CHAPTER-I

INTRODUCTION.

In Section 1 of this chapter, the problem of acceptance sampling is described. In section 2 an example of each of the variable sampling plans in whcih the effective sample size is random and non-random is given. Section 3 deals with the concept of operating characteristic function of a single sampling plan and some related definitions. In section 4 the OC function of a single sampling plan by attributes is derived. These are compared with single sampling plans by variables through out this dissertation. The single sampling plan by variables is derived in case of uniform distribution and minimum sample size required for the OC function to pass through the given producer's and consumer's risk points is computed. In section 5 these plans are compared with the altribute plan. In the last section a brief description of the remaining chapters is given.

1.1 The problem of acceptance sampling :

Suppose that a lot of N items is given. The lot is to be accepted or rejected, according as whether it is of satisfactory quality or not. Suppose that the quality of the lot is defined as the proportion defective in the lot That is, if θ denotes the lot quality, θ is given by

$$\theta = D/N$$

where, D is the number of defectives in the lot.

Suppose that a lot is considered to be of satisfactory quality if its quality Θ is less than or equal to Θ_0, a given number. Naturally Θ_0 should be a number such that $0 \leq \Theta < 0.5$. If Θ is known for a given lot, one can accept the lot if $\Theta \leq \Theta_0$ and reject if $\Theta > \Theta_0$. But, for a given lot, Θ is unknown and varies from lot to lot. However, it is known that Θ takes one of the values in the set $\left[0, \frac{1}{N}, \frac{2}{N}, \ldots 1\right]$. In order to know the quality of the lot and to take a decision, one can inspect all the items in the lot. This procedure of inspecting all the items in the lot is called 100 percent inspection or screening of the lot.

However, 100 percent inspection of a lot may not be possible or may not be desirable because it is time consuming, costly and may not even guarantee perfect inspection of each item, which implies that the quality of the lot is wrongly assessed. Also, in some cases inspection may be destructive.

So, the alternative is to inspect only a part of the lot which is called sampling inspection. In this procedure only some items from the lot are inspected. The quality of the lot is estimated by the proportion defective found in the sample. Based on this proportion the lot is either accepted or rejected as the proportion of defective in the sample is 'small' or 'large'.

The problem is to decide about the number of items to be inspected and about the creteria of acceptance or rejection. Naturally it may not be possible to inspect many items, because of the reasons mentioned above. Also if too less items are inspected the fraction defective in the sample may not be a good estimate of the true lot quality Θ. Thus the decision to accept or reject a lot depends on many quantities in general and sample size in particular.

Once it is decided to use the sampling inspection, one has to specify,

i) the number of items ''n'' to be selected from the lot. This number is called the sample size.

ii) the method of selecting the n items from the lot, that is with replacement or without replacement, whether all the n items are selected at a time or in stages.

iii) whether to inspect all the n items in the sample and take a decision or to take a decision even before all the n items in the sample are inspected. In the second case the procedure is called curtailed inspection.

iv) the criteria for accepting or rejecting the lot.

Any specification of (i) to (iv) above is called a sampling plan.

Sampling plans may be broadly classified as sampling plans by attributes and sampling plans by variables. In either case each item is classified as good or defective, in order to determine the quality of the lot. However if the criteria of accepting or rejecting the lot is in terms of fraction defective in the sample then, the plan is called a sampling plan by attributes, otherwise the plan is called a sampling plan by variables.

Now, we consider an example to illustrate the difference between an attribute plan and a variable plan.

Example 1.1 : Suppose that it is possible to measure each item in the lot on a certain scale. The measurement on items vary from each other and is assumed to be a random variable X. An item is considered to be defective, if the measurement X on it is less than or equal to L (a given number). The quality of the lot is defined as the proportion of defectives in the lot, that is the proportion of items in the lot whose measurements are less than or equal to L.

In order to decide whether the lot is of acceptable quality or not, a sample of size n items is taken from the lot, and each item is measured.

Suppose the measurements are $X_1, X_2, .., X_n$. Define the variable

$$Z_i = \begin{cases} 1, & \text{if } X_i \leq L \\ 0, & \text{if } X_i > L \end{cases}$$

for $i = 1,2,..,n$. Note that the Z_i represents whether the i-th item is defective or not.

Now suppose the following two criteria are available for accepting or rejecting the lot.

Criteria 1 : Accept the lot if and only if

$$\bar{Z} \leq C \quad \text{and}$$

Criteria 2 : Accept the lot if and only if

$$\bar{X} \leq C^*$$

where C and C* are given constants.

In the first case the criteria is in terms of fraction defective in the sample, so it is an attribute sampling plan. The second one uses the actual measurements on the items in the sample. It is an example of a variable plan.

Now, we shall consider two examples where the use of attribute plan is appropriate or convenient.

Example 1.2 : Suppose that a lot consisting of N glass sheets is given. Here a glass sheet is considered to be defective or non-defective according as it is damaged or not. Since the state of the glass sheet can be described only by an attribute, we use an attribute plan.

Example 1.3 : Now, suppose that a lot of N items is given. Suppose that the quality of an item is in terms of its length and diameter. If any of the measurements deviates from the standard specified, then the item is said to be defective. When there is more than one character to be taken into account, it is convenient to classify the item as defective or good according as any one or more of the measurements lie outside the standards specified or not and use an attribute plan.

Once it is decided to use either an attribute plan or a variable plan, the number of items inspected in the sample to reach a decision about the lot is called the <u>effective sample size</u> and is denote by S.

Usually, S is a random variable. If S is degenerate, the sampling plan is called a single sampling plan. The expected value of S denoted by E(S) is called the <u>average sample number</u> (ASN). This is a function of lot quality θ.

S could be a random variable either because (i) Sampling inspection is curtailed (ii) a multistage sampling plan is used or (iii) a sequential sampling plan is used. In the following section we describe two variable sampling plans to illustrate the cases where S is degenerate and non-degenerate.

1.2 Two examples of sampling plans by variables :

Consider a lot of items such as, Suppose that a measurement of interest on an item in the lot is a random variable X, having distribution $F(x; \eta)$, where η is unknown but it is known that $\eta \in \Theta$, Θ is called the parameter space. Suppose that an item is considered to be defective if its measurement $X \leq L$ (a given number) otherwise good. Then the quality Θ of the lot is measured by the probability of an item being defective, that is

$$\Theta = P_{\eta}[X \leq L]$$
$$= F(L; \eta).$$

Here Θ has two interpretations firstly it is the probability that an item chosen at random from the lot is defective. Secondly, if all the items in the lot are classified as defective or not according as the measurements are $\leq$ or $>$ L ; then Θ is interpreted as proportion defective. In the second case, if lot is considerably 'large' Θ gives a good estimate of $F(L; \eta)$.

If η were known, Θ is known and hence one can decide whether to accept or reject the lot. However, η is unknown, so one has to estimate Θ based on measurements made on a sample of items chosen from the lot.

The following specifications (i) to (iv) described in (1.1) give different variable plans.

1.2.1 Single sampling plan by variables :

Suppose that a sample of n items are chosen at random without replacement from the lot. All the n items are measured. Let $X_1, X_2, \ldots, X_n$ be the measurements. Using $X_1, X_2, \ldots, X_n$ an estimate $\hat{\Theta}$ of $\Theta = F(L;\eta)$ is obtained. If $\hat{\Theta} \leq \Theta_0$ (a given number) then the lot is accepted;

otherwise the lot is rejected. Here $\hat{\theta}$ is a function of $X_1, X_2, .., X_n$ which is not necessarily the fraction defective in the sample. Also the effective sample size is a constant n. Hence this is a single sampling plan by variables.

1.2.2 Double sampling plan or two-stage sampling plan :

Suppose that a lot of size N items is given. Firstly choose n_1 items at random without replacement from the lot. Let $X_1, X_2, \ldots, X_{n_1}$ be the measurements. Using $X_1, X_2, \ldots, X_{n_1}$ on estimate $\hat{\theta}_1$ of $\theta = F(L:\eta)$ is obtained.

If $\hat{\theta}_1 \leq \theta_o'$ (a given number) . Then the lot is accepted. If $\hat{\theta}_1 \geq \theta_o''$ then the lot is rejected, where $\theta_o'' > \theta_o'$ otherwise, that is, if $\theta_o' < \hat{\theta}_1 < \theta_o''$, then choose an additional n_2 items from the lot. Let $X_{n_1+1}, \ldots, X_{n_1+n_2}$ be the measurements on the n_2 items. Using all the n_1+n_2 observations $X_1, X_2, .., X_{n_1}, X_{n_1+1}, X_{n_1+2}, \ldots, X_{n_1+n_2}$ an estimate of $\hat{\theta}_2$ of $\theta = F(L;\eta)$ is obtained.

If $\hat{\theta}_2 \leq \theta_o'''$, then lot is accepted ; otherwise it is rejected, where θ_o''' is a given number such that $\theta_o' \leq \theta_o''' \leq \theta_o''$. The effective sample size is a random variable taking the values either n_1 or n_1+n_2 according as a decision is taken on the first sample or on the first and second samples.

Thus in order, to determine the lot quality based on a sampling inspection of the lot, alternative sampling plans are available.

Even if, one restricts to single sampling plans, either a single sampling plans by attributes or a single sampling plans by variables can be used. In this dissertation only single sampling plans by variables are considered and their performance is compared with appropriate single sampling plans by attributes.

It is to be observed that even when one restricts to single sampling plans by variables, different plans arise depending on the values of the sample size n and critical lot quality θ_0. These two quantities are called the parameters of the single sampling plan by variables. Hence in order to find an appropriate plan, it is necessary to compare the performance of various plans and choose n and θ_0.

In the following section some criteria of comparing different plans is described.

1.3 Operating characteristic function of a sampling plan :

In this section the concepts of operating characteristic (OC) function, Acceptable Quality Level (AQL), Producer's risk, Lot Tolerance Proportion Defective(LTPD) and Consumer's risk are introduced.

Suppose that SP be a fixed sampling plan. The sampling plan SP gives a criteria to accept or to reject the lot. The probability of accepting the lot when the sampling plan SP is used, will be a function of the unknown lot quality θ. This function is useful in comparing different plans.

Definition 1.3.1 : The probability of accepting the lot, which is a function of θ, based on the sampling plan SP, is called the operating characteristic (OC) function of the sampling plan SP. The OC function of the plan is denoted $L_{SP}(\theta)$.

In the ideal case, all lots with proportion defectives $\theta \leq \theta_o$ should have a probability of acceptance $L_{SP}(\theta)=1$; that is all lots $\theta \leq \theta_o$ are accepted, and those lots with proportion defectives $\theta > \theta_o$ should have the probability of acceptance $L_{SP}(\theta) = 0$. The Ideal OC function can be shown graphically as follows

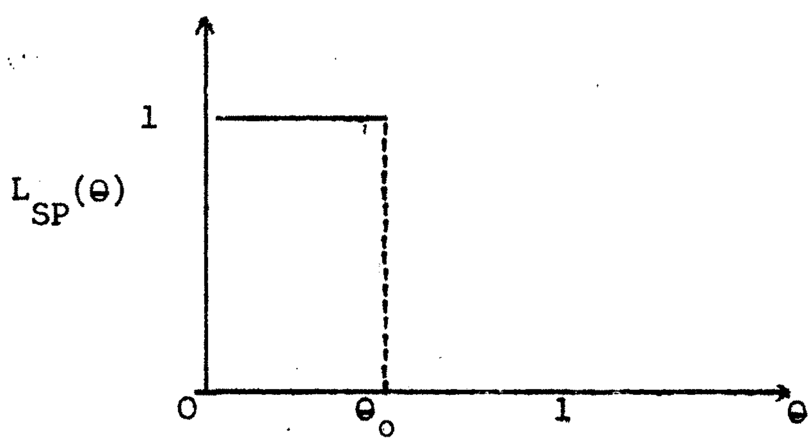

Fig 1 : The Ideal OC-Curve

Ideal OC function can be achieved only by 100 percent perfect inspection. But the 100 percent inspection may not be possible because of the reasons mentioned in section (1.1). Thus an alternative desirable form of the OC curve, one has to make the specifications such as, all lots with proportion defective $\Theta \leq \Theta_1$ should have a probability of acceptance $L_{SP}(\Theta) = 1$, and those lots with proportion defectives $\Theta > \Theta_2$ should have the probability of acceptance $L_{SP}(\Theta) = 0$.

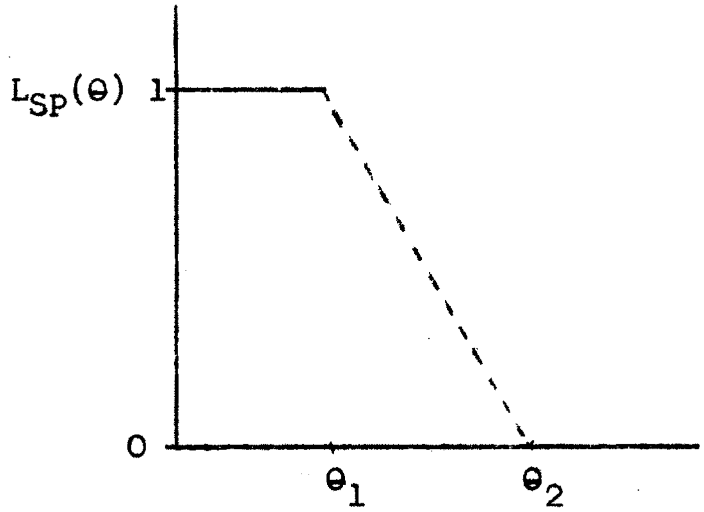

Fig 2 : Modified form of OC curve.

Note that $\Theta_1 < \Theta_2$, the region $(\Theta_1\ \Theta_2)$ is indifference region for consumer and producer. Usually Θ_1 is specified by the consumer and is called the acceptable quality level and Θ_2 by the producer and is called the Lot tolerance proportion defective. Even this alternative form of Ideal OC curve is not possible to achieve without 100 percent perfect inspection. Thus, one may try to find a sampling plan whose OC function is as close to the modified form of the OC function as possible.

The general shape of the OC function corresponding to a sampling plan which is different from 100 percent perfect inspection is as given below :

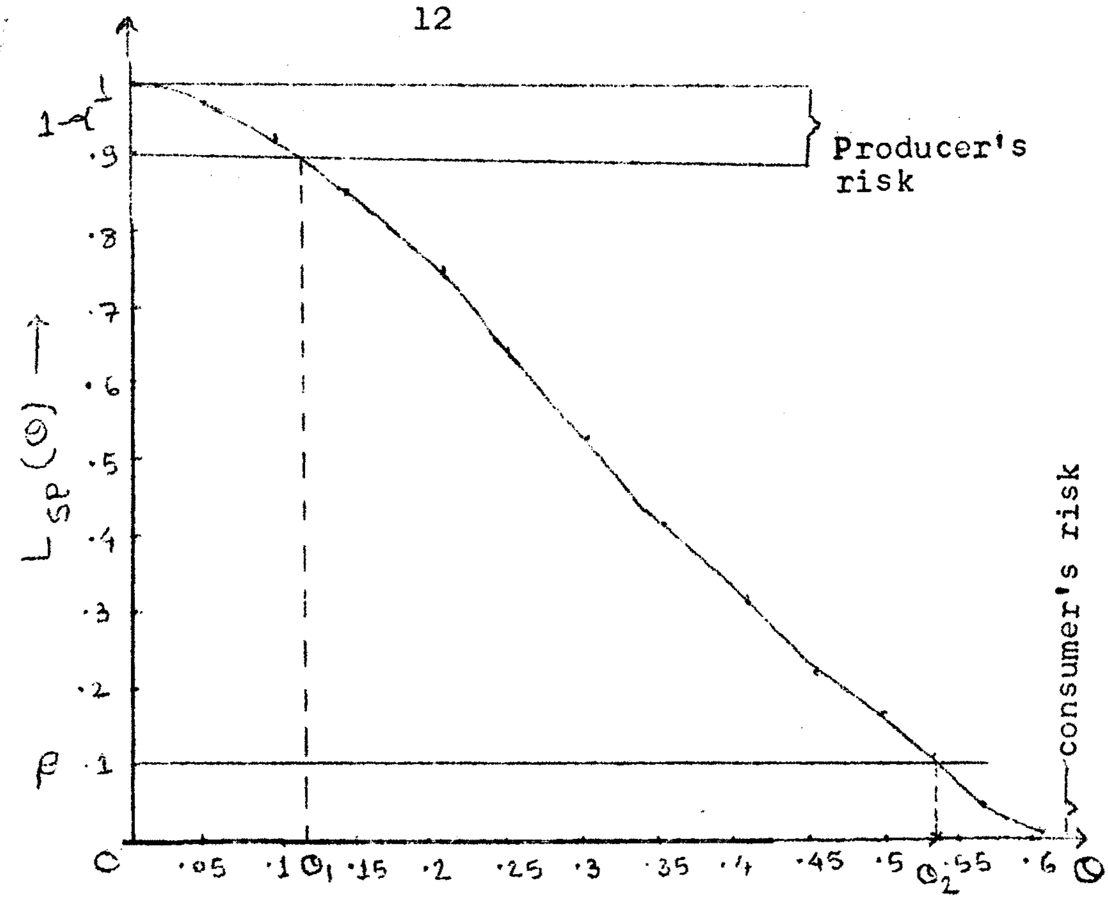

Fig 3 : General shape of OC function.

To choose an appropriate sampling plan whose OC function as close to the one given in fig 2, we should know the values of θ_1 and θ_2,($\theta_1 < \theta_2$) specified by the consumer and producer respectively.

Definition 1.3.2 : The value of the lot quality θ_1 which the consumer considers to be satisfactory is called the Acceptable Quality Level (AQL).

Definition 1.3.3 : The probability of rejecting the lot of AQL quality is called the producer's risk (PR), which is denoted by α. The point (θ_1, 1-α) is called the producer's risk point.

Definition 1.3.4 : The lot quality θ_2 which is considered to be unsatisfactory by the producer and so any lot of quality $\theta \geq \theta_2$ should be rejected is called the lot Tolerance Proportion Defective (LTPD).

Definition 1.3.5 : The probability of accepting a lot of LTPD quality is called the Consumer's risk (CR) and it is denoted by β. The point (θ_2,β) is called the consumer's risk point, we note that $\theta_1 < \theta_2 < .5$. The consumer's and producer's risk points are shown in Fig.3.

1.4 Designing the single sampling plan by attributes :

In this dissertation, only single sampling plans by variables are considered and are compared with the appropriate attribute plan, in the sense that both the plans have OC functions passing through the same producer's risk point $(\theta_1, 1-\alpha)$, and consumers' risk point (θ_2,β). Thus the comparison between variables plans and attribute plans is in terms of the minimum sample size required, for the resulting OC function of the plans passes through the given points.

In this section, a single sampling plan by attributes is described and an approximate method of obtaining the parameters of the plan is given.

1.4.1 The OC function of a single sampling plan by attributes:

Definition 1.4.1 : Suppose that a lot of size N items is given. Take a sample of size n items at random without replacement from the lot. Inspect all items in the sample, if the number of defectives in the sample is less than or equal to c, accept the lot; otherwise reject the lot. Here c is called the rejection number. The quantities n and c are called the parameters of the single sampling attribute plan.

Now, we obtain an approximate expression for the OC function of the above plan with parameters (n, c).

Let N be the lot size and θ be the lot quality, so that $N\theta$ is the number of defectives in the lot.

Let D_n be the number of defective found in the sample of size n.

The probability of accepting a lot of qulaity θ is,

$$L(\theta) = \text{Prob}[\text{ accepting the lot when the number of defectives in the lot are } N\theta\ ]$$

$$= \text{Prob}[\ Dn \leq c \mid \text{when the lot quality is } \theta\ ]$$

$$= \sum_{d=o}^{c} \text{Prob}[\ Dn = d \mid \text{when the lot quality is } \theta\ ]$$

$$= \sum_{d=o}^{c} \binom{N\theta}{d} \binom{N-N\theta}{n-d} \Big/ \binom{N}{n} \qquad (1.4.1)$$

If, N is large the hypergeometric distribution can be approximated by binomial distribution. Duncan (1970). So, (1.4.1) can be written as,

$$L(\theta) \simeq \sum_{k=o}^{c} \binom{n}{k} \theta^k (1-\theta)^{n-k} \qquad (1.4.2)$$

Replacing the binomial probabilities by Poisson probabilities having the same mean, then (1.4.2) can be written as,

$$L(\Theta) \simeq \sum_{k=o}^{c} \frac{e^{-n\Theta}(n\Theta)^k}{k!} \qquad (1.4.3)$$

Now, in view of the following lemma the expression (1.4.3) can be written as,

$$L(\Theta) = \frac{1}{\lceil c+1} \int_{n\Theta}^{\infty} e^{-t}\, t^c\, dt$$

<u>Lemma 1.4.2</u> :

$$\frac{1}{n!} \int_m^{\infty} e^{-t}\, t^n\, dt = \sum_{n=o}^{k} \frac{e^{-m} m^n}{n!}$$

<u>Proof</u> : Let

$$I_n = \frac{1}{n!} \int_m^{\infty} e^{-t}\, t^n\, dt \qquad (1.4.4)$$

By integrating the right hand side by parts, we get

$$= \left[\frac{-e^{-t} t^n}{n!} \right]_m^{\infty} + \frac{n}{n!} \int_m^{\infty} e^{-t}\, t^{n-1}\, dt$$

$$I_n = \frac{e^{-m}\, m^n}{n!} + I_{n-1}$$

Ther-ofre,

$$I_n - I_{n-1} = \frac{e^{-m}\, m^n}{n!} \quad \text{for all } n,$$

Hence

$$\sum_{n=1}^{k} (I_n - I_{n-1}) = \sum_{n=1}^{k} \frac{e^{-m}\, m^n}{n!}$$

$$I_k = I_o + \sum_{n=1}^{k} \frac{e^{-m} m^n}{n!} \quad \text{, since } I_o = e^{-m} \text{ (by 1.4.4)}$$

and hence that

$$L(\Theta) = P[G_{c+1} \geq n\Theta] \qquad (1.4.5)$$

where G_l is a random variable having gamma distribution with parameter l and scale parameter 1.

It is known that 2 G_l has a chi-square distribution with 2l degrees of freedom (d.f.). Hence (1.4.5) can be written as,

$$L(\theta) = P[\chi^2_{2(c+1)} \geq 2n\theta] \tag{1.4.6}$$

where χ^2_n is a chi-square variate with n d.f.

1.4.2 Determination of the Parameters n and c :

We use equation (1.4.6) to find the parameters n and c of the single sampling plan by attributes, so that the resulting plan has OC function passing through the producers' risk point $(\theta_1, 1-\alpha)$ and consumer's risk point (θ_2, β). Since in equation (1.4.2) n and c are integers, we determine n and c such that

$$\sum_{k=o}^{c} \binom{n}{k} \theta_1^k (1 - \theta_1)^{n-k} \geq 1-\alpha \tag{1.4.7}$$

and

$$\sum_{k=o}^{c} \binom{n}{k} \theta_2^k (1 - \theta_2)^{n-k} \leq \beta \tag{1.4.8}$$

The n and c which satisfy the constraints (1.4.7) and (1.4.8) assume the producer 's risk of at least 1-α and consumer's risk of at most β. From equation (1.4.6) the inequalities (1.4.7) and (1.4.8) can be written as

$$P[\chi^2_{2(c+1)} \geq 2\, n\theta_1] \geq 1-\alpha \tag{1.4.9}$$

and

$$P[\chi^2_{2(c+1} \geq 2n\theta_2] \leq \beta \qquad (1.4.10)$$

Let $\chi^2_{2(c+1),p}$ denote the lower p-th quantile of chi-square distribution with 2(c+1) d.f.

That is

$$P[\chi^2_{2(c+1)} \leq \chi^2_{2(c+1),p}] = p \qquad (1.4.11)$$

In view of (1.4.11) inequalities (1.4.9) and (1.4.10) imply that

$$P[\chi^2_{2(c+1)} \geq 2n\theta_1] \geq P[\chi^2_{2(c+1)} > \chi^2_{2(c+1),\alpha}] \qquad (1.4.12)$$

and

$$P[\chi^2_{2(c+1)} \geq 2n\theta_2] \leq P[\chi^2_{2(c+1)} > \chi^2_{2(c+1),\beta}] \qquad (1.4.13)$$

so that

$$2n\theta_1 \leq \chi^2_{2(c+1),\alpha} \qquad (1.4.14)$$

$$2n\theta_2 \geq \chi^2_{2(c+1),\,1-\beta} \qquad (1.4.15)$$

Taking the ratio of (1.4.14) and (1.4.15) we get that

$$\frac{2n\theta_2}{2n\theta_1} \geq \frac{\chi^2_{2(c+1),1-\beta}}{\chi^2_{2(c+1),\alpha}}$$

That is,

$$\frac{\theta_2}{\theta_1} = \frac{\chi^2_{2(c+1),1-\beta}}{\chi^2_{2(c+1),\alpha}}$$

Let

$$r(c) = \frac{\chi^2_{2(c+1),\,1-\beta}}{\chi^2_{2(c+1),\,\alpha}} \qquad (1.4.16)$$

From equation (1.4.16) it is observed that for various values of α and β such that $1-\beta > \alpha$, $r(c)$ is a decreasing function of c. This result is clear from Tables I to IV. We also tried to prove this result analytically. But we could not prove the result. Since θ_2/θ_1 is given, we choose c such that

$$r(c-1) > \theta_2/\theta_1 \geq r(c) \qquad (1.4.17)$$

The ratio (1.4.17) is tabulated in Cameron (1952) for some chosen values of c, α and β. These tables may be used in reverse to find c for the desired values of α and β.

Having determined c, n can be found from (1.4.14) and (1.4.15) which give the condition that

$$\frac{\chi^2_{2(c+1),1-\beta}}{2\,\theta_2} \leq n \leq \frac{\chi^2_{2(c+1),\alpha}}{2\,\theta_1} \qquad (1.4.18)$$

If there is no n satisfying the constraint (1.4.18), then increase the value of c untill such a n can be found.

Wetherill (1977) has given tables of c and r(c) for various α equal to 0.1, 0.05, 0.25, 0.01 and $1-\beta$ equal to 0.9, 0.95, 0.975 and 0.99. The same table is reproduced here as table I-IV together with the values of c and r(c) for the above values of α and $1-\beta = .995$. The method of computation is illustrated below.

Suppose that $\alpha = .01$ and $1-\beta = .995$. Then for fixed c, r(c) is computed using the cumulative probability points for the chi-square distribution with 2(c+1) degrees of freedom given by Burr (1976). The procedure is as follows :

Take c = 0. Then using the chi-square distribution with 2 d.f. and table II of Burr (19[illegible]), we get

$$r(0) = \frac{10.597}{.211}$$

$$= 50.2227.$$

Similarly we compute r(c) for different values of c. These values for c = 1,2,.....20, are given in table I to IV for various α and $1-\beta$.

TABLE I

Values of $r(c) = \chi^2_{1-\beta} / \chi^2_{\alpha}$ with d.f. = 2(c+1)

α = .1	1 - β				
c	0.900	0.950	0.975	0.990	0.995
0	21.85	28.43	35.01	43.71	50.22
1	7.31	8.92	10.48	12.48	13.96
2	4.83	5.71	5.02	7.63	8.41
3	3.83	4.44	5.02	5.76	6.29
4	3.29	3.76	4.21	4.77	5.17
5	2.94	3.34	3.70	4.16	4.48
6	2.70	3.04	3.35	3.74	4.02
7.	2.53	2.82	3.10	3.44	3.67
8	2.39	2.66	2.90	3.20	3.41
9	2.28	2.52	2.75	3.02	3.21
10	2.19	2.42	2.62	2.87	3.04
11	2.12	2.33	2.51	2.74	2.90
12	2.06	2.25	2.42	2.64	2.79
13	2.00	2.18	2.35	2.55	2.69
14	1.95	2.12	2.28	2.47	2.60
15	1.91	2.07	2.22	2.40	2.52
16	1.87	2.03	2.17	2.34	2.46
17	1.84	1.99	2.12	2.29	2.40
18	1.81	1.95	2.08	2.24	2.34
19	1.78	1.92	2.04	2.19	2.29
20	1.76	1.89	2.01	2.15	2.25

TABLE II

Values of r(c) = $\chi^2_{1-\beta} / \chi^2_{\alpha}$ with d.f.= 2(c+1)

α=0.05	$1 - \beta$				
c	0.900	0.950	0.975	0.990	0.995
0	44.89	58.40	71.97	89.78	102.88
1	10.95	13.35	15.68	18.68	20.90
2	6.51	7.70	8.84	10.28	11.34
3	4.89	5.67	6.42	7.35	8.03
4	4.06	4.65	5.20	5.89	6.39
5	3.55	4.02	4.47	5.02	5.41
6	3.21	3.60	3.98	4.44	4.76
7	2.96	3.30	3.62	4.02	4.30
8	2.77	3.07	3.36	3.71	3.95
9	2.62	2.89	3.15	3.46	3.68
10	2.50	2.75	2.98	3.27	3.46
11	2.40	2.63	2.84	3.10	3.28
12	2.31	2.53	2.73	2.97	3.13
13	2.24	2.44	2.63	2.85	3.01
14	2.18	2.37	2.54	2.75	2.90
15	2.12	2.30	2.47	2.66	2.80
16	2.07	2.24	2.40	2.59	2.72
17	2.03	2.19	2.34	2.52	2.64
18	1.99	2.15	2.29	2.46	2.57
19	1.95	2.10	2.24	2.40	2.51
20	1.92	2.07	2.20	2.35	2.46

TABLE III

Values of $r(c) = \chi^2_{1-\beta} / \chi^2_{\alpha}$ with d.f. = 2(c+1)

α=0.025	$1 - \beta$				
c	0.900	0.950	0.975	0.990	0.995
0	90.95	118.33	145.70	181.89	207.78
1	16.06	19.59	23.00	27.41	30.70
2	8.60	10.18	11.68	13.59	14.99
3	6.13	7.11	8.04	9.22	10.07
4	4.92	5.64	6.31	7.15	7.57
5	4.21	4.77	5.30	5.95	6.42
6	3.74	4.21	4.64	5.18	5.56
7	3.41	3.81	4.18	4.63	4.96
8	3.16	3.51	3.83	4.23	4.51
9	2.96	3.28	3.56	3.92	4.17
10	2.81	3.09	3.35	3.67	3.89
11	2.68	2.69	3.17	3.47	3.67
12.	2.57	2.81	3.03	3.30	3.48
13	2.48	2.70	2.90	3.15	3.33
14	2.40	2.61	2.80	3.03	3.19
15	2.33	2.53	2.71	2.92	3.07
16	2.27	2.46	2.62	2.83	2.97
17	2.21	2.39	2.55	2.74	2.88
18	2.16	2.33	2.49	2.67	2.80
19	2.12	2.28	2.43	2.61	2.73
20	2.08	2.24	2.38	2.55	2.66

TABLE IV

Values of $r(c) = \chi^2_{1-\beta} / \chi^2_{\alpha}$ with d.f. = 2(c+1).

α=0.015	1 - β				
c	0.900	0.950	0.975	0.990	0.995
0	229.10	298.07	367.04	458.24	529.85
1	26.18	31.93	37.51	44.69	50.03
2	12.21	14.44	16.57	19.28	21.27
3	8.12	9.42	10.65	12.20	13.33
4	6.25	7.16	8.01	9.07	9.84
5	5.20	5.89	6.54	7.34	7.92
6	4.52	5.08	5.60	6.25	6.72
7	4.05	4.52	4.96	5.51	5.89
8	3.70	4.12	4.49	4.96	5.29
9	3.44	3.80	4.14	4.55	4.84
10	3.23	3.56	3.85	4.22	4.48
11	3.06	3.35	3.63	3.96	4.19
12	2.92	3.19	3.44	3.74	3.95
13	2.80	3.05	3.28	3.56	3.75
14	2.69	2.93	3.14	3.40	3.58
15	2.60	2.82	3.02	3.27	3.44
16	2.52	2.73	2.29	3.15	3.31
17	2.45	2.65	2.83	3.05	3.20
18	2.39	2.58	2.75	2.96	3.10
19	2.34	2.52	2.68	2.87	3.01
20	2.29	2.46	2.61	2.80	2.93

The use of the Table I to IV in determining (n,c) for given α, β, θ_1, θ_2 is illustrated below.

Example 4.2.1 :

Suppose the parameter values are ($\theta_1 = 0.01$, $\alpha = 0.05$), ($\theta_2 = 0.04$, $\beta = 0.05$). Then $\theta_2/\theta_1 = 4$ from table II, we find that c = 6. Using (1.4.17), we get

$$4.02 > 4 > 3.60$$

so that c = 6.

1.5 Single sampling plan by variables in case of uniform Distribution :

In this section a single sampling plan by variables is obtained when the measurements of the items in the lot follow a uniform distribution. The OC function of the plan is computed and the parameters of the plan are determined. The sample size required is compared with the attribute plan. (This example is given here, because of it's simplicity). The assumptions are as follows :

i) A large lot is presented for acceptance or rejection.

ii) Lower specification limit $L>0$ is specified.

iii) An item is considered to be defective if the measurement X on the item is $\leq L$; otherwise the item is considered to be good. Thus L is a lower specification limit.

iv) X is a random variable having uniform distribu-

-tion on $(0, \sigma)$, where $\sigma > 0$.

Under the above assumptions, the lot quality Θ, which is the probability of an item being defective is given by,

$$\Theta = P\ [\ X \leq L\ ]$$

$$= \begin{cases} 0, & \text{if } L < 0 \\ L/\sigma & \text{if } 0 \leq L \leq \sigma \\ 1 & \text{if } L > \sigma \end{cases}$$

That is

$$\Theta = \min\ [\ L/\sigma\ ,\ 1\ ] \qquad (1.5.1)$$

From the above, it is clear that there is a one-to-one correspondence between Θ and σ and Θ is a decreasing function of σ. So we treat σ itself as lot quality instead of Θ and write the OC function as a function of σ. Consider the following.

Take a sample of n items at random from the lot. Measure all the n items. Suppose that $X_1, X_2, \ldots, X_n$ be the measurements.

We obtain any apprpriate estimater of σ in order to determine the lot quality. We can take the estimater $\hat{\sigma}$ to be either the maximum likelihood estimater (MLE) $X_{(n)} = \max\ (X_1, X_2, \ldots, X_n)$ or the minimum variance unbiased estimater (MVUE) $n^{-1}(n+1)\ X_{(n)}$. In the following we take $\hat{\sigma} = X_{(n)}$.

Let

$$\hat{\theta} = \min [L/X_{(n)}, 1] \quad (1.5.2)$$

The criteria of accepting or rejecting the lot is as follows.

If $\hat{\theta} \leq \theta_o$ accept the lot ; otherwise reject the lot. Since $\theta_o < 1$, we express this criteria equivalently in terms of $\hat{\sigma}$ itself as: Accept the lot iff,

$$X_{(n)} \theta_o \geq L . \quad (1.5.3)$$

From (1.5.3) the OC function of the plan is

$$\begin{aligned} L(\sigma) &= P_\sigma \text{ [Accepting the lot]} \\ &= P_\sigma [X_{(n)} \geq L/\theta_o] \\ &= P_1 [X_{(n)}/\sigma \geq L/\theta_o \sigma] \end{aligned}$$

because, if $X_1, X_2, \ldots, X_{(n)}$ are i.i.d. uniform $(0, \sigma)$. Then X_1/σ , X_2/σ ,...,X_n/σ is i.i.d uniform on (0,1). Then

$$L(\sigma) = 1 - \left(\frac{L}{\theta_o \sigma} \right)^n, \quad \text{if } 0 < L \leq \theta_o \sigma \quad (1.5.4)$$

$$= 0 \qquad , \quad \text{otherwise}$$

Using equation (1.5.4) we find n and θ_o, so that the resulting plan has OC function passing through the producer's risk point$(\theta_1, 1-\alpha)$ and consumers' risk point (θ_2, β). We have the two equations.

$$\begin{aligned} L(\sigma_1) &= 1 - \left(\frac{L}{\theta_o \sigma_1} \right)^n \\ &= 1-\alpha \end{aligned} \quad (1.5.5)$$

and

$$\begin{aligned} L(\sigma_2) &= 1 - \left(\frac{L}{\theta_o \sigma_2} \right)^n \\ &= \beta \end{aligned} \quad (1.5.6)$$

Equations (1.5.5) and (1.5.6) can be written as

$$\theta_o = \frac{L}{\sigma_1 \alpha^{1/n}} \qquad (1.5.7)$$

and

$$\theta_o = \frac{L}{\sigma_2 (1-\beta)^{1/n}} \qquad (1.5.8)$$

From (1.5.7) and (1.5.8) we get,

$$\frac{\sigma_2}{\sigma_1} = (\alpha/1-\beta)^{1/n}$$

Here $\theta = \theta_1$ correspond to $\sigma = \sigma_1$ and $\theta = \theta_2$ correspond to $\sigma = \sigma_2$ so that $\theta_1 < \theta_2$ implies that $\sigma_1 > \sigma_2$.

Hence,

$$n = \left[\frac{\log(\alpha/1-\beta)}{\log(\sigma_2/\sigma_1)} \right] + 1 \qquad (1.5.9)$$

where $[x]$ is the largest integer $\leq x$.

Having determined n by(1.5.9), θ_o can be found by substituting the values of n either in equation (1.5.7) or (1.5.8). It is found from (1.5.7) that the resulting OC function of the plan passes through the producer's risk point $(\theta_1, 1-\alpha)$ and if it is found from (1.5.8) it passes through the consumer's risk point (θ_2, β).

Th minimum sample size n required for the variable plan given in (1.5.9) is compared with the minimal sample size required for the attribute plan. For this purpose, the same producer's risk and consumer's risk points are used.

Example 1.5.1 :

Suppose that, the following quantities are given,
$\alpha = .05$, $\beta = .05$, $\theta_1 = .01$ and $\theta_2 = .15$

Sample size for attribute plan :.

Using (1.4.17) and table II, we find,

$$r(0) > \theta_2/\theta_1 > r(1)$$

where $r(0) = 58.40$

$r(1) = 13.35$

so that $c = 1$

Then from (1.4.18) we get n such that

$$\frac{9.49}{.30} \leq n \leq \frac{.711}{.02}$$

$$31.67 \leq n \leq 35.55$$

The four integers 32, 33, 34 and 35 satisfy the above inequality. We can take n to be any of them. But we choose the minimum value 32 as n. Hence the desired sampling plan is $n = 32, c = 1$.

Now, we consider the case, that the inequality (1.4.18) can not be satisfied.

Suppose that $\alpha = .05$, $\beta = .05, \theta_1 = .01$ and $\theta_2 = .0567$. Using the inequality (1.4.17) and table II, we get,

$$r(2) > \theta_2/\theta_1 > r(3)$$

where $r(2) = 7.70$

$r(3) = 5.67.$

so that $c = 3$.

Then from (1.4.18) we get,

$$\frac{15.51}{.1134} \leq n \leq \frac{2.73}{.02}$$

That is,

$$136.772 \leq n \leq 136.5$$

The inequality (1.4.18) can not be satisfied for n. Then choose c such that the inequality (1.4.18) can be satisfied.

Sample size for variable plan :

For the variable plan n can be computed as follows.

Suppose that $\alpha = .05$, $\beta = .05$, $\sigma_1 = 100$, and $\sigma_2 = 7.4906$. Then using (1.5.9) we get,

$$n = 1.129701 + 1$$

that is

$$n = 2$$

θ_1^* and θ_2^* can be computed by using (1.5.7) and (1.5.8) respectively.

That is

$$\theta_1^* = .044721$$

and

$$\theta_2^* = .1390189.$$

Taking the average of θ_1^* and θ_2^* we will get θ_o.

TABLE VII

$\alpha = .1, \quad \beta = .1, \quad \theta_1 = .01$

θ_2	Attribute plan parameters		2	Variable paln parameters	
	n	c		n	θ_o
.0383	174	3	26.10	2	.03595
.0329	243	4	30.39	2	.0331
.0253	465	7	39.52	3	.02155
.0206	863	12	48.54	4	.0191
.0176	1536	20	56.81	4	.01785

TABLE VIII

$\alpha = .01, \beta = .05, \theta_1 = .01$

θ_2	Attribute plan parameters		2	Variable plan parameters	
	n	c		n	θ_o
.0942	82	3	10.61	3	.0707
.0716	127	4	13.96	3	.059605
.0412	350	8	24.27	4	.0358
.0356	476	10	28.08	4	.03315
.0319	609	12	31.34	4	.03155
.0305	677	13	32.78	5	.0276
.02[illegible]	746	14	34.12	5	.027005
.0258	1034	18	38.75	5	.02532
.0252	1106	19	39.68	5	.025005
.0246	1181	20	40.65	6	.02485

TABLE V

$\alpha = .05,\ \beta = .05,\ \theta_1 = .01$

θ_2	Attribute plan parameters		2	Variable plan parameters	
	n	c		n	θ_o
.1335	35	1	7.49	2	.0915
.077	47	2	12.98	2	.061
.0465	197	4	21.50	2	.042
.0275	616	10	36.36	3	.0275
.0263	692	11	38.02	4	.02387
.0253	768	12	39.52	4	.02372
.0237	923	14	42.19	4	.022570
.0215	1241	18	46.51	4	.02120
.0207	1403	20	48.30	5	.01805

TABLE VI

$\alpha = .1,\ \beta = .05,\ \theta_1 = .01$

θ_2	Attribute plan parameters		2	Variable plan parameters.	
	n	c		n	θ_o
.0376	243	4	26.59	2	.03475
.0334	314	5	29.94	2	.0327
.0242	700	10	41.32	3	.02307
.0225	864	12	44.44	4	.02221
.0189	1537	20	52.91	4	.0188

1.6 SUMMARY OF THE DISSERTATION:

The summary of the first chapter is given in the introduction. In chapter two the acceptance sampling by variables for normal distribution is considered. In section 2.2, variable plan when mean is unknown and variance is known with lower and upper specification limit is done for finding the parameters n and k. These parameters are compared with attribute plan parameters in tables I to IV. In section 2.3 the case of mean is known and variance is unknown with lower and upper specification limit for finding the parameters n and k is considered. In tables V to VIII the variable plan parameters are compared with attribute plan parameters. In section 2.4 both mean and variance are unknown is considered for lower and upper specification limit to find n and k. Here two methods are given for finding n and k, one is approximate method and second is exact method. In tables IX to XII the parameters of approximate distribution is compared with the parameters of exact distribution.

Chapter 3 is devoted for exponential distribution with one parameter case. In section 3.2 lower and upper specification limit is considered for finding the parameters n and k. In tables I to IV the variable plan parameters are compared with attribute plan parameters.