CHAPTER II

ANTENNA ARRAYS AND RELEVANT THEORY

2.1 INTRODUCTION :

When greater directivity is required like in the cases Of point to point communication one makes use of a system of Identical antennas, similarly oriented to enable the use of wave interference phenomenon to produce greater directivity than a single antenna. Such a system of identical antennas arranged in a regular geometric pattern is an array. At higher frequencies it is required to produce a narrow beam of energy, then we make use of linear arrays where all the identical antennas are spaced at uniform distance along a straight line.The array is called as uniform linear array when these antennas are fed by currents of same amplitude undergoing an uniform progressive phase shift along the line in which the antennas are located.

The techniques of producing directive beams by means of arrays of radiators that are suitably spaced and driven with appropriate relative amplitudes and phases has been used widely at the longer wavelengths.

The arrays that have been designed to date can be grouped into two general cases

(1)End-fire arrays producing a beam directed along the axis of the array and

(2)Broadside arrays producing beams the peak intensity of which is in a direction normal to or nearly normal to the axis.

In our case high directivity is required. To design high directive broadside antenna, we shall discuss some theory of the array antennas. First let us develop the theory of the thin linear antenna and next theory of the array antenna.

2.2 THE THIN LINEAR ANTENNA :

Let us develop expressions for the far-field patterns of thin linear antennas. Assume that the antennas are symmetrically fed at the center by a balanced two wire transmission line. The antennas may be of any length, but it is assumed that the current distributions is sinusoidal.

The retarded value of the current at any point z on the antenna referred to a point at a distance S is (Fig 2.1)

$$I = I_0 \sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2} \pm z\right)\right] e^{j\omega[t-(r/c)]} \qquad \text{\_\_\_\_\_\_\_\_\_\_} \{2.1\}$$

In the eqn {2.1} the function,

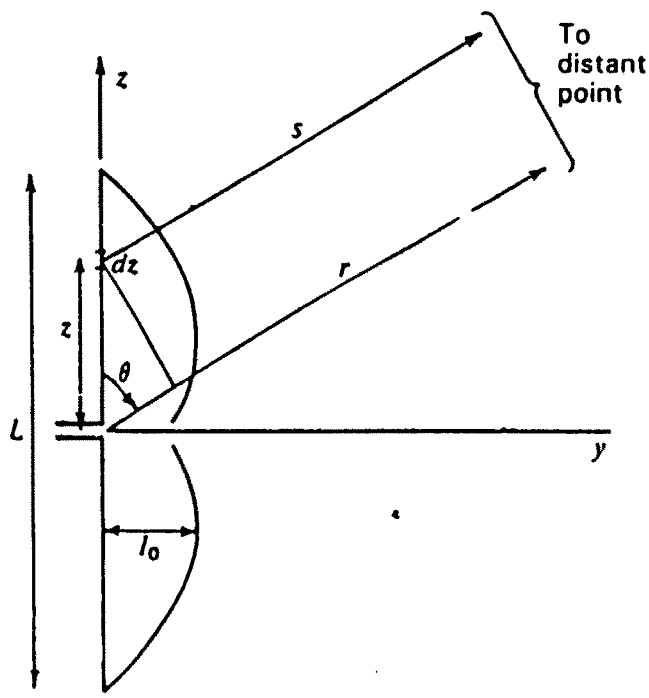

Fig. 2.1 – Shows the radiation for symmetrical,thin, linear, center fed antenna of length L.

$$\sin\frac{2\pi}{\lambda}\left[\left(\frac{L}{2}\pm z\right)\right]$$

is the form factor for the current on the antenna. The expression $\left(\frac{L}{2}\pm z\right)$ is used when z < 0 and L/2-z is used when z > 0. By regarding the antenna as made up of a series of infinitesimal dipoles of length dz, the field of the entire antenna may be obtained by integrating the fields from all of the dipoles making up the antenna. The far fields dE_υ and dH_ϕ at a distance S from the infinitesimal dipole dz are

$$dE_\theta = \frac{j60\pi I \sin\theta\, dz}{S\lambda} \qquad \text{\{2.2\}}$$

$$dH\phi = \frac{jI \sin\theta\, dz}{2S\lambda} \qquad \text{\{2.3\}}$$

The value of the magnetic field H_ϕ for the entire antenna is the integral of eqn {2.3} over the length of the antenna.

Thus,

$$H_\phi = \int_{-L/2}^{L/2} dH\phi \qquad \text{\{2.4\}}$$

Introducing the value of I from {2.1} into {2.3} and substituting this into {2.4}, we have,

$$H_\phi = \frac{jI_\theta \sin\theta e^{i\omega t}}{2\lambda}\left\{\begin{array}{l} \int_{-L/2}^{0} \frac{1}{S} Sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2}+z\right)\right] e^{-j\omega z/c} dz \\ + \int_{0}^{L/2} \frac{1}{S}\left[\frac{2\pi}{\lambda}\left(\frac{L}{2}-z\right)\right] e^{-j\omega z/c} dz \end{array}\right\} \qquad \text{\{2.5\}}$$

In eqn {2.5}, 1/S affects only the amplitude, and hence at a large it may be regarded as a constant. Also at a large distance, the difference between S & r can be neglected in its effect on the amplitude although its effect on the phase must be considered.

From, Fig{2.1},

$$S = r - z\cos\theta \qquad \text{\{2.6\}}$$

Substituting {2.6} into {2.5} and also r for S in the amplitude factor, {2.5} becomes,

$$H_\phi = \frac{jI_0 \sin\theta e^{j\omega\left[t-\left(\frac{r}{c}\right)\right]}}{2\lambda r}\left\{\begin{aligned}&\int_{-L/2}^{0} \sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2}+z\right)\right] e^{j(\omega\cos\theta)\frac{z}{c}}dz\\&+\int_{0}^{L/2} \sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2}-z\right)\right] e^{j(\omega\cos\theta)\frac{z}{c}}dz\end{aligned}\right\} \qquad \text{\{2.7\}}$$

Since $\beta = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ and $\frac{\beta}{4\pi} = \frac{1}{2\lambda}$,

Eqn. {2.7} may be written as

$$H_\phi = \frac{j\beta I_0 \sin\theta e^{j\omega\left(t-\frac{r}{c}\right)}}{4\pi r}\left\{\begin{aligned}&\int_{-L/2}^{0} e^{j\beta z\cos\theta} \sin\left[\beta\left(\frac{L}{2}+z\right)\right]dz\\&+\int_{0}^{L/2} e^{j\beta z\cos\theta} \sin\left[\beta\left(\frac{L}{2}-z\right)\right]dz\end{aligned}\right\} \qquad \text{\{2.8\}}$$

The integrals are of the form

$$\int e^{ax} \sin(c+bx)dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(c+bx) - b\cos(c+bx)\right] \qquad \text{\{2.9\}}$$

Where for the first integral

$a = j\beta\cos\theta$
$b = \beta$
$c = \beta L/2$

For the second integral a and c are the same as in the first integral, but $b = -\beta$, carrying through the integrations, adding the results and simplifying yields.

$$H_\phi = \frac{jI_0}{2\pi r}\left[\frac{\cos[(\beta L\cos\theta)/2] - \cos(\beta L/2)}{\sin\theta}\right]$$

$$H_\phi = \frac{jI_0}{2\pi r}\left[\frac{\cos[(\beta L\cos\theta)/2] - \cos(\beta L/2)}{\sin\theta}\right] \quad \text{\_\_\_\_\_\_\_\_ \{2.10\}}$$

Multiplying H_ϕ by $z = 120\ \pi$ gives E_θ as

$$E_\theta = \frac{j60I_0}{r}\left[\frac{\cos[(\beta L\cos\theta)/2] - \cos(\beta L/2)}{\sin\theta}\right] \quad \text{\_\_\_\_\_\_\_\_ \{2.11\}}$$

Equations {2.10} & {2.11} are expressions for the far fields, H_ϕ and E_θ of a symmetrical centerfed thin linear antenna of length L.

In our case we want field pattern for full wave dipole antenna. Since the amplitude further is independent of the length only the relative field patterns has given by the pattern factor will be compared. Let us compare the field pattern for half wave and full wave dipole antenna.

CASE 1: λ/2 ANTENNA

When L = λ/2, the pattern factor becomes

$$E = \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \qquad \text{\{2.12\}}$$

This pattern is shown in Fig.(2.2a),

The beam width between half- power points of the λ/2 antenna is 78^0.

CASE 2: FULL WAVE ANTENNA

When L =λ, the pattern factor becomes

$$E = \frac{\cos(\pi\cos\theta) + 1}{\sin\theta}$$

This pattern is shown in Fig.(2.2b),

The half power beam width is 47^0.

So, we need high gain low beam width antenna to study cosmic radio noise, so full wave dipole antenna is necessary for our study.

2.3 LINEAR ARRAYS :

For point to point communication at the higher frequencies the desired radiation pattern is a single narrow lobe or beam. To obtain such a characteristic a multielement linear is used. An array is linear when the elements of the array are spaced equally

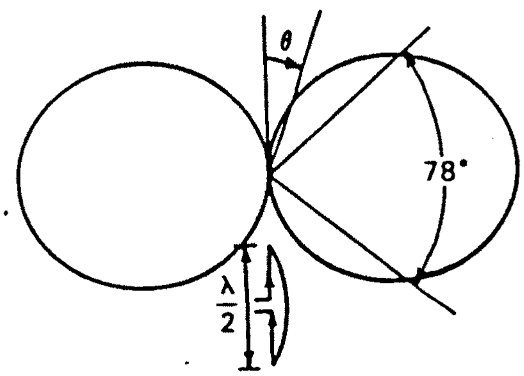

Fig. 2.2a – Shows far field pattern of λ/2 antenna.

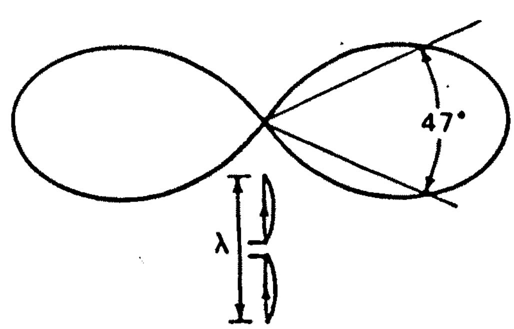

Fig. 2.2b – Shows far field pattern of full wave antenna.

along a straight line Fig {2.3}.

In a uniform linear array the elements are fed with currents of equal magnitude and having a uniform progressive phase shift along the line. The pattern of such an array can be obtained by adding vectorially the field strengths due to each of the elements. For a uniform array of non-directional elements the field strength would be

$$E_T = E_0\left[1 + e^{i\varphi} + e^{i2\varphi} + e^{i3\varphi} + ----+e^{i(n-1)\varphi}\right] \quad \_\_\_\_\_\_ \{2.13\}$$

Where, $\varphi = \beta d \cos\varphi + \alpha$ and α is the progressive phase shift between elements.(α is the angle by which the current in any element leads the current in the preceding element).The eqn {2.13} may be viewed as a geometric progression and written in the form

$$\frac{E_T}{E_o} = \left|\frac{1-e^{in\varphi}}{1-e^{i\varphi}}\right| = \left|\frac{\sin\frac{n\psi}{2}}{\sin\psi/2}\right| \quad \_\_\_\_\_\_ \{2.14\}$$

The maximum value of this expression is n and occurs when $\psi = 0$. This is the principal maximum of the array.

Since $\psi = \beta d \cos\varphi + \alpha$ the principal maximum occurs when,

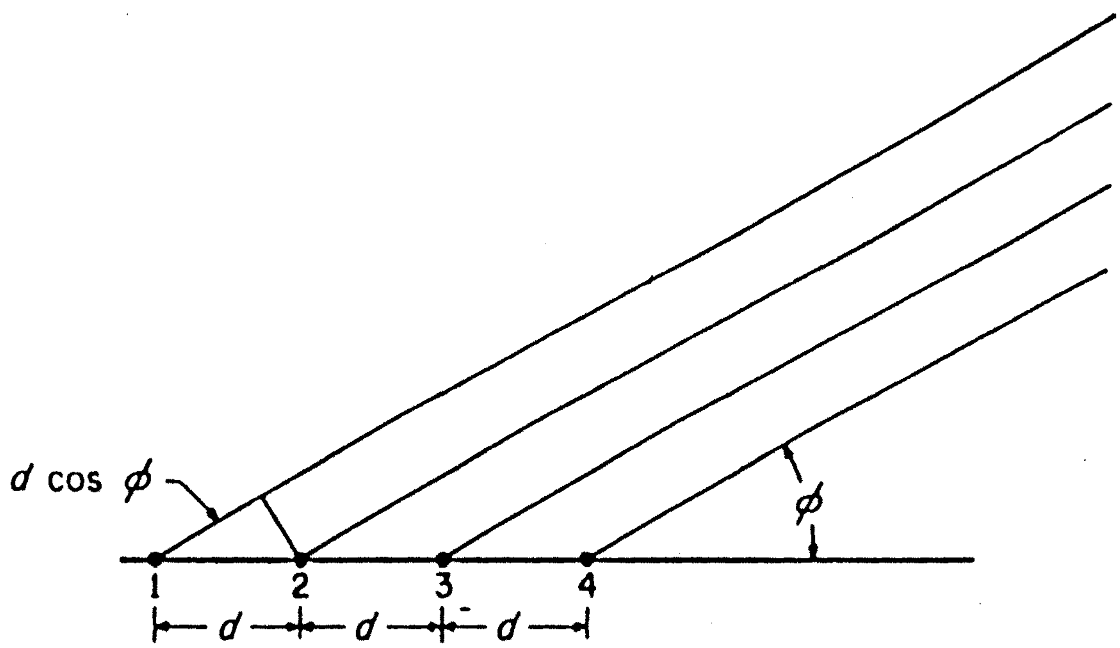

Fig. 2.3 – Shows four element linear array of non directional radiators.

$$\cos\varphi = -\frac{\alpha}{\beta d}$$

For a broadside array the maximum radiation occurs perpendicular to the line of the array at φ = 90 degrees, so α = 0 degrees. The expression {2.14} is zero when

$$\frac{n\psi}{2} = \pm k\pi \qquad k = 1,2,3,\text{--------}$$

These are the nulls of the pattern. Secondly maxima occur approximately midway between the nulls, when the numerator of expression {2.14} is maximum, that is when

$$\frac{n\psi}{2} = \pm(2m+1)\pi/2 \qquad m = 1,2,3,\text{--------}$$

The first secondary maximum occurs when

$$\frac{\psi}{2} = +\frac{3\pi}{2n}$$

The amplitude of first secondary lobe is

$$\left|\frac{1}{\sin(\psi/2)}\right| = \left|\frac{1}{\sin\left(\frac{3\pi}{2n}\right)}\right| \approx \frac{2n}{3\pi} \qquad \text{For large n}$$

The amplitude of the principal maximum was n so the amplitude ratio of the first secondary maximum to principal maximum is $\frac{2}{3\pi} = 0.212$.

This means that the first secondary maximum is about

13.5 db below the principal maximum, and this ratio is independent of the number of elements in the uniform array, as long as the number is large.

For all odd values of n, the smallest lobe (at $\psi = \pi$) has an amplitude of unity.

The width of the principal lobe, measured between the first nulls, is twice the angle between the principal maximum and first null. This latter angle is given by

$$\frac{n\psi_1}{2} = \pi \quad \text{or} \quad \psi_1 = \frac{2\pi}{n}$$

For a broadside array

$$\cos\phi = \frac{\psi}{\beta d}$$

and the principal maximum occurs at

$$\phi = \frac{\pi}{2}$$

The first null occurs at an angle $\left[\left(\frac{\pi}{2}\right) + \Delta\phi\right]$ where,

$$\cos\left(\frac{\pi}{2} + \Delta\phi\right) = \frac{\psi_1}{\beta d}$$

$$= \frac{\lambda}{dn}$$

If $\Delta\phi$ is small, it is given approximately by

$$\Delta\phi = \frac{\lambda}{nd}$$

The width of the principal lobe is

$$2\Delta\phi = \frac{2\lambda}{nd}$$