CHAPTER – 1

A REVIEW OF RAY ANALYSIS OF PLANAR OPTICAL WAVEGUIDES

1.1 INTRODUCTION

The propagation of electromagnetic radiation as a guided wave in dielectric medium forms the basis of integrated optics (IO). The basic concept of IO was first proposed by Anderson[4] in 1965 and considerable progress has been made since then.

If any processing of the signal transmitted using light waves is required, the signal usually has to be converted into an electrical one before this can be done. The aim of the integrated optics is to be able to do as much processing as possible directly on the optical signal itself. It is envisaged that a family of optical and electro-optical elements in thin film planar form will be used, allowing the assembly of a large number of such devices on a single substrate. Most of the device elements are expected to be based on single mode planar optical waveguides.

Light can be guided and confined in very thin films (with dimensions wave length of light) of transparent materials deposited on suitable substrates. By proper choice of substrates and films and proper configuration of the waveguides, one can perform a wide range of operations like modulation, switching, multiplexing, filtering or generation of optical waves. Due to the miniature size of these components, it is possible to obtain a high density of optical components in space unlike the case in bulk optics. In addition, the confinement of light energy in small regions of space leads to an efficient interaction of the optical energy with the applied electric field or as acoustic wave, thus leading to much more efficient electro-optic and acoustic-optic modulators requiring very low drive powers.

The variety of active and passive thin film devices which are available at present are based upon one or the other physical property of thin films. For example, devices like thin film transistors and ferroelectric picture device (FERPIC) utilise electrical properties, whereas antireflection coatings, dichronic beam splitters (useful in colour TV) and low loss thin film laser mirror coatings make use of optical properties.

1.2 THIN FILMS

The term 'thin films' is applied to a wide variety of physical structures[5] such as self supporting solid sheets called foils, the films obtained by stripping a deposited layer from its substrate etc. The substrate can be an optical element having important role of its own or it can serve simply as a support for the films. The substrates can be planar or curved on which the supported films can be deposited by various methods like vacuum evaporation, cathode spraying and various chemical surface reactions in a controlled atmosphere or electrolyte. The thickness of such films varies from less than one atomic monolayer to a few microns. Thin films exhibit different physical properties like mechanical, electrical, optical etc. which differ from those of the bulk material.

Thin film optics has emerged as an essential division of modern optics. Thin film optics can be divided into two broad divisions.[6] The first s optical coatings and the second one is light guides and associated devices.

1.2.1 Optical Coatings :

Almost all thin film coatings are characterized by propagation of light across the film thickness involving interference effect. Optical coatings are normally multilayered structures. The constructive interference of light reflected at the outer and inner surfaces is possible when the path difference is an integral multiple of wavelengths. The path difference depends on the layer thickness and the angle of incidence. The interference causes the appearance of different colours which are seen to vary with layer thickness and with the angle at which these are viewed. Thin film coatings are either named after the function which they perform e.g. beam splitters, polarisers, long wave pass filters, band pass filters or called by their construction like quarter wave stack, quarter-half-quarter coating etc.

The intensities of beams involved in the interference can be adjusted by choosing the refractive indices of different layers in a thin film assembly; while the phases of beams can be altered by varying the layer thickness. Thus number of parameters in a multilayer is twice the number of layers which can be suitably chosen to get the required performance. However, there are limitations due to spectral width of source, minimum bandwidth of a narrow band filter, the ripple in the pass band, the range of usable angles of incidence etc. A film in optical coating can be thin or thick depending on the detection of interference effects, the illumination, the nature of incident light, detection arrangements etc. In general, a film of several wavelength thickness supported by a substrate of 1mm thickness is regarded as a thin film.

1.2.2 Thin Film Light Guides[7-10] :

A thin transparent film sandwitched between two surfaces of lower refractive indices is capable of propagating the e.m. wave parallel to its boundaries. Such a film is called a light guide in which the energy of the wave is subjected to losses due to scattering and absorption. It works on the total internal reflection at both of its surfaces. Light incident at sufficiently large angle is multiply reflected inside and the net propagation occurs along the film. The angle of incidence depends on the optical constants of the guide and its surrounding media, the thickness of guide and states of polarisation of the wave. The effective phase velocity is higher for smaller angle of incidence. Thus for each angle of incidence we have a different waveguide mode. The lowest permissible angle of incidence corresponds to the highest order mode.

1.3 RAY PATHS IN PLANAR OPTICAL WAVEGUIDES[11-14]

Let us consider a medium in which the refractive index depends only on the x co-ordinate.

$$n = n(x) \qquad \ldots \quad 1.1$$

A medium with continuously varying refractive index given by the above equation can be thought of as a limiting case of a medium consisting of a set of thin layers, each characterized by a specific value of the refractive index [Figure 1.1(a)]. To trace the rays through such a stack of thin layers we can use Snell's law according to which

$$n_1 \sin \phi_1 = n_2 \sin \phi_2 = n_3 \sin \phi_3 = \ \ldots \text{constant}$$

where ϕ_1, ϕ_2,.... are the angles of incidence at various interfaces as shown in figure 1.1(a). If ϕ_1, ϕ_2 .... are the corresponding angles that the rays make with the z axis, then

$$n_1 \cos \theta_1 = n_2 \cos \theta_2 = n_3 \cos \theta_3 = \ldots \text{constant} \qquad \ldots \quad 1.2$$

When the refractive index variation is continuous, the thickness of each layer becomes infinitesimally small and the piecewise straight lines form a continuous curve as shown in figures 1.2 (a) and (b). From equation (1.2) we can infer that the rays bend in such a way that the product n (x) cos θ (x) remains constant, which we denote by $\bar{\beta}$.

Thus

$$n(x)\cos\theta(x) = \bar{\beta} \text{ (variant of the ray path)} \qquad \ldots \quad 1.3$$

Now if ds represents the infinitesimal arc length along the ray path, then from figure 1.2(b) we have

$$(ds)^2 = (dx)^2 + (dz)^2$$

or

$$\left(\frac{ds}{dz}\right)^2 = \left(\frac{dx}{dz}\right)^2 + 1 \qquad \ldots \quad 1.4$$

Since

$$\cos\theta = \left(\frac{dz}{ds}\right)$$

we obtain

$$\left(\frac{ds}{dz}\right) = \frac{1}{\cos\theta(x)} = \frac{n(x)}{\bar{\beta}}$$

Substituting in equation (1.4) we get

$$\left(\frac{dx}{dz}\right)^2 = \frac{n^2(x)}{\beta^2} = -1 \qquad \ldots \qquad 1.5$$

which represents the rigorously correct ray equation for n^2 depending only on the x - co-ordinate.

For a given refractive index profile, a ray will become parallel to the z axis and turns back toward the axis when $dx/dz = 0$, that is, at the value $x = x_t$ satisfying

$$n(x_t) = \bar{\beta} \qquad \ldots \qquad 1.6$$

The point $x = x_t$ is known as the turning point of the ray.

Equation (1.5) can be put in a more convenient form by differentiating it with respect to z

$$2\frac{dx}{dz}\frac{d^2x}{dz^2} = \frac{1}{\bar{\beta}^2}\frac{dn^2}{dx}\frac{dx}{dz}$$

or

$$\frac{d^2x}{dz^2} = \frac{1}{2\bar{\beta}^2}\frac{dn^2}{dx} \quad \text{ray equation} \qquad \ldots \qquad 1.7$$

which is another form of the ray equation.

1.4 RAY PATHS IN SQUARE LAW MEDIA

A square law medium is characterized by the following refractive index distribution [Figure 1.2 (a)].

$$n^2(x) = n_1^2\,[1-2\,\Delta\,(x/a)^2], \quad |x| < a \quad \text{film}$$

$$= n_1^2\,[1-2\,\Delta] = n_2^2, \quad |x| > a \quad \text{substrate} \qquad \ldots \quad 1.8$$

We consider ray paths in the film region of the waveguide. Substituting for $n^2(x)$ from equation (1.8) in equation (1.7) we obtain

$$\frac{d^2x}{dz^2} + \Gamma^2 x(z) = 0 \qquad \ldots \quad 1.9$$

where

$$\Gamma = \frac{n_1\sqrt{2\Delta}}{\bar{\beta}\,a} \qquad \ldots \quad 1.10$$

The general solution of equation (1.7) is given by

$$x(z) = A \sin \Gamma z + B \cos \Gamma z \qquad \ldots \quad 1.11$$

which represents the general ray path. In a parabolic index medium the ray paths are given by equation (1.11) and, without any loss of generality, we may assume

$$x(0) = 0$$

which implies B = 0. Thus

$$x(z) = A \sin \Gamma z$$

If the ray makes an angle θ_1 with the z - axis at z = 0, then

$$\tan \theta_1 = \frac{dx}{dz}\Big|_{z=0} = A\Gamma$$

or

$$A = \frac{\bar{\beta} a \tan\theta_1}{n_1\sqrt{2\Delta}} = \frac{a \sin\theta_1}{\sqrt{2\Delta}}$$

$$= \frac{a}{\sqrt{2\Delta}}\left[1-\left(\frac{\bar{\beta}}{n_1}\right)^2\right]^{1/2} \qquad \ldots \qquad 1.12$$

where we have used the fact that

$$\bar{\beta} = n_1 \cos \theta_1$$

We therefore obtain

$$x(z) = \frac{a \sin\theta_1}{\sqrt{2\Delta}} \sin\left(\frac{n_1\sqrt{2\Delta}}{a\bar{\beta}} z\right) \qquad \ldots \qquad 1.13$$

The periodic length of the sinusoidal ray path is given by

$$z_p = \frac{2\pi}{\Gamma} = \frac{2\pi a\bar{\beta}}{n_1\sqrt{2\Delta}} \qquad \ldots \qquad 1.14$$

Typical ray paths are shown in figure [1.2(b)]; each ray corresponds to a specific value of $\bar{\beta}$ and bends in such a way that the product n (x)

$\cos\theta$ (x) remains constant. The ray paths given by equation (1.13) are valid only for $|x| < a$ since the refractive index profile is parabolic only in the region $|x| < a$. Now if $\bar{\beta}$ is such that the turning point $x = x_t$ lies between $x = +a$ and $x = -a$, then the ray will propagate by periodically oscillating around the z axis as shown in figure (1.3). Such rays will form guided rays. For this to happen $\bar{\beta}$ must be $> n_2$ since the turning point is determined by $n(x_t) = \bar{\beta}$ and the refractive index reduces monotonically from the axis. Also from equation (1.3) and the fact that the maximum value of n is n_1, we must have $\bar{\beta} \leq n_1$. Hence for guided rays we must have

$$n_2 < \bar{\beta} < n_1 \text{ guided rays} \qquad \ldots \qquad 1.16$$

If $\bar{\beta} < n_2$ then the ray will intersect the film-substrate interface at a finite angle and will be transmitted into the substrate / superstrate (Figure 1.2). Further since the substrate has uniform refractive index, the ray will travel in a straight line. Hence, such rays are not guided and are called refracted rays.

1.5 SUMMARY

At the outset the importance of integrated optics in light guidance is brought out. This is followed by the explanation of terms like thin films and optical coatings illustrating the role of thin films as light guides. Next the determination of ray paths in planar optical waveguides is discussed in somewhat details. The variant of ray paths is defined and the ray equation is established. The solution of this equation is obtained for a thin film with parabolic index distribution. The expression for the periodical length of ray path is given and the condition for the guided rays is explained.

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FIG. 1-1 (a) RAY PATH IN A LAYERED STRUCTURE.

(b) RAY PATH IN A CONTINUOUSLY VARYING REFRACTIVE INDEX DISTRIBUTION.

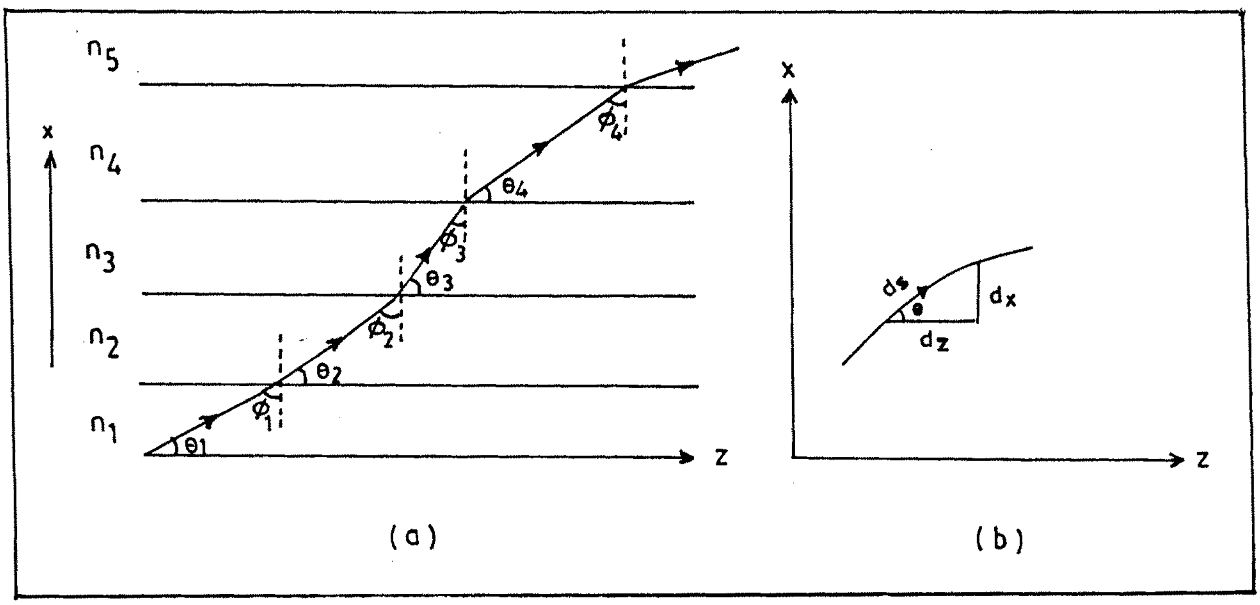

FIG. 1·2 (a) PARABOLIC VARIATION OF REFRACTIVE INDEX.
(b) RAY PATHS IN A PARABOLIC INDEX SLAB WAVEGUIDE.