CHAPTER - 0

INTRODUCTION AND SUMMARY

1. INTRODUCTION.

In reliability models the life time distributions where no aging is present are important. In univariate case, the situation were no aging is present corresponds to the exponential distribution. The exponential distribution is the most fundamental distribution in reliability theory. The probability properties of the exponential distribution are useful in probabilistic reliability models, and in statistical applications. A property of the exponential distribution which makes it especially important in reliability theory and applications is that the remaining life of a used component is independent of its initial age. This property is called as "the memory less" property or "the lack of memory" (LMP) property. Formally a random variable X is said to have LMP if,

$P(X > s+t \mid x > t) = P(X > s)$, $s,t \geq 0$, or equivalently if

$S(s+t) = S(s)\,S(t)$, $s,t \geq 0$, where $S(t) = P(X > t)$. Due to this property of the exponential distribution, it forms a boundary of some classes of distributions based on aging properties such as IFR & DFR, IFRA & DFRA etc. Thus lack of memory property (LMP) is a

full forms ?

fundamental concept in reliability theory.

In many situations the component lifetimes will be dependent and will have some joint probability distribution. For example the failure times of two or more identical engines or hydraulic systems in an airplane, failure times of the components working parallel in a system of many engineering industries, failure of electricity distribution systems, failure of nuclear power station etc. In order to study such situations it is important to extend univariate LMP to higher dimensions.

We have discussed possible extensions of LMP for bivariate case and multivariate case. In particular a weaker form of bivariate LMP and multivarite LMP proposed by Marshall and Olkin (1967) is discussed in detail. Hencefourth we refer to this version of bivariate LMP as bivariate lack of memory property (BLMP) and its multivariate extension as multivariate lack of memory property (MLMP). It is seen that the only distribution possessing BLMP with
Bivariate Exponential
exponential marginals is BVE given by Marshall and Olkin (1967).

9 abbreviat.

We also study, an extension of lack of memory property namely the 'Setting the Clock Back to Zero' (SCBZ) property for univariate case given by Rao and Talwalkar (1989). This property can be viewed as a generalization of LMP of exponential distribution, in the sense that for exponential distribution, the conditional distribution of additional survival time is exactly the same as the original distribution, while for SCBZ property it belongs to the same family, but may be with a changed value of the parameter. We present some distributions possessing SCBZ property and study some consequences of having SCBZ property. It is seen that the mean residual life function of a life distribution having SCBZ property takes a particularly simple form. If the form of expected life time for the distribution having SCBZ property is known, its MRLF can be easily obtained. We have obtained the life expectancy of some distributions possessing SCBZ property. We also discuss the closure property under competing risk set-up for the distributions possessing the SCBZ property and study characterization of SCBZ property in terms of mean residual life function and failure rate function.

The SCBZ property for discrete case given by Nair and Mini (1999) is also presented. Its Characterizations of failure rate function and mean residual life function and closure properties are discussed. Some discrete distributions possessing SCBZ property are presented.

At the end we discuss the extension of SCBZ property to bivariate case given by Rao and Damaraju (1993). We also propose an another version of SCBZ property for bivariate case. We show that the mean residual life function and percentile life function takes a simpler form for distribution possessing the SCBZ properties, and we present some distributions possessing the SCBZ properties.

2. Chapterwise Summary.

In chapter I, We have discussed possible extensions of LMP for bivariate case and multivariate case i.e. Bivariate lack of memory property (BLMP) and Multivariate lack of memory property (MLMP). In section 2, we discuss some extensions of LMP for Bivariate and Multivariate case and its interpretation. In section 3, we present characterizations of BLMP given by Marshall and Olkin (1967), Block and Basu (1974) and Kulkarni and Prasad. (1997). In section 4, we discuss characterizations of MLMP given by Ghurye and Marshall (1984) and Kulkarni (1998). In section 5, some supplementary results are presented. It is shown that the only distribution having BLMP with exponential marginals is BVE given by Marshall and Olkin (1967). Conditions on marginals of a bivariate distribution having BLMP are discussed. Some distributions having BLMP are presented.

Ref ?

In chapter II, We also study, an extension of lack of memory property namely the 'Setting the Clock Back to Zero' (SCBZ) property for univariate case given by Rao and Talwalkar (1989). In section 2, we show that the 'Setting the Clock Back to Zero' (SCBZ) property is a generalization of Lack of Memory property (LMP). In section 3, we present some distributions possessing the SCBZ property. It is observed that Weibull family dose not possesses the SCBZ property. In section 4, we describe some consequences of having SCBZ property namely the discuss the simplified form of mean residual life function as a consequent of SCBZ property. Also we discuss the closure of SCBZ property in competing risk set-up. In section 5, we present some characterizations of SCBZ property in terms of failure rate function and mean residual life function.

In Chapter III, we discuss the 'Setting the Clock Back to Zero' property and its characterizations for discrete case. In section 2, we present the formal definition of SCBZ property for discrete case and some discrete distributions possessing the SCBZ property. In section 3, we discuss some characterizations of SCBZ property in terms of the failure rate function and mean residual life function. In section 4, we show that the SCBZ property of a random variable X is preserved in competing risk set-up.

In chapter IV, we discuss the extension of SCBZ property to bivariate case given by Rao and Damaraju (1993). In section 2, we define SCBZ property given by Rao and Damaraju (1993). We also propose an another version of SCBZ property for bivariate case. In section 3, we shown that the mean residual life function and percentile life function takes a simpler form for distribution possessing SCBZ property. In section 4, we present some bivariate distributions satisfying these definitions ~~are presented~~.

3. OUR ORIGINAL CONTRIBUTION:

In this dissertation the following~~s~~ are our original contribution.

(i) We prove characterizations of SCBZ property in terms of failure rate function and mean residual function for univariate continuous distribution in chapter II.

(ii) We prove closure property of SCBZ property under competing risk set-up for discrete distribution in chapter III.

(iii) We introduce a variant of SCBZ property for bivariate distribution in chapter IV.

—— O X O ——