# Chapter 0

## INTRODUCTION AND SUMMARY

### 0.1 Introduction:

The most common way of comparing two random variables or two distribution functions is by the comparison of associated means. However, such comparison is based on only two single numbers i.e. means, and therefore it is often not very informative, the means sometimes do not exist. When one wish to compare two distribution functions that have the same mean, one is usually interested in the comparison of the dispersion of these distributions. The simplest way of doing it is by the comparison of the associated standard deviation. Again this comparison is based on two single numbers and therefore it is often not very informative, the standard deviation sometimes does not exist. Suppose that one wants to compare two brands of electric bulbs with regard to their lifetimes. Some bulbs of the first brand may be having lifetime smaller than the lifetime of the second brand. The problem is how to compare the lifetimes of bulbs of these two brands?

In most cases one can express various forms of knowledge about the underlying distributions in terms of their survival function, hazard rate function etc and other suitable functions of probability distributions. These methods are much more informative than those based only on few numerical characteristics of distributions. Comparisons of random variables based on such functions usually establish partial orders among them. We call them a Stochastic Order. Generally speaking, stochastic order means a partial order among random variables (or their distributions) defined in a suitable way. The concept of usual stochastic order conveys the information about one random variable or distribution dominates the other in a natural sense. In many situations, such an ordering or domination is intuitively obvious or can be argued on a rational basis. For example it is generally agreed that women live longer than men do.

## 0.2 Importance:

In the recent years, a variety of stochastic orders have been applied to various fields such as Queue theory, Inference, Reliability, Bio-statistics, Economics etc. Stochastic ordering concept is so important that it is highly desirable to have powerful statistical tests available for testing stochastic ordering as null/alternative hypothesis. Although non-parametric tests, such as the one sided Mann-Whitney-Wilcoxon and Kolmogorov-Smirnov tests are frequently used to reject equality of distributions in favour of stochastic ordering. These tests are quite general in scope and are lacking in strong power characteristics.

## 0.3 Literature Survey:

Hardy, Littlewood and Polya published stochastic orders in 1934 in their well-known monograph on inequalities. They introduced a majorization relationship between two non-negative vectors. Lehman (1955) introduced the concept of stochastic order. He defined a random variable X with distribution function F to be stochastically smaller than a random variable Y with distribution function G, when $F_X(x) \geq G_Y(x)$, for all x. Bagai and Kochar (1986) have studied some implications between dispersive ordering and hazard rate ordering of two probability distribution functions. Bartoszewicz (1986) has discussed some results related to the dispersive ordering and the total time on test transformation.

Dykstra, Kochar and Robertson (1991) have derived some useful applications of stochastic ordering. They have shown that uniform stochastic ordering is quite tractable in matters of statistical inference. Block, Langberg, Savits (1993) have compared the number of repairs for various repair replacement policies. That is, if a repair replacement policy is considered for two components whose lifetimes are stochastically ordered showing the intuitive result that using the component with the longer life time leads to fewer. In Bio-statistics, the concept 'Stochastic ordering of Epidemics' has been discussed by Lef'evre and Picard (1993) and others. Their study was devoted to the analysis of the spread of infectious diseases. Boland, Neweihi and Proschan (1994) have given comparison of hazard rate functions for k-out of-n systems and other. A new hazard rate ordering of vectors of discrete dependent random lifetimes has been introduced and illustrated by Shaked, Shanthikumar and Valdez-Torres (1994). Kochar (1996) has given some results of dispersive orderings of order statistics. Bartoszewicz (1997) has discussed some results related to dispersive functions and stochastic orders. Stochastic orders and inequalities are very useful tool in various areas like Economics and Finance. Kijima and Ohnishi (1999) have discussed the main results obtained by using the idea of stochastic orders in financial optimization. Khaledi and Kochar (1999) have established some results of stochastic ordering between distributions and their sample spacings. Khaledi and Kochar (2000a) have discussed few results on dispersive ordering among order statistics in one-sample and two-sample problems.

Nanda and Shaked (2001) have given some interesting results related to hazard and reversed hazard rate orders with application to order statistics. They have pointed out a simple observation that can be used successfully in order to translate results about the uniform stochastic order (hazard rate order) into results about the reversed hazard rate order. Nanda and Gupta (2001) have established some results with respect to reversed hazard rate (RHR) ordering between the exponentiated random variables. They have addressed some properties of order statistics under reversed hazard rate ordering. Lillo, Nanda and Shaked (2001) have discussed some interesting results regarding likelihood ratio orders by order statistics. Kochar and Khaledi (2002) have explained some results related to stochastic ordering among order statistics and sample spacings. Bartoszewicz and Skolimowska (2004) have discussed some results related to stochastic orderings for weighted distributions. They have represented weighted distribution by the Lorenz Curve to obtain some results concerning their relations with life distributions. Preservation of increasing failure rate average (IFRA) and decreasing failure rate average (DFRA) and also new better than used (NBU), new worse than used (NWU) classes of life distributions under weighting has been discussed by Bartoszewicz and Skolimowska (2004). Barmi and Mukarjee (2004) have studied the Inference under a stochastic ordering constraint the K-Sample case. Bartoszewicz (2005) has given some comparative results of dispersive ordering between order statistics and spacings from an increasing reversed failure rate (IRFR) distribution. Ahmad, Kayid and Li (2005) have studied the new better than used in the total time on test transform order (NBUT) class of life distributions and discussed comparison of lifetime of a component with its residual life at different ages. Cheng and Zhou (2005) have derived some applications of stochastic orders to actuarial science.

## 0.4 Chapter-wise Summary:

The chapter 1 contains various stochastic orders like, usual stochastic order, hazard rate order, reversed hazard rate order, likelihood ratio order, mean residual life order and dispersive order. Section 1 of this chapter is introductory, which gives the meaning and scope of stochastic orderings. Section 2 is devoted to the usual stochastic order and some basic results. In this section, we have defined the usual stochastic order and discussed various implications of the same. Some examples to illustrate the usual stochastic ordering have also been given. Relation between stochastic ordering and ordering between the means is discussed. A counter example in which ordering in means does not imply stochastic ordering is also provided. The hazard rate order or uniform stochastic order is stronger than usual stochastic order, which we have introduced in section 3. The continuous and discrete versions of hazard and reversed hazard rate order with example are also included in this section. Section 4 is devoted to the some other stochastic orders. This section includes the definitions of likelihood ratio order, mean residual life order and dispersive order with example. The likelihood ratio order is the strongest stochastic order among all stochastic orders. The relationship among various stochastic orders is shown by implication in section 5 of this chapter. The proofs of some essential theorems are given to show the relationship among stochastic orders.

Chapter 2 contains some results on stochastic orderings. Section 1 of this chapter is introductory. Section 2 begins with some important and essential definitions like increasing failure rate (IFR), decreasing failure rate (DFR) distributions and their examples. Some results related to the IFR and DFR distribution have been proved, which are helpful for deriving further results given in present and next chapter. Section 3 contains stochastic comparisons of order statistics in one-sample problem. In this section, we discuss some results related to the order statistics in one-sample problem. The most of the results are related to likelihood ratio ordering between order statistics. The hazard rate ordering is stronger than usual stochastic ordering. Section 4 is devoted to the hazard rate ordering. We discuss hazard rate ordering is preserved under series system but not under parallel system. In section 5 we discuss the comparisons of hazard rate functions for k-out-of-n systems. The principle result of this application is the uniform stochastic or hazard rate ordering for the lifetime of a k-out-of-n system with i.i.d. component lifetime distribution related to preservation of hazard rate ordering. These results are due to Boland, Neweihi and Proschan (1994).

Chapter 3 is devoted to the tolerance limits for the class of IFR distributions. In this chapter we study application of stochastic ordering to obtain tolerance limits for the class of an IFR distribution. Section 1 of this chapter is introductory. Section 2 contains some results related to star-shaped function, convex function and IFR/ DFR distribution. Lower tolerance limits for an

IFR distribution is discussed in section 3. Section 4 is devoted to the application of lower tolerance limits for Weibull and Exponentiated exponential distribution.

Chapter 4 is devoted to the selection of better component in bivariate lifetime models. Section 1 of this chapter is introductory. The criterion of betterness is studied with respect to the mean life of components. We can compare two random variables by some other criterion. We discuss some other criterion and compare these criteria through the probability of correct selection for some bivariate models. In section 2 the procedure of selecting better component has been discussed by applying procedure of mean and stochastic orders. Section 3 is devoted to the procedure based on counts for better selection of component. Procedure based on samp.e mean of selecting better component is discussed in section 4. Section 5 of this chapter is devoted to procedure based on maximum likelihood estimators of selecting better component. Asymptotic relative efficiency (ARE) is discussed in the last section. At the end of the chapter 4, we discuss areas for future research.