APPENDIX-1

sometimes, expressions of deviance functions can be simplified. The following theorem is useful for this purpose.

Theorem : In generalised linear models with constant terms, if the link function is either

$$\underline{T} = (\ln(\mu_1), \ln(\mu_2), \ldots, \ln(\mu_n))',$$

or

$$\underline{T} = (\mu_1^{\gamma}, \mu_2^{\gamma}, \ldots, \mu_n^{\gamma})',$$

then

$$\sum_i \left\{ \frac{(y_i - \hat{\mu}_i)\hat{\mu}_i}{V_{ii}(\hat{\mu}_i)} \right\} = 0,$$

where the notations have their usual meaning.

Proof :

Case (a) : Suppose the link function is,

$$\underline{T} = (\ln(\mu_1), \ln(\mu_2), \ldots, \ln(\mu_n))'.$$

Therefore,

$$dT_i/d\mu_i = \mu_i^{-1}, \text{ for } i=1,2,\ldots,n. \qquad (1)$$

We know. $\hat{\mu}_i$ and $V_{ii}(\hat{\mu}_i)$ $(i=1,2,\ldots,n)$ are maximum likelihood estimates of the corresponding parameters, these are the solutions of estimating equations,

$$\partial l/\partial \beta_j = 0, \text{ for } j=0,1,\ldots,k.$$

Thus,

$$\left[\frac{\partial l}{\partial \beta_j}\right]_{\underline{\mu}=\hat{\underline{\mu}}} = 0 \;;\; \text{for } j=0,1,\ldots,k. \qquad (2)$$

Hence, for j=0, equation (2) along with the equation (2.4-22) gives,

$$\alpha(\phi) \sum_i \frac{(y_i - \hat{\mu}_i)\hat{\mu}_i}{V_{ii}(\hat{\mu}_i)} = 0. \tag{3}$$

This completes the proof of case (a).

<u>Case (b)</u> : Here the link function is,

$$\underline{T} = (\mu_1^{\gamma} \quad \mu_2^{\gamma} \qquad \mu_n^{\gamma})'.$$

Hence,

$$dT_i / d\mu_i = \gamma\ \mu_i^{(\gamma-1)}, \text{ for } i=1,2,\ldots,n. \tag{4}$$

Therefore, equation (4) along with equation (2.4-22) gives,

$$\sum_i \left\{ \frac{(y_i - \hat{\mu}_i)}{V_{ii}(\hat{\mu}_i)} \left[\hat{\mu}_i^{(\gamma-1)}\right]^{-1} x_{ij} \right\} = 0,$$

which imply,

$$\sum_i \left[\frac{(y_i - \hat{\mu}_i)\hat{\mu}_i}{V_{ii}(\mu_i)(\hat{T}_i)} \right] = 0. \tag{5}$$

Consider,

$$\sum_i \left[\frac{(y_i - \hat{\mu}_i)\hat{\mu}_i}{V_{ii}(\hat{\mu}_i)} \right] = \sum_i \left[\frac{\hat{T}_i (y_i - \hat{\mu}_i)\hat{\mu}_i}{V_{ii}(\hat{\mu}_i)(\hat{T}_i)} \right]$$

$$= \sum_j \beta_j \left[\sum_i \left\{ \frac{(y_i - \hat{\mu}_i)\hat{\mu}_i}{V_{ii}(\hat{\mu}_i)} x_{ij} \right\} \right]. \tag{6}$$

Equations (5) and (6) combinedly completes the proof of case (b).