C H A P T E R - 0

INTRODUCTION AND SUMMARY

0.1 Introduction :

In this section, we discuss the historical survey of chi-square distribution. The symbol 'χ^2' used for chi-square variable, was first introduced by Pearson (1896) where it is written in place of $X'R^{-1}X$ where X is a vector and R is a positive definite variance covariance matrix of size n). Pearson's contributions to statistical theory were numerous but the greatest of them was the chi-square test of goodness of fit, which has remained one of the most useful of all statistical tests. Pearson (1904) also studied contingency tables which are used to test the independence of two attributes. The work of De-Moivre and Laplace established the asymptotic normality of the standard variable

$$\chi = (m - NP)/(Npq)^{1/2}$$

that is, $\chi^2 = (m - Np)^2/Npq$

where m is observed number of successes in N independent trials with constant probability p of success at each trial. The above expression was generalized by Pearson (1900a) to

$$\chi^2 = \sum_{i=1}^{n} (a_i - Np_i)^2/Np_iq_i$$

where a_i is the observed number in the i^{th} cell of multinomial distribution.

R.A.Fisher made many contributions to the theory and

application of χ^2. Fisher (1928b and 1935b) gave the distribution of the "central" or theoretical χ^2 and (1928c) gave the distribution of noncentral χ^2. Fisher (1924) gave the first proof of the asymptotic distribution of χ^2 in the general case when parameters are estimated from the data. Poisson (1828 and 1832), Bienayme (1852, 1858, 1868,& 1938), Sheppard (1898a) also contributed to chi-square distribution.

0.2 Chapterwise Summary

This dissertation contains five chapters a brief account of which is presented below. The first chapter numbered by '0' is devoted to a general introduction and historical survey of chi-square distribution and chapterwise summary. This dissertation is extremely useful to the undergraduate students of this University.

Chapter-1 contains elementary properties of a chi-square distribution. In section 1.2, definitions and examples related to a chi-square distribution is given. Some important properties of chi-square distribution is also given. In section 1.3, we introduce the relationship of a chi-square distribution to other important distributions. In last section of this chapter model sampling from a chi-square distribution is given . Also we have presented some tables of random numbers from chi-square distribution with n = 1,2, 4,8,16 d.f. which are prepared on computer.

Chapter-2 deals with estimation, testing of hypothesis and confidence intervals for the parameter of chi-square distribution. In section 2.2, we discuss the estimation of unknown parameter θ from a chi-square distribution. Also we have shown that $\hat{\theta}$ is minimum variance unbiased estimate, as well as MSE of any other estimator is smaller than MVUE of θ. In section 2.3, some testing of hypothesis problem is considered that is problems related to simple null hypothesis against simple alternative hypothesis and composite alternative hypothesis. Also composite null hypothesis against composite alternative hypothesis is described. Section 2.4 introduce shortest length confidence intervals for the parameter of a chi-square distribution based on pivotal quantity method. Also we have given some examples which are related to negative exponential in which it has a chi-square distribution. A Table of multipliers for the shortest confidence intervals (a/2y, b/2y) is given. Section 2.5, contains analysis of contingency tables while the last section deals with log likelihood ratio statistics which has an asymptotically chi-square distribution.

Chapter 3 describes selection problems for chi-square populations. In section 3.2, we have explained selection problem by giving suitable example while in section 3.3, probability of correct selection is described. Section 3.4 leads to determine the minimum sample size to achieve the required probability. Section 3.5 describes to find the

probability of correct selection for selection criteria in terms of minimum value of mean and section 3.6 deals with probability of correct selection for selection criteria based on variance of normal populations. Last section contains some applications where the chi-square distribution is the appropriate distribution model.

Chapter 4 is devoted to Applications of a chi-square distribution to industrial statistics. In Section 4.2, we investigate the applications of chi-square distribution to compute the probability of accepting a lot of quality θ. In the same section chi-square distribution is used to find the sample size n and rejection number c for the single sampling plan by attributes. Section 4.3 and 4.4 contains applications of chi-square distribution to find the sample size n and constant K in case of acceptance sampling by variables for exponential distribution and variable plan for normal distribution with mean μ (known) and variance σ^2 (unknown) respectively.

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