C H A P T E R - 2

THE PROBLEM OF STRUCTURE

Rutherford's α- ray scattering experiments revealed that the atom consists of positively charged nucleus. Thus the 'elements' revealed an internal structure consisting of positively charged nucleus and negatively charged electrons moving around it.

But are the proton, neutron and electron really elementary ? or they consist of some kind of inner structure ? To answer this question one must bombard these particles with high energy probes. From the results of such experiments, one can infer wheather the particles are pointlike or have inner structure.

(1) When high energy electrons are bombarded on protons, it was found that the electrons are scattered with large transfer of momentum.

(2) The analysis of experiments suggests that the proton contains discrete scattering centres within it.

(3) The distribution of the scattered electrons in energy and angle suggests a phenomenon called 'Scale Invariance'.

2.1 Scale Invariance :[2]

When a lepton scatters from a target it transfers energy and momentum to target via a virtual current (photon for electromagnetic and W-boson for weak interactions).

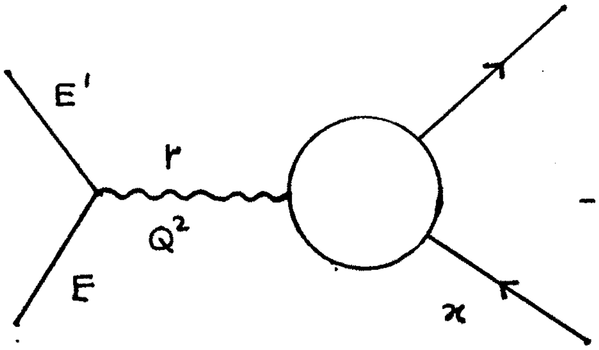

Fig. 2.1 Electromagnetic scattering of electron by hadron.

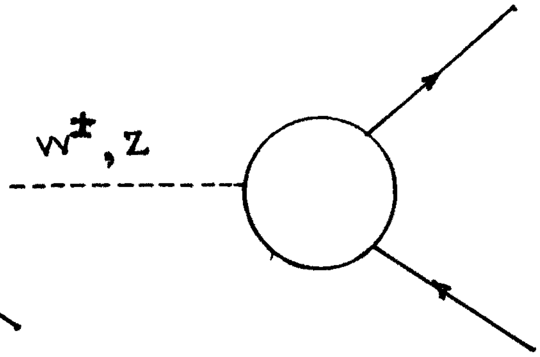

Fig. 2.2 Weak scattering of electron by hadron.

Let the target be in laboratory rest frame. Let the initial energy of electron be E, final E'. Then the momentum transfered to proton is

$$E' - E = \frac{q^2}{2M} = \frac{-Q^2}{2M}$$

Where M is mass of virtual propagator field quantum (photon or W bosons).

It is found that the interaction of the current with a parton is a function of x only, where x is ratio of parton's energy to the current energy. It is independent of the invariant squared mass Q^2 of the virtual current probe.

The dependence on only the dimensionless ratio x is known as scale invariance or scaling, since no energy or length scale governs the interaction, i.e. the partons are 'pointlike'.

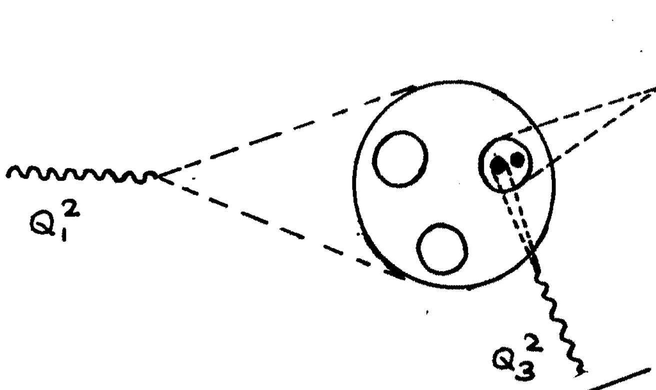

Fig. 2.3(a) Prequarks $:Q_1{}^2<Q_2{}^2<Q_3{}^2$ yields coherent proton, quark structure and finally prequark structure as successive granular layers are revealed.

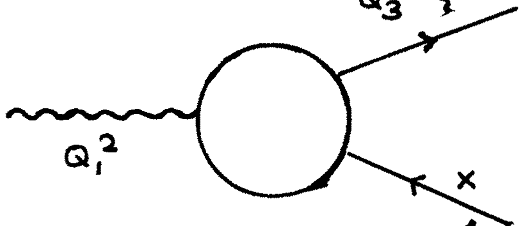

Fig.2.3(b) Small Q_1^2 sees a parton of momentum x.

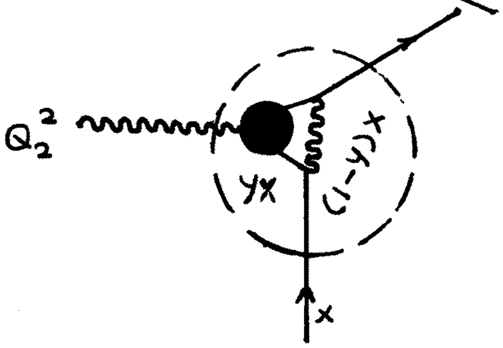

Fig.2.3(c) at larger Q^2 we see that the quark has radiated a gluon and lowered its momentum to xy.

If on the other hand, partons (quarks) have a structure themselves, then this may be resolved if a current of larger Q^2 probes the system. The parton carrying momentum fraction x will be seen as a system of pre-partons each carrying some fraction y of the momentum x.

As Q^2 is increased and this new structure is resolved, there will be a violation of scaling. The size of the parton clouds sets an intrinsic scale of length which is resolved when $Q^2 \gtrsim Q^2_{\text{critical}}$.

2.2 Partons

From the analysis of e-p scattering experiments, it is seen that the protons and neutrons are not elementary, but consist of 'partons'. There appear to be two types of partons :

(1) Electrically neutral particles called gluons. These are massless vector particles and propagators of strong interactions.

(2) Spin half-fermions called quarks which have electrical charges which are fractions of proton charge ($\pm 2/3$, $\pm 1/2$). 3

Earlier it was thought that quarks come in three types or flavours : p, n, λ or u, d, s (up, down and strange).
But now three other flavours have been confirmed, viz .C(charm),B(beauty) and T(truth).

As opposed to the hadrons, the leptons do not show any internal structure. The quarks and leptons can be grouped in three families in grand unified theories.
Each family consisting of four members :

Electron family : u, c, e, ν_e

Muon family : d, s, μ, ν_μ

Tau family : b, t, τ, ν_τ

2.3 Charm and Charmed Particles [2]

Charm is the fourth flavour of a quark.

The three flavours of quarks in SU(3) give a triplet representation in I_3 - Y space, it is a triangle. The fourth quark C with charge 2/3, isospin and strangeness is zero and one unit of charm is added to the SU(3) triplet. It generates the fundamental quartet representation of SU(4). The weight diagram is now a pyramid.

Fundamental Representation :

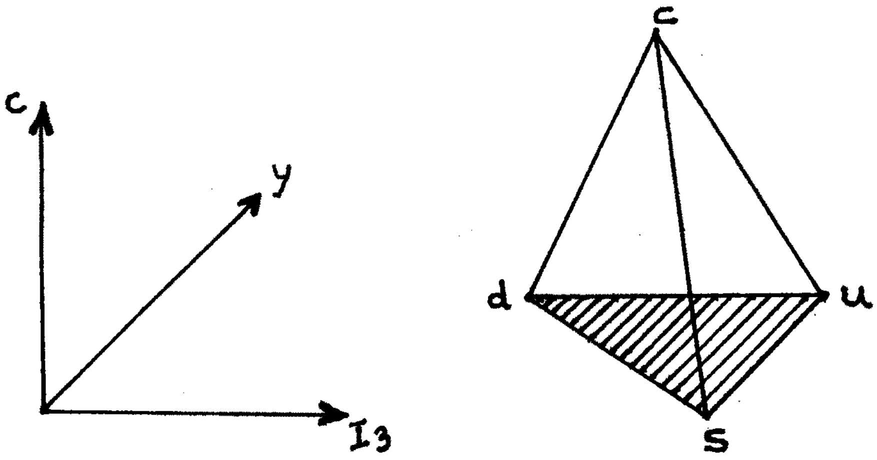

Fig. 2.4 Fundamental representation of SU(4).

The four triangle surfaces of the pyramid show the four SU(3) subgroups (uds), (udc), (dsc) and (usc) contained in the basic SU(4).

Mesons :

They are formed from $q\bar{q}$

$$4 \otimes \bar{4} = 1 \oplus 15$$

Hence in addition with nonet of SU(3), seven new mesons must exist. These can be grouped in following way :

(1) Three states with charm 1 forming a $\bar{3}$ (antitriplet) of SU(3).

$c\bar{u}$ This is denoted as (D^{o}) or (D^{o*})

$c\bar{d}$ This is denoted as (D^{+}) or (D^{+*})

$c\bar{s}$ This is denoted as (F^{+}) or (F^{+*})

(2) Three states with charm-1 forming a 3 (triplet) of SU(3).

$\bar{c}u$ $(\bar{D}^{o})$ or $(\bar{D}^{o*})$

$\bar{c}d$ (D^{-}) or (D^{-*})

$\bar{c}s$ (F^{-}) or (F^{-*})

(3) A state with charm zero i.e. C=0 (hidden charm) which is singlet of SU(3).

$$C\bar{C} \rightarrow \eta_C \quad \text{or } \psi$$

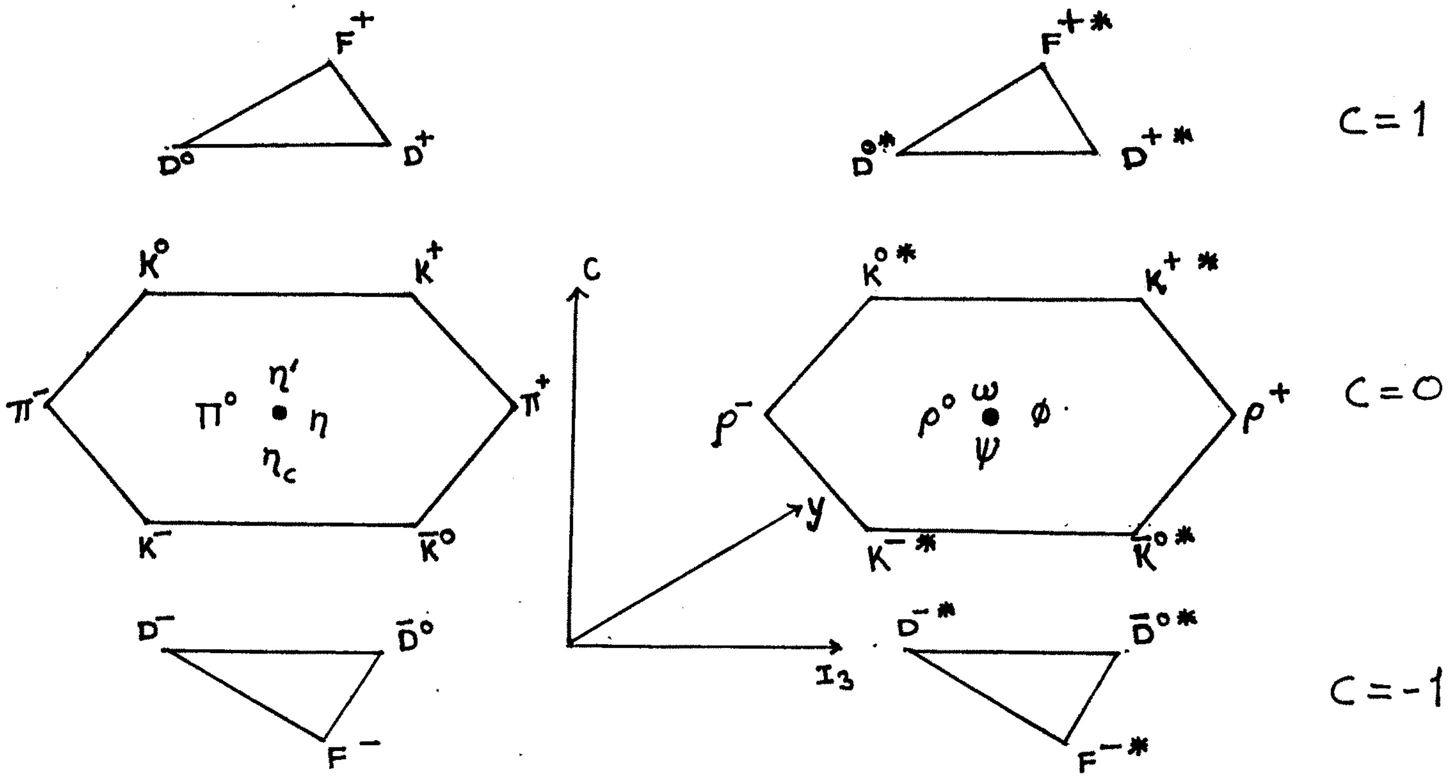

Fig. 2.5

0^- Pseudoscalar Mesons. 1^- Vector Mesons

Representations of SU(4).

2.4 Charmonium Spectrum[3]

In analogy with positronium i.e. e^-e^+ bound state or ($q\bar{q}$-bound state), we can define the fourth quark bound state $C\bar{C}$ as charmonium. Each charm quark has spin 1/2. When these particles are bound in different relative angular momentum and spin states, we get the charmonium spectrum or different states. A charmonium state is represented in a spectroscopic notation as $n^{2s+1}\,L_J$.

Where,

n is radial quantum number.

S is spin.

L is relative angular momentum.

J is total angular momentum,

$$J = L + S.$$

			J^{PC}	
e.g. $S = 0,\ L = 0,\ n = 1$	$\Longrightarrow 1\ {}^1S_0$	$\chi(2830)$	0^{-+}	Pseudoscalar Mesons. $\uparrow\downarrow$
$S = 0,\ L = 0,\ n = 2$	$\Longrightarrow 2\ {}^1S_0$	$\chi(3455)$	0^{-+}	

Vector Mesons : J=1

$S = 1,\ L = 0,\ n = 1$	$\Longrightarrow 1^3S_1$	$\psi(3095)$	1^{--}	Vector Mesons. $\uparrow\uparrow$
$S = 1,\ L = 0,\ n = 2$	$\Longrightarrow 2\ {}^3S_1$	$\psi(3684)$	1^{--}	
$S = 1,\ L = 2,\ n = 1$	$\Longrightarrow 1\ {}^3D_1$	$\psi(3772)$	1^{--}	

J=2 Mesons Components

$S = 1,\ L = 1$	$\Longrightarrow 1\ {}^3P_0$	$\chi(3415)$	0^{++}	Scalar mesons
$S = 1,\ L = 1$	$1\ {}^3P_1$	$\chi(3510)$	1^{++}	Axial vector mesons
$S = 1.\ L = 1$	$1\ {}^3P_2$	$\chi(3550)$	2^{++}	Tensor mesons

Production of hidden Charm mesons[4]

The charmonium $C\bar{C}$ are hidden charm mesons. They are produced in high energy collisions of e^+e^-.

e.g. $e^+ + e^- \longrightarrow J/\psi(3095)$ + anything.

$e^+ + e^- \longrightarrow \psi'(3684)$ + anything

$\longrightarrow \psi''$ (3772) + anything

$\longrightarrow \psi$ (4028) + anything

Decay :

The massive mesons ψ'', ψ (4028) can decay and pair produce a charmed mesons.

e.g. $\psi'' \longrightarrow D + \bar{D}$

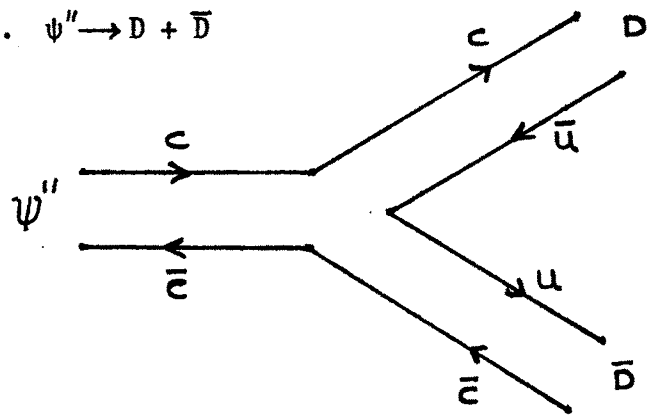

Fig. 2.6 Decay of ψ''

But the masses of lighter varieties ψ and ψ' lie below the threshold of charm production. Hence they decay into uncharmed mesons. This process can be represented as follows :

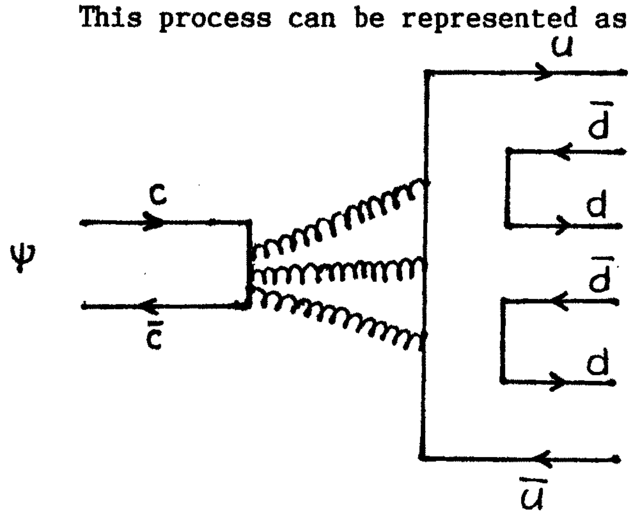

Fig. 2.7 Decay of ψ

Zweig Rule

The decay of hadrons is governed by Zweig rule. It is also called as OZI (Okubo, Zweig Iizuka) rule.

It states that, "Disconnected quark diagrams are suppressed relatively to connected once."

In other words, Hadronic reactions are suppressed when their quark diagrams are such that one cannot trace a continuous quark line from the initial hadrons to the final once.

Thus a diagram 2.6 above is OZI allowed, while diagram 2.7 is forbidden.

This rule was first discovered for ϕ decay. The ϕ particle is an $S\bar{S}$ bound state.

The ϕ particle decays into :

$$\phi \longrightarrow K^+ + K^-$$
$$\not\longrightarrow \pi^+ + \pi^0 + \pi^-$$

eventhough both interactions conserved the quantum numbers.

Let us represent the quark diagram :

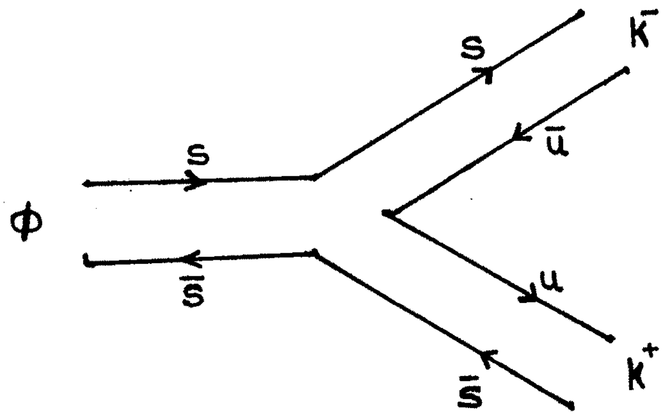

Fig. 2.8 ϕ - decay - OZI allowed (continuous quark diagram).

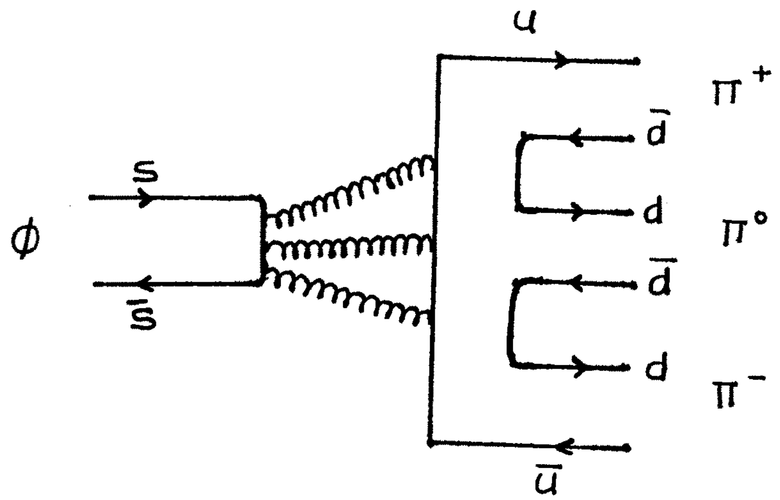

Fig. 2.9 ϕ -decay : OZI-forbidden (disconnected quark diagram).

$\phi \longrightarrow 3\pi$ is suppressed compared to $\phi \rightarrow K^+K^-$

Thus one would expect that, J/ψ in analogy with ϕ will decay into charmed and anticharmed mesons D and $\bar{D}$. But D is lightest charmed meson having a mass of 1863 MeV. Therefore, J/ψ(3097) as well as ψ'(3684) lie below the threshold for decay into D and $\bar{D}$.

Hence they must decay via OZI forbidden process.

The only way the initial quark-antiquark pair can interact with the final quarks is by exchange of gluons. Single gluon exchange is not possible because a gluon carries a colour and the final hadrons should be colourless. Similarly, two gluon exchange is also forbidden as it violates charge conjugation symmetry. Therefore, J/ψ and ψ' must decay into uncharmed mesons by OZI forbidden process through exchange of three gluons.

2.5 Evidence of new flavours as revealed in the meson spectroscopy or Quarkonia as bound states of flavours[3]

The meson quark structure can be understood in analogy with positronium which is bound state of e^- and e^+.

In positronium, the electron (e^-) and positron (e^+) can couple their spins to 1 (triplet state) or 0 (singlet state). In addition they can have relative angular momentum 0,1,2..... etc. called S(sharp), P(principle), D(diffuse), F(fundamental) etc. states.

The total angular momentum is given by,

$$J = L + S$$

The resulting energy levels are lebelled as ${}^{2S+1}L_J$

$\therefore$ The lowest energy levels are ${}^1S_0$, ${}^3S_1$.

Further we have, ${}^1P_1$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$ and so on

If there is only one flavour of quark, then we will have a similar series of levels like positronium for quarkonium i.e. bound state of a quark and antiquark. These will be identified with following mesons.

$$\left.\begin{matrix} {}^1S_0 \\ \pi \end{matrix}\right\} \left.\begin{matrix} {}^3S_1 \\ \rho \end{matrix}\right\}; \left.\begin{matrix} {}^1P_1 \\ B \end{matrix}\right\} \left.\begin{matrix} {}^3P_0 \\ \delta \end{matrix}\right\} \left.\begin{matrix} {}^3P_1 \\ A_1 \end{matrix}\right\} \left.\begin{matrix} {}^3P_2 \\ A_2 \end{matrix}\right\}$$

The following figure shows the low-lying spectrum of a hypothetical $q\bar{q}$ system with a quark mass that is a million times greater than the electron mass.

Bohr radius of this $q\bar{q}$ system is about 7.7×10^{-18}m.

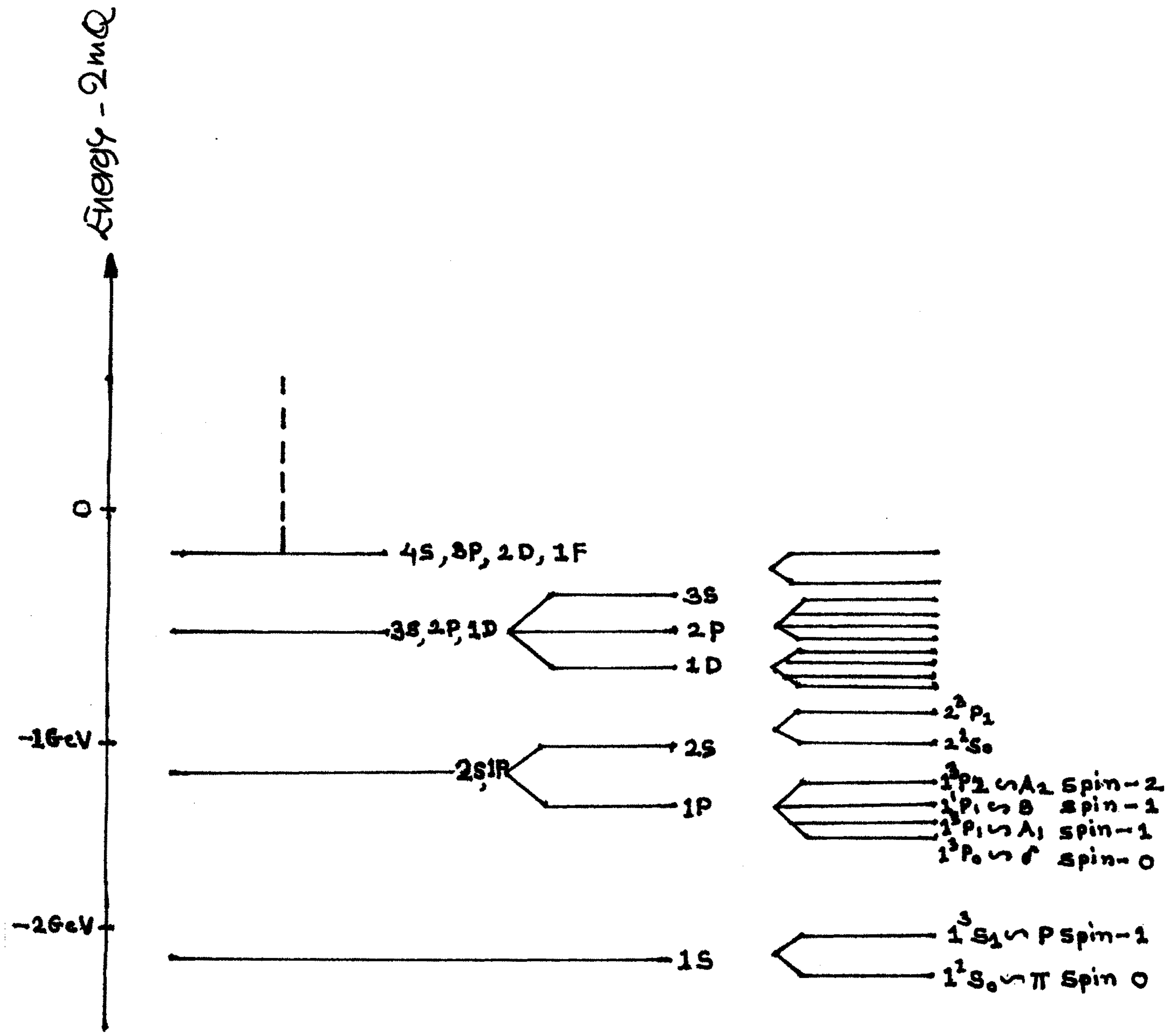

Fig. 2.10

Heavy quarkonium. This diagram shows the low-lying energy levels in a hypothetical heavy quark-antiquark system.

The forces determining the positronium spectrum are Electro-magnetic where as in quarkonium they are "color-electric" and "color magnetic". At short distances these forces are very analogous in form to EM interactions.

But we know that π and other respective particles come in three charge states i.e. π^+, π^0; π^- and in addition another neutral particle η° having $^1S_0$ structure accompanies them. Same pattern is repeated at every energy level.

e.g. at $^3S_1$ we have ρ^+, ρ^0, ρ^- and ω° and so on.

This shows that there exist two flavours of quark u and d.

Further, the discovery of strange particles shows that there are in fact, nine mesons at each level. Thus nine is natural number of mesons at each level if quarks come in three flavours : u, d and s.

The results could be displayed in tabular form as below:

$^1S_0$	$^3S_1$	$^1P_1$?	$^3P_0$	$^3P_1$?	$^3P_2$
π^+	ρ^+	B^+	δ^+	A^+_1 ?	A^+_2
π^o	ρ^o	B^0	δ^o	A^0_1 ?	A^0_2
π^-	ρ^-	B^-	δ^-	A^-_1 ?	A^-_2
η^o	ω^o	?	ε	D ?	f^0
$\eta^{o\prime}$	ϕ^o	?	S^*	E ?	$f^{0\prime}$
K^+	K^{*+}	Q^+_B ?	K^+	Q^+_A ?	K^{**+}
K^0	K^{*0}	Q^0_B ?	K^0	Q^0_A ?	K^{**0}
K^-	K^{*-}	Q^-_B ?	K^-	Q^-_A ?	K^{**-}
$\bar{K}^0$	$\bar{K}^{*o}$	$\bar{Q}^0_B$?	$\bar{K}^0$	$\bar{Q}^0_A$?	$\bar{K}^{**0}$

Table 2.1 QuarkoniumMeson Spectroscopy

L-S coupling $J = L + S$

S states $J = 0 + 0 \Longrightarrow {}^1S_0$

$J = 0 + 1 \Longrightarrow {}^3S_1$

P states $J = 1 + 1 = 2 \Longrightarrow {}^3P_2$

$1 \Longrightarrow {}^3P_1$

$0 \Longrightarrow {}^3P_0$

$J = 1 - 1 = 0 \Longrightarrow {}^1P_1$

Currently, it is thought that quarks come in six flavours i.e. u, d, S, C, b, t. The number mesons that are expected at each level is $6 \otimes 6 = 36$ particles.

2.6 Quarkonium Bound State Physics[5]

When an electron and positron annihilate at a total centre of mass energy close to the mass of the particle V (vector mesons)

[having conserved quantum numbers of photon i.e. charge = 0, $J^{PC} = 1^{--}$]

We observe the resonant behaviour in the cross-section for any final state F into which the particle V can decay.

Thus, $e^+e^- \longrightarrow \gamma \longrightarrow V \longrightarrow F(\text{hadrons})$

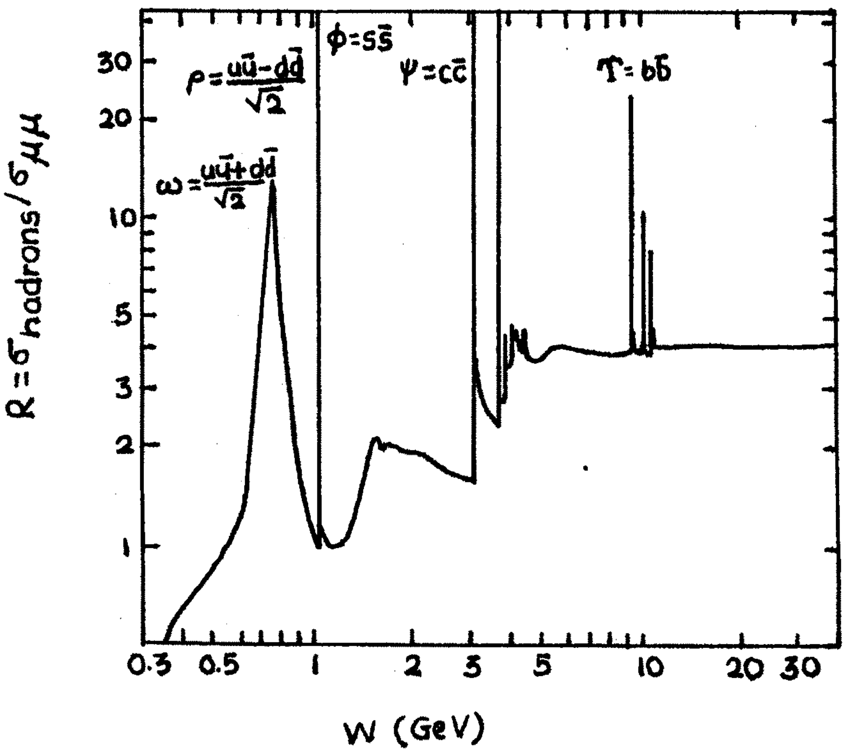

Fig. 2.10

The graph shows the total cross-section for e^+e^- annihilation into hadrons divided by the muon pair production cross-section, and plotted as a function of the centre of mass energy.

<u>Spectroscopy</u>

The vector mesons (ρ°, ω, ϕ, ψ, γ) are each only the lowest state in a sequence of 1^{--} resonances seen in the e^+e^- annihilations. We can add higher resonance of these particles to the group. We assume that they are bound states of $q\bar{q}$ with quantum numbers of photon.

The negative parity of these particles implies that,

$$L = 0, 2, 4, \dots\dots\dots\dots$$

Negative charge conjugation $C = 1 = (-1)^{L+S}$

$\therefore S = 1.$

Since, J=1, L must be either 0 or 2.

Hence in spectroscopic notation, the resonant states must be $^3S_1$ or $^3D_1$.

The peaks only indicate existence of quarkonia bound states at higher energies, it is only the decay of these states which confirm that they correspond to new flavours.

2.7 <u>The $q\bar{q}$ Potential: Empirical form :</u>

The typical spacing of levels in $b\bar{b}$ and $c\bar{c}$ systems is less than 600 MeV. If this is interpreted as K.E. of bound quarks with mass C(1.8 GeV), b(5 GeV). Then their motion is slow enough to be treated nonrelativistically.

Therefore, we can define a binding potential which when used in the nonrelativistic Schrodinger equation will give the observed energy spacings of $^3S_1$ states.

If QCD is correct and quanta of $q\bar{q}$ binding carry no flavour, then the potential function must be same for $c\bar{c}$ and $b\bar{b}$.

A Coulomb potential $V(r) = -\frac{a}{r}$, gives a spectrum

$$E_n = \frac{-K}{n^2} ,$$

For the states of radial quantum number n. This will predict a spacing ratio

$$\frac{E_3 - E_1}{E_2 - E_1} = 1.18 .$$

A harmonic oscillator potential $V = Kr^2$ gives equally spaced levels and a spacing ratio equal to 2.

since the observed spacing ratio for the first three upsilon $3S_1$ states is 1.59, the actual potential is intermediate between r^{-1} and r^2.

If we assume a power law potential $V = a\,r^{\nu}$ then the observed ratio of $E_2 - E_1$ for $\frac{\Upsilon}{\psi}$ is equal to $\frac{561}{589} \sim 1$, which implies $\nu \sim 0$.

The best fit is $\nu = 0.1$.

This is the empirical form of potential.

2.8 Other Flavours - Beauty and Truth [5,6]

The bottom quark : (b)

Along with two lepton doublets, ℓ_1, (e, ν_e), ℓ_2 (μ, ν_μ) two quark doublets $q_1(d, u)$, $q_2(s, c)$ were nicesly fitted in the theory. But in 1975 the third lepton doublet $\ell_3(\tau, \nu_\tau)$ was discovered which upset the balance between quarks and leptons. Hence the third quark doublet $q_3(b,t)$ (bottom and top) was postulated. The corresponding flavours were called beauty and truth.

In 1977, three states were observed at 9.4, 10.0 and 10.4 GeV in Proton(P) + Berilium(Be) collisions. They were named as Υ (upsilon), Υ', Υ''. Further, these states were also observed in $e^+e^- \longrightarrow$ hadrons collision.

In 1980, four states were observed.

Υ (1S) - 9460 MeV.

Υ (2S) -10023.5 MeV.

Υ (3S) -10355.5 MeV

Υ (4S) -10573 MeV.

In 1985, two more states were observed.

Υ (5S) -10870 MeV.

Υ (6S) -11020 MeV.

along with χ_b(1P) & χ_b(2P) states.

By now, more states have been observed in the Υ(upsilon)-family than in positronium.

Decay :

The dominant weak decay chain for b-quark is

$$b \longrightarrow C \longrightarrow S \longrightarrow u,d$$

The top quark(t)

It was anticipated to be found at 30 GeV, but it was not seen. This was against the standard model. Some nonstandard 'topless' models which contained five quarks were also suggested. But in 1984, $P\bar{P}$ collider at CERN, Geneva has registered six events consistent with following scheme :

$$P\bar{P} \longrightarrow WX$$

$$W \rightarrow t\bar{b}$$

$$t \rightarrow b\,\ell\nu$$

where, t-quark has a mass between 30 and 50 GeV.