This arises because the phases of waves reflected from different parts of the region will some times augment and sometimes diminish the signal intensity received at any given place on the ground. In the same way signals received at a distant station, by reflection from the E layer show a fluctuation in intensity (fading). This occurs even when there are no multiple paths possible for the signal and there is insufficient ground wave to

give fading by interference of rays which have followed very different paths.

Suppose now a record is made of the strength of the received signal for some particular reflected frequency. It might have an appearance rather like that shown in Fig. 4.19 (a). It is found in practice that sometimes the patterns of signal intensity for two neighbouring ionospheric stations show a marked similarity, with the difference that the features are displaced in time, occurring later at one station than at the other e.g. Fig. 4.16

#### 4.9 Approximation at high frequencies.

At frequencies of 50 mc/sec or more, and for reasonable values of the ionospheric electron density, say of the order of  $10^{12}$  meter<sup>-3</sup> or less, we can establish a simple approximation for the group path in the same manner as that used for the phase path.

Thus we write the index of refraction in the form of eq. 4.7, introduce it in to eq. 4.47 (a) and apply the binomial expansion to obtain a first approximation.

$$P^1 = \int_{\text{path}} ds + [K^2 / 2f^2] \int_{\text{path}} N ds \quad 4.59$$

Where the first integral is clearly the arc length of the ray.

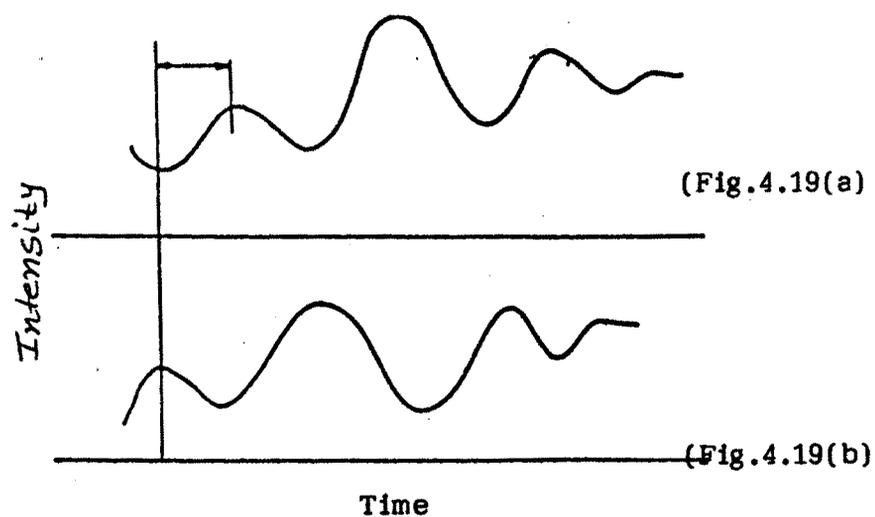
We can introduce the effect of a horizontally stratified layer by writing.

$$ds = \mu ds / \sqrt{\mu^2 - \sin^2 \theta_0} \quad 4.60$$

Before applying binomial expansion.

In this case, the approximation for  $P^1$  becomes

$$P^1 (1/\cos \theta_0) \int_0^z dz + [K^2/2f^2] \cos^3 \theta_0 \int_0^z N dz \quad 4.61$$



Displacement between similar signal strength records obtained at neighbouring stations.

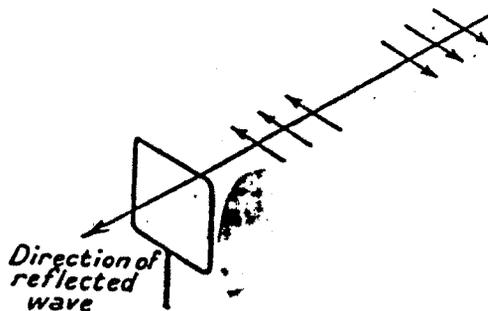


Fig. 4.20 Polarization of the emergent ray

4.10 The equivalent height of reflection for linear profile of electron density :

Suppose that the electron number density  $N$  increases linearly with height  $Z$  in the ionosphere. Let the base of the ionosphere be at height  $Z = h_0$  above the ground. Then since  $f_N^2$  is proportional to  $N$ ,

$$f_N^2 = \alpha (Z - h_0) \quad \text{Where } Z > h_0 \quad 4.62$$

Where  $\alpha$  is constant. The level of reflection  $Z_0$  is given by

$$Z_0 = h_0 + (f^2 / \alpha) \quad 4.63$$

hence eq 4.62 give

$$L^1(f) = h_0 + (h_0 \int_{Z_0}^{Z_0} \{1 - [\alpha(Z-h_0) / f^2]\}^{-1/2} dz).$$

$$\text{Where, } h^1(f) = h_0 + (2 f^2 / \alpha) \quad 4.64$$

4.11 The ray path for a linear gradient of electron density.

Let the base of the ionosphere be at height  $Z = h_0$  above the ground, and suppose that the electron number density  $N$  increases linearly with height above this. Then  $f_N^2$ , which is proportional to  $N$ , is given by (4.62)

so that

$$q^2 = c^2 - [\alpha (Z - h_0) / f^2] \quad \text{where } Z \geq h_0 \quad 4.65$$

$$q = c \quad \text{where } Z \leq h_0$$

Where  $c = \cos\theta_I$  and  $\theta_I$  is the angle of incidence of the wave packet on the ionosphere. In this case eq.4.10 can be written as,

$$\tan \theta = - dq / ds = S / q,$$

Where S is direction cosine of wave normal.

$$\text{and } q^2 = \mu^2 - S^2 \quad 4.66$$

(When earth's magnetic field is neglected).

The equation for the ray path is

$$x = S \int f^2 dz / q \quad 4.67$$

Referring eq 4.66 and 4.67 becomes

$$x = (h_0 \tan \theta_I) + (f^2 \sin^2 \theta_I / \alpha) - \left[ \frac{2f^2 \sin^2 \theta_I}{\alpha} \right] \left\{ \cos^2 \theta_I - \alpha(Z-h_0) / f^2 \right\}^{1/2} \quad 4.68$$

Which shows that within the ionosphere the ray path is a parabola. The total horizontal range D traversed by the wave packet, when it returns to the ground is twice the value of x, when the top of its trajectory is considered where  $dx / dz = \alpha$  (infinity) that is where  $q = 0$  and at this point,

$$x = h_0 \tan \theta_I + \left[ \frac{f^2 \sin^2 \theta_I}{\alpha} \right] \quad 4.69$$

hence

$$D = 2 h_0 \tan \theta_I + \left[ \frac{2 f^2 \sin^2 \theta_I}{\alpha} \right] \quad 4.70$$

The equivalent path P' may now be found by Breit and Tuve's theorem eq 4.28.

$$\text{Snell's law gives, } \mu \sin \theta = \sin \theta_0 = S \quad 4.71$$

Where  $\theta$  be the inclination of the ray if wave normal to the vertical,

Hence

$$p^1 = D/S$$

Let  $\delta_s$  be an element of the ray path, and  $\delta_x$  its horizontal

projection.

$$\delta_x = \delta_s \sin \theta = (\delta_s / \mu) \cdot S = S \cdot \mu^1 \delta_s$$

Where  $\mu^1$  is group reflective index defined in eq. 4.56. Now the wave packet travels with the group velocity  $c/\mu^1$  and hence  $\mu^1 \delta s = C \delta t$  where  $\delta t$  is the time taken to travel distance  $\delta s$ . Hence  $\delta x = S c \delta t$ , and when this is integrated, we obtain  $(x = S \cdot P)_{4.72}$ .

Eqs. 4.67 and 4.72 may be combined to give

$$P^1 = \int_0^Z (dz/q)$$

Where (4.72) with Snell's law gives

$$D = 2 h_0 \sec \theta_I + [(4f^2 \cos \theta_I) / \alpha] \quad 4.73$$

#### 4.13 Fading

The fact that fading depends on frequency causes different frequency components in the sideband of a modulated wave to fade differently. This results in distortion of the modulation envelope which is called selective fading. Single sided band (ssb) are however less distorted and remain quite intelligible. The sky wave signal strength diminishes with distance because of two reasons. The first reason is the spreading of the rays as they leave the transmitter (spatial spreading) causing the signal strength to be inversely proportional to the distance. The second reason causing reduction of the signal strength is the loss of energy in the ionized regions as a result of collisions between the vibrating electron and the gas molecules.

The signal strength at any time depends upon absorption along

the path, path focusing, polarization, and phase of the waves.

Polarization matching (or mismatching) : we know within ionosphere two progressive waves can exist (ordinary and extraordinary) On entry into the ionosphere a wave with arbitrary polarization will excite ordinary and extraordinary waves by different amounts depending on the propagation angle at the bottom of ionosphere. Thus from the point of view of a single magneto-ionic wave, there is a loss of power, in general, on entry into the ionosphere.

Due to the curvature of the ionosphere and/or due to the change of ground range with angle of elevation, the power flux at any point on the earth may be enhanced by focusing or diminished by defocusing.

On leaving the ionosphere the polarization for a particular magneto-ionic wave will have a certain configuration depending on the angle between the wave normal and the earth's magnetic field. Since the angle between the magnetic field and the emergent wave differs from the angle the magnetic field and the incident wave, the polarization of the incident wave and the polarization of the emergent wave will, in general be different. Refer Fig. 4.20.

The reflected wave is often not twisted through a given amount, but is set spinning so that the plane of polarization is continually rotating. Therefore, at a distant point, the electric field is vertical at one moment and horizontal a short time

afterwards, and will continue to change slowly and more or less regularly in this manner. When wave is normally polarized, it will affect a wireless receiver in this ordinary way, where as when it is horizontally polarized no signals will be induced. The signal strength, therefore, may vary from quite a strong one down to nothing in the space of a few minutes or even less.

Finally, there are ohmic losses in the receiving antenna in its transmission line.

The instantaneous field strength may fluctuate widely about the mean value and amplitude variations of the order of '10' to '1' can occur in the course of a few seconds.

The period of a fading cycle depends largely on the cause of the fading. Thus the fading period of interference and polarization fading may vary from a fraction of a second to a few seconds, absorption fading may have a period of the order of an hour or longer, where as focusing may be of the order of 15 to 30 min. Signals can fade-in or fade-out if the frequency of the signal is near the maximum frequency and the critical frequency is changing with time. Fig. 4.21

It is therefore highly irregular as far as period is concerned and may occur only in early morning and late afternoon (fade-in and fade-out).

Some examples of (a) random (b) periodic and (c) double periodic fading of WWV - 20 as received in Boulder, Co 10, are shown in Fig. 4.2D (a), (b), (c).

The periodic fading in Fig. 4.21 (b) that is sharp minima and blunt maxima, is the resultant of two sine waves of almost equal amplitude whose phase difference is changing at a constant rate. In Fig. 4.21 (c) the short period fading could be due to high and low rays and the long period fading could be due to beating between ordinary and extraordinary waves.

Vary bad interference fading is experienced in cases where the sky-ground wave and sky-wave amplitude are comparable. This combination produces much more severe fading than is usually experienced with sky-waves alone. A somewhat related type of interference fading is experienced primarily on low frequencies, where radio transmission is relatively stable. Near sunrise and sunset the heights of the reflecting D-layer change rather rapidly, and the sky wave arrive alternatively in and out of phase with the ground waves. This produces fading with relatively long period.

#### 4.13

#### Scintillations

:

This refers to the amplitude and phase variation of high frequency signals transmitted through the ionosphere from outside the earth. The two common sources are cosmic noise from outer space and signals from artificial satellite. When a radio wave travels through an irregular ionosphere the later behaves as a diffraction grating so that there is redistribution in the amplitude of the wave with position which results in fading due to ionospheric movement. There are also irregular fluctuations

in the apparent position of a radio source viewed through ionosphere. These two processes are usually described as amplitude and angular scintillations respectively.

In order to estimate the magnitude of the irregular component in electron density which is necessary to produce scintillation, we must first compute the total change in phase path length which is imposed on a radio wave in traversing the ionosphere, then consider the fractional variation in this path which will result in phase differences of about one radian in waves emerging at places separated by approximately the fresnel zone radius. The total phase path length  $P$  from source to receiver is given by

$$\int_0^S \mu ds \quad 4.74$$

Where  $\mu$  is refractive index and  $S$  the source distance. Hence, the difference in phase path length introduced by the ionosation is

$$\int_0^S ds - \int_0^S \mu ds = \Delta P \quad 4.75$$

Now, when the wave frequency is high compared with the plasma frequency ( $f_N$ ) we have

$$\mu \approx 1 - \frac{1}{2} (f_N / f)^2,$$

and hence

$$\Delta P \approx \frac{1}{2f^2} \int_0^S f_N^2 ds = (40.5/f^2)^S f_0 N ds \quad 4.76$$

Taking the typical value  $10^{17}$  electrons /  $m^2$  column for  $f_N ds$  and assuming a radio wave frequency of 40 Mc/s ( $4 \times 10^7$  c/s), we get  $\Delta P \approx 2500$  m. This corresponds to about 2000 rad at 40 Mc/s.

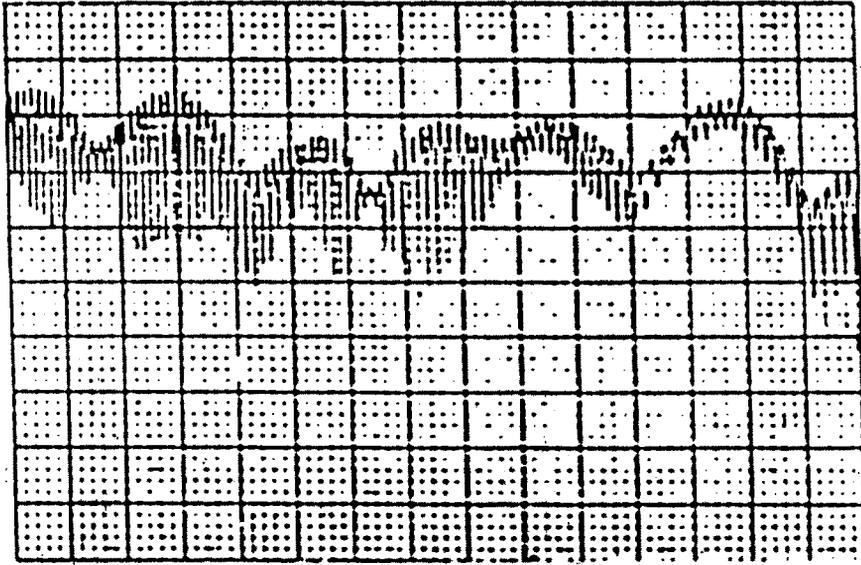


fig.4.20(c) double periodic fading

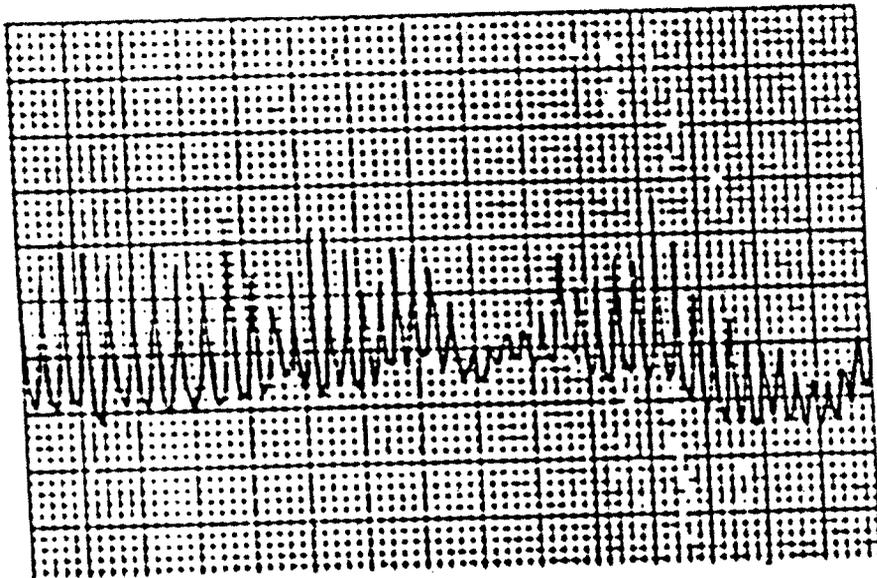


Fig.4.20(a) Periodic fading

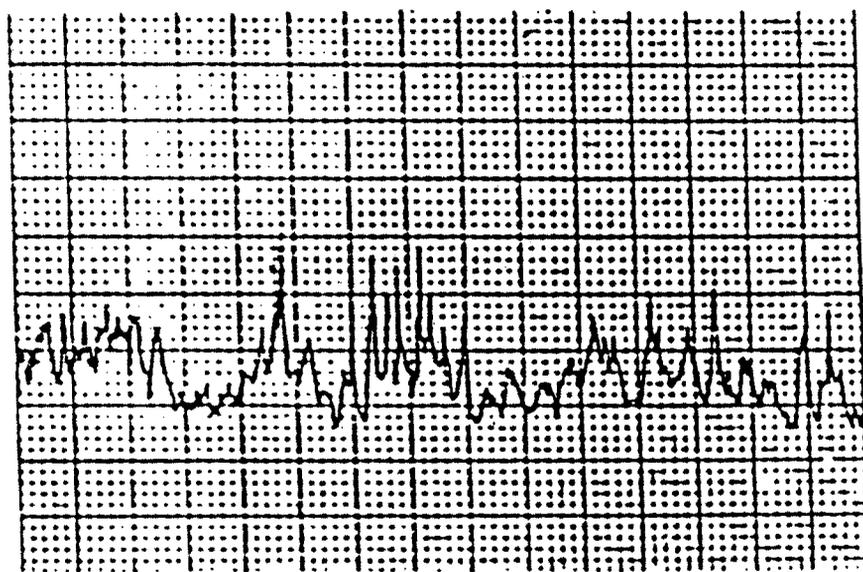


fig.4.20(b) Random fading<sup>b)</sup>

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