CHAPTER 1

INTRODUCTION TO PROCESS CAPABILITY INDICES

1.1 INTRODUCTION

In this Chapter we first introduce the first generation indices namely, C_p and C_{pk}. Section 1.2 and Section 1.3 are devoted to introduction of C_p and C_{pk}, their relationship with the probability of non-conformance and their weaknesses. Though these two indices are being widely used in practice, they may give misleading results. Their limitations had motivated the development of the second generation index C_{pm}. The third generation PCI C_{pmk} has been developed by combining the modifications to C_p that produce C_{pk} and C_{pm}. Section 1.4 introduces the PCI's C_{pm} and C_{pmk}. Relationships among the PCI's C_p, C_{pk} and C_{pm} have been reported in Section 1.5. Section 1.6 discusses an unifying approach to the PCI's through the use of weight function due to Spiring (1997). The index C_{pv} given by Spiring (1997) generates the PCI's C_p, C_{pk}, C_{pm} and C_{pmk} for suitable choices of the weight function 'ω'. The Chapter is concluded by providing generalizations of the PCI's for the processes whose target is not the midpoint of the specification limits.

1.2 THE INDEX C_p

Juran et. al. (1974) has introduced the first capability index C_p . It is defined as

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{d}{3\sigma} \tag{1.2.1}$$

where USL is the upper specification limit, LSL is the lower specification limit, $d = (USL - LSL)/2$ and σ is the process standard deviation. Process measurements outside the specification limits are termed as 'Non-Conforming' (NC).

While defining such an index following assumptions were made.

1. The process measurements are normally distributed.
2. The target or nominal value is the midpoint of the specification limits.
3. The process mean is located at the target.

That is, the process is as shown in Figure 1.1.

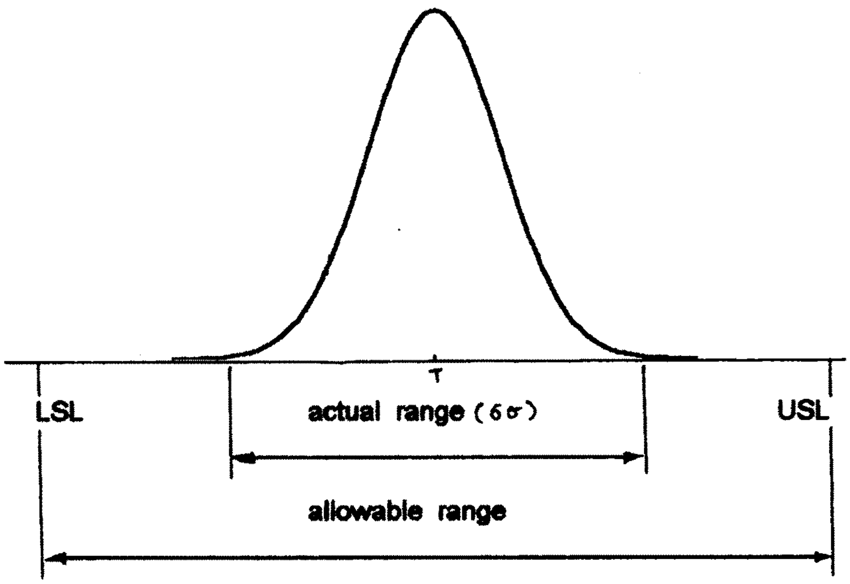


Figure 1.1

The motivation for the multiplier 6 in the denominator is that, since the process measurements are normally distributed almost all (99.73%) process data falls in the middle spread of length 6σ. While the numerator defines the allowable range of process measurements. Hence C_p can be redefined as

$$C_p = \frac{\text{allowable range of measurements}}{\text{actual range of measurements}}$$

It is obvious that the above index will be smaller than one if a considerable portion of the product is not contained in the interval (USL,LSL). This is not a desirable situation as this indicates that a considerable part of the product is not conforming to the specifications. On the other hand if the index is equal to one then it can be concluded that the process is capable in the sense that the product is conforming to the required specification to a significant extent. A larger value of the index is desirable as this indicates that a yet larger portion is within these limits.

In the following we discuss the relation between C_p and probability of non-conformance.

Lemma 1.2.1:

Let the random variable X denotes the quality characteristic and p be the probability of non-conformance (i.e. the probability of NC material) associated with C_p. Then

$$p = 2\Phi(-3C_p) \tag{1.2.2}$$

Proof:

We have

$p = P[\, X > USL \,] + P[\, X < LSL \,]$

$= 2\,P[\, X < LSL \,]$ (Since X is assumed to be symmetric about the target, $T = (LSL + USL)/2$.)

$$= 2\,P\left[\frac{X-\mu}{\sigma} < \frac{LSL-\mu}{\sigma}\right]$$

$$= 2\Phi\left(-\frac{d}{\sigma} \right)$$

$$= 2\Phi(-3C_p) \quad \blacksquare$$

To illustrate the relationship between C_p and probability of non-conformance, Kocherlakota (1992) has provided Table 1.1 as follows:

Table 1.1: *Probability of non-conformance associated with C_p.*

USL - LSL	C_p	$p = 2(-3C_p)$	
6σ	1.00	0.2700×10^{-2}	(2700 ppm)
8σ	1.33	0.6334×10^{-4}	(63 ppm)
10σ	1.67	0.5733×10^{-6}	(0.57 ppm)
12σ	2.00	0.1973×10^{-8}	(0.002 ppm)

The trend now is to report proportion NC in terms of parts per million (ppm) defective. Juran et. al. (1974) have noted that a reject rate of one part per million equals to C_p value of approximately 1.63.

Naturally one may raise an obvious question: why should one not resort directly to the observed proportion outside the specification limits? This would indeed a preferable and more easily understood measure for practioners. The difficulty with this approach is that if it is required to make this proportion small (e.g. in the vicinity of 0.27% or less), then a massive amount of data would be required to estimate this proportion with sufficient accuracy. The assumption of a specific parametric model (normality in the present case) enables one to estimate the proportion NC with far less data for a given precision by using the estimates of the parameters of the assumed distribution.

Montgomery (1996) has recommended minimum values of C_p as 1.33 for an existing process and 1.5 for a new process. Also for the characteristic related to essential safety, strength or performance feature, the recommended minimum values are 1.5 for existing process and 1.67 for a new process.

We recall that all above discussion about C_p is true only when all the three assumptions about the process stated earlier hold good. Now let us see what happens when the third assumption is not valid. Consider the five process having distributions viz, $N(\mu_i, \sigma^2)$; i= 1,2,...,5 as shown in Figure 1.2. Note that all the processes have same set of the specification limits. It is clear that all the five processes possess the same value of C_p, since they are having same value of variances. Because the variances of the processes appear much smaller than

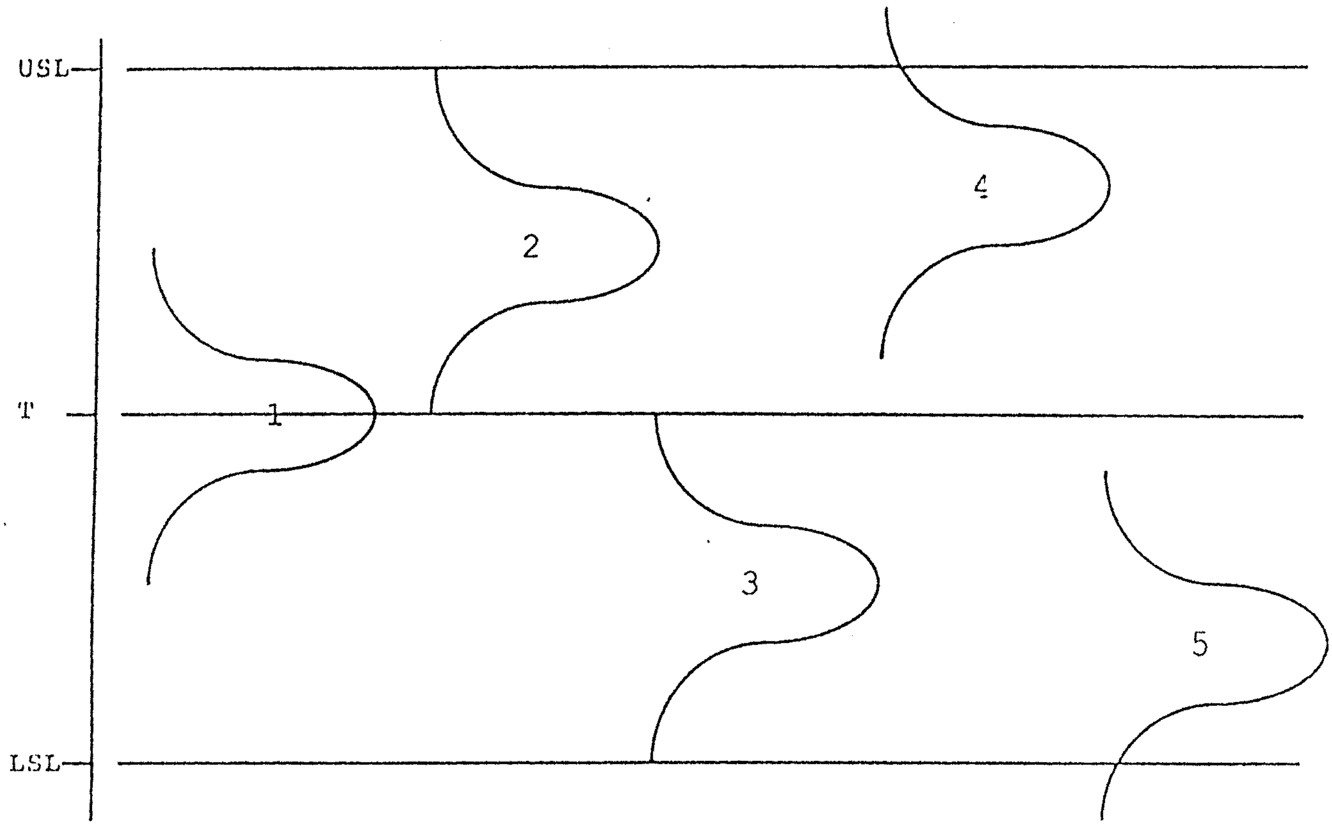


Figure 1.2: *(Source: Chan et al.(1988), pp 163)*

the specification range, the corresponding value of C_p would be fairly large, suggesting that the processes are all capable. But it can be easily seen that as process mean deviates from the target value the capability of the process decreases. However, C_p fails to reveal this fact. That is, C_p fails to take into account proximity to the target value in its assessment of a process capability . Also note that for the process whose mean is deviated from the target, relation (1.2.2) between the value of

C_p and the probability of non-conformance does not hold. Hence there is no exact interpretation for C_p value of such offcentered process in terms of probability of non-conformance. Due to this inherent inability of C_p it is useless to compare off-centered processes.

To overcome this drawback of C_p, several indices have been proposed that attempt to take the target value T into account. For example, when only a single specification limit is given one can use the indices CPU or CPL, which are defined as

$$CPU = \frac{USL - \mu}{3\sigma} \quad : \text{when only USL is given} \tag{1.2.3}$$

$$CPL = \frac{\mu - LSL}{3\sigma} \quad : \text{when only LSL is given,} \tag{1.2.4}$$

where μ is the process mean which satisfies the condition $LSL \leq \mu \leq USL$.

Minimum of these two is the another index, we discuss below.

1.3 THE INDEX C_{pk}

The index C_{pk} is defined as

$$C_{pk} = \min(CPU, CPL) \tag{1.3.1}$$

From (1.2.3) and (1.2.4) we have

$$C_{pk} = \frac{\min(USL - \mu\ ,\ \mu - LSL)}{3\sigma}$$

$$= \frac{d - |\mu - M|}{3\sigma} \qquad \text{(where } M \text{ (LSL + USL)/2)}$$

$$= \left\{ 1 - \frac{|\mu - M|}{d} \right\} \frac{d}{3\sigma}$$

$$= (1 - k)C_p \qquad (1.3.2)$$

where $k = \dfrac{|\mu - M|}{d}$.

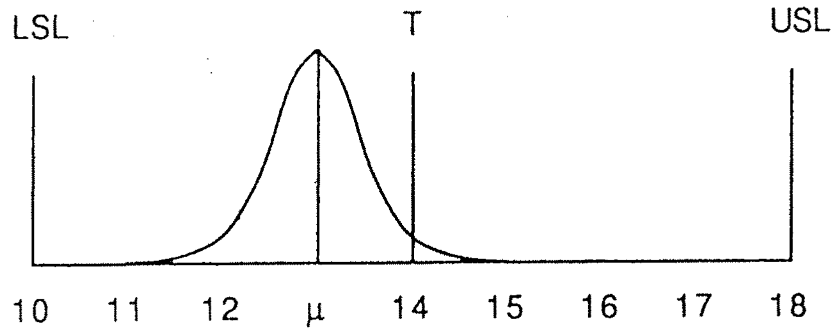
From (1.3.2) it follows that $C_{pk} \leq C_p$, with equality if and only if $\mu = M$.

Figure 1.3 illustrates how the indices CPU, CPL, C_p react to the departure of process meam from the target.

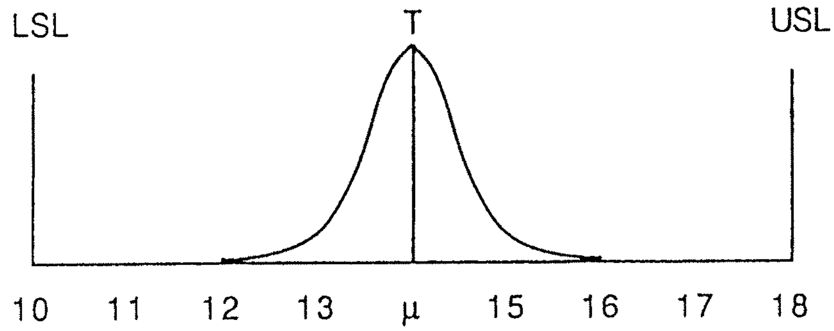
Unlike C_p, C_{pk} does not give exact probability of non-conformance but limit it. The following lemma gives limits on probability of non-conformance associated with given value of C_{pk}.

Lemma 1.3.1:

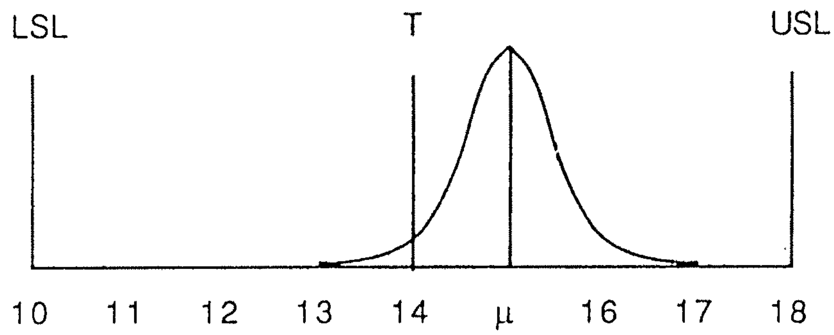
$$\Phi(-3C_{pk}) \leq p \leq 2\Phi(-3C_{pk})$$



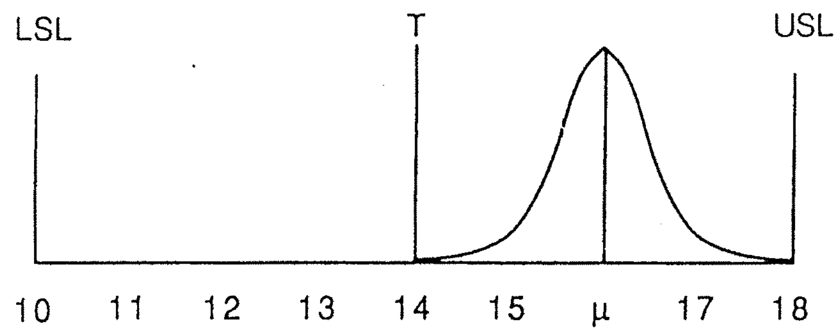
μ	Cp	CPL	CPU	Cpk	Cpm
13	2	1.5	2.5	1.5	1.11



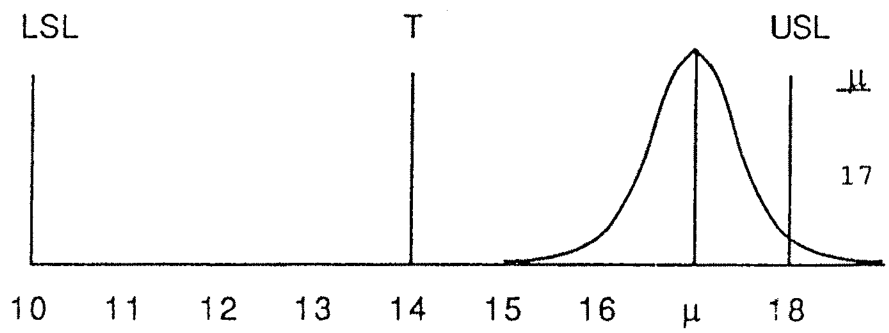
μ	Cp	CPL	CPU	Cpk	Cpm
14	2	2.0	2.0	2.0	2.00



μ	Cp	CPL	CPU	Cpk	Cpm
15	2	2.5	1.5	1.5	1.11



μ	Cp	CPL	CPU	Cpk	Cpm
16	2	3.0	1.0	1.0	0.63



μ	Cp	CPL	CPU	Cpk	Cpm
17	2	3.5	0.5	0.5	0.43

Figure 1.3: *(Source: Chan et al. (1988), pp 165)*

Proof:

we see that if $M \le \mu \le USL$, then

$$C_{pk} = \frac{USL - \mu}{3\sigma}$$

and $$\frac{LSL - \mu}{3\sigma} = \frac{(USL - \mu) - (USL - LSL)}{3\sigma}$$

$$= C_{pk} - 2C_p$$

$$\le -C_{pk}, \tag{1.3.3}$$

which follows from the fact that $C_p \ge C_{pk}$.

Hence probability of non-conformance (p) associated with given C_{pk} is

$$p = \Phi\left(\frac{LSL - \mu}{\sigma}\right) + \left[1 - \Phi\left(\frac{USL - \mu}{\sigma}\right)\right]$$

$$= \Phi[-3(2C_p - C_{pk})] + \Phi(-3C_{pk}) \tag{1.3.4}$$

$$\le \Phi(-3C_{pk}) + \Phi(-3C_{pk}) \qquad \text{(using (1.3.3))}$$

$$= 2\Phi(-3C_{pk})$$

Also from (1.3.4) $p \ge \Phi(-3C_{pk})$

Thus, we have

$$\Phi(-3C_{pk}) \le p \le 2\Phi(-3C_{pk})$$

The case where $LSL \le \mu \le M$ can be treated similarly.■

It has been found that C_{pk} is not also an absolute measure of process capability. To demonstrate this we consider the following example due to Boyles (1991).

Let A, B, and C be the processes with the following characteristic:

A : $\mu = 50$ $\sigma = 5$

B : $\mu = 57.5$ $\sigma = 2.5$

C : $\mu = 61.25$ $\sigma = 1.25$

The specification limits for all the three processes are, USL = 65 and LSL = 35. The processes are as shown in Figure 1.4.

Note that all the three processes have the same value of C_{pk}, that is 1, but the processes differ in capability. This shows that C_{pk} is also an inadequate measure of process capability. A large value of C_{pk} does not imply that the process

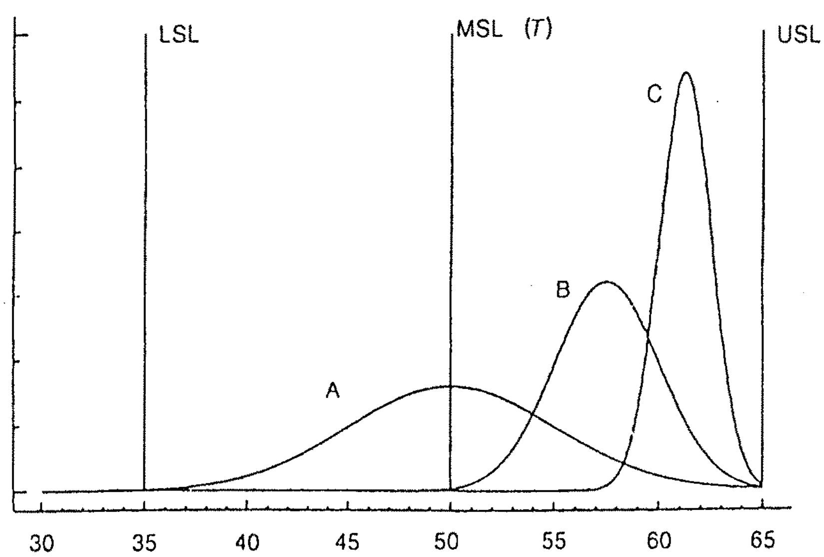


Figure.1.4: *(Source: Kotz and Lovelace (1998), pp 49.)*

is really good. Observe that for any fixed value of μ in the interval from LSL to USL, C_{pk} depends inversely on σ and becomes large as σ approaches zero. This characteristic makes C_{pk} unsuitable.

Thus, both the indices C_p and C_{pk}, though are being widely used in practice, suffer from some or other drawback. This has motivated to develope second and third generation indices. Some of them have been discussed below.

1.4 THE INDICES C_{pm} AND C_{pmk}

1.4.1 The Index C_{pm}

The first of second generation PCI's C_{pm} was introduced independently by Hsiang and Taguchi (1985) in a lecture at a meeting, and later formally by Chan et al. (1988). It is defined as

$$C_{pm} = \frac{USL - LSL}{6\sigma'} \tag{1.4.1}$$

where $\sigma'^2 = E(X - T)^2$.

Note that C_{pm} is a minor alteration of C_p, essentially examining square-deviations from the target rather than from the process mean.

Since $\sigma'^2 = E(X - T)^2 = E(X - \mu)^2 + (\mu - T)^2 = \sigma^2 + (\mu - T)^2$, we have

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

$$= \frac{C_p}{\sqrt{1 + \dfrac{(\mu - T)^2}{\sigma^2}}}$$

$$= \frac{C_p}{\sqrt{1 + \tau^2}}, \qquad (1.4.2)$$

where $\tau = |\mu - T|/\sigma$.

From (1.4.2) it follows that, $C_{pm} \le C_p$ with equality if and only if $\mu = T$.

Apprantly it feels that C_{pm} will possess the necessary properties required for assessing the capability. If the process variance increases, the denominator of (1.4.1) increases and C_{pm} will decrease. If the process drifts from the target (i. e. if μ moves away from T) the denominator of (1.4.1) will again increase causing C_{pm} to decline. In the case where both the process mean and process variance will change, the C_{pm} index will reflect these changes as well.

Figure 1.3 (page 20) illustrates for fixed variance, how C_{pm} reacts to departures from the target value. It follows that C_{pm} reacts to the departure in a similar way as does C_{pk}. But it is more sensitive than C_{pk} for small departures and less sensitive than C_{pk} for large departures.

To illustrate how C_{pm} reacts to both the changes simultaneously, again consider the three processes A, B and C, discussed in section 1.3. It is clear that the processes are capable in the order C : B : A. The values of C_{pm} for the processes A, B, C are respectively 1, 0.63 and 0.44, perceiving that the order of capability is A : B :C, which is misleading. From this example it follows that while assessing the capability, C_{pm} gives greater importance to the departure of process mean than to the change in process variance.

The motivation behind the index C_{pm} came not from an examination of the probability of non-conformance of a process, but from the ability of the process to meet target values. C_{pm} attempts to take the attention away from conformance to specifications and refocus on the optimal product quality, achived only when critical dimensions are made to target. However, upper bounds of the probability of non-conformance can be estimated for specific values of C_{pm}.

Govaerts (1994) found that the relationship $p \leq 2\Phi(-3C_{pm})$ holds only for sufficiently large C_{pm} and no longer holds at small values of C_{pm}.

Define, without loss of generality, USL + LSL = 0, where USL = -LSL = d. With these choices, Kotz and Johnson (1993) found that the probability of non-conformance associated with given value of C_{pm} is

$$p = \Phi\left(\frac{-d - \mu}{\sqrt{\lambda^2 - \mu^2}} \right) + 1 - \Phi\left(\frac{d - \mu}{\sqrt{\lambda^2 - \mu^2}} \right)$$

where

$$\lambda = \frac{d}{3C_{pm}} .$$

In the following we introduce the third generation index C_{pmk}.

1.4.2 The Index C_{pmk}

Pearn et al. (1992) have constructed a third generation PCI C_{pmk}, by combining the modifications to C_p that produced C_{pk} and C_{pm} as

$$C_{pmk} = \frac{\min(USL - \mu,\ \mu\ \ LSL)}{3\sigma'} \tag{1.4.3}$$

$$= \frac{d - |\mu - T|}{3\sigma'} .$$

C_{pmk} can be written in terms of C_{pk} and C_{pm} as

$$C_{pmk} = \frac{C_{pk}}{\sqrt{1 + \frac{(\mu - T)^2}{\sigma^2}}} = \frac{C_{pk}}{\sqrt{1 + \tau^2}} \qquad (1.4.4)$$

$$= \left(1 - \frac{|\mu - T|}{d} \right) C_{pm} = (1 - k) C_{pm} \qquad (1.4.5)$$

The term τ^2 in the denominator of (1.4.4) may be viewed as an additional penalty to the process quality for the departure of process mean from the target. Same is the interpretation for the term k in (1.4.5). This penalty ensures that C_{pmk} will be more sensitive to the departures than C_{pk} and C_{pm}.

The following section reports relationship among C_p, C_{pk} and C_{pm}.

1.5 SOME RELATIONS AMONG C_p, C_{pk} AND C_{pm}

Lemma 1.5.1:

$$C_p \geq \max(C_{pk}, C_{pm}).$$

Proof:

Since $C_p \geq C_{pk}$ and $C_p \geq C_{pm}$, we have

$C_p \geq \max(C_{pk},\ C_{pm})$. ■

Lemma 1.5.2:

For a fixed value of C_p, C_{pk} and C_{pm} have a one-to-one relationship.

Proof:

Since $C_{pk} = C_p(1 - k)$ and because $C_p = C_{pm}\sqrt{1 + \tau^2}$ (See (1.3.2) and (1.4.2).), we have

$$C_{pk} = (1 - k)C_{pm}\sqrt{1 + \tau^2}$$

Hence the relationship is one-to-one. ■

Lemma 1.5.3:

$C_{pk} \geq C_{pm}$ if and only if $C_p^2 \geq \dfrac{2 - k}{9k(1 - k)^2}$.

Proof:

Since $C_{pk} = (1 - k)C_p$ and $C_{pm} = \dfrac{C_p}{\sqrt{1 + \tau^2}}$,

we have $C_{pk} \geq C_{pm}$

$$\Leftrightarrow \quad (1 - k) \geq \left(\sqrt{1 + \tau^2} \right)^{-1}$$

$$\Leftrightarrow \quad (1 - k)\sqrt{1 + \tau^2} \geq 1$$

$$\Leftrightarrow \quad (1 - k)\sqrt{1 + \left(\frac{d}{\sigma} \right)^2 k^2} \geq 1 \qquad \left(\because \frac{|\mu - T|}{d} = k \right)$$

$$\Leftrightarrow \quad 1 + \left(\frac{d}{\sigma} \right)^2 k^2 \geq \frac{1}{(1 - k)^2}$$

$$\Leftrightarrow \quad \left(\frac{d}{\sigma} \right)^2 \geq \frac{1 - (1 - k)^2}{k^2 (1 - k)^2}$$

$$= \frac{2 - k}{k(1 - k)^2}$$

$$\Leftrightarrow \quad C_p^2 \geq \frac{2 - k}{9k(1 - k)^2} \qquad \blacksquare$$

In the following we discuss an unified way to define a wide class of PCI's as given by Spiring (1997).

1.6 UNIFIED APPROACH TO PCIs

Spiring (1997) has proposed a unifying approach that ties the various PCIs together while illustrating some of the statistical properties associated with each. Through the use of, what he calls, a weight function the relationship that exists among the measures C_p, C_{pk}, C_{pm} and C_{pmk} can be illustrated.

Consider the PCI C_{pv} as

$$C_{pv} = \frac{USL - LSL}{6\sqrt{\sigma^2 + \omega(\mu - T)^2}} \tag{1.6.1}$$

where ω represents a weight function. Allowing the weight function to assume different values permits C_{pv} to assume equivalent computational algorithms as those for C_p, C_{pk}, C_{pm} and C_{pmk} as well as a host of other potential measures of capability.

Lemma 1.6.1:

Setting

(i) $\omega = 0$, C_{pv} becomes C_p;

(ii) $\omega = 1$, C_{pv} becomes C_{pm};

(iii) $$\omega = \begin{cases} \dfrac{k(2 - k)}{(1 - k)^2\tau^2} & ; \ 0 < k < 1 \\ 0 & ; \ \text{otherwise} \end{cases} \tag{1.6.2}$$

or $$\omega = \begin{cases} \dfrac{(6C_p - \tau)}{(3C_p - \tau)^2\tau^2} & ; \ 0 < \tau/3 < C_p \\ 0 & ; \ \text{otherwise,} \end{cases} \tag{1.6.3}$$

C_{pv} becomes C_{pk};

$$\text{(iv)} \quad \omega = \begin{cases} \dfrac{\tau^2 + 2k - k^2}{(1 - k)^2 \tau^2} & ;\ 0 < k < 1 \\ 0 & ;\ \text{otherwise,} \end{cases} \qquad (1.6.4)$$

C_{pv} becomes C_{pmk}.

Proof:

Proofs of (i) and (ii) are obvious.

Setting $\tau = \dfrac{|\mu - T|}{\sigma}$ and $k = \dfrac{|\mu - T|}{d}$ we have

$$C_{pk} = (1 - k)C_p$$

$$= (1 - k)\,\frac{USL - LSL}{6\sigma}$$

$$= \frac{USL - LSL}{\sqrt{\dfrac{\sigma^2}{(1 - k)^2}}} \qquad (1.6.5)$$

Now

$$\frac{\sigma^2}{(1 - k)^2} = \frac{\sigma^2}{(1 - k)^2} - \sigma^2 + \sigma^2$$

$$= \sigma^2 + \left[\frac{1}{(1 - k)^2} - 1 \right]\sigma^2$$

$$= \sigma^2 + \left[\frac{1 - (1 - k)^2}{(1 - k)^2} \right] \frac{\sigma^2}{(\mu - T)^2}\,(\mu - T)^2$$

$$= \sigma^2 + \frac{k(2 - k)}{(1 - k)^2} \frac{1}{\tau^2} (\mu - T)^2$$

Substituting in (1.6.5), we have

$$C_{pk} = \frac{USL - LSL}{\sqrt{\sigma^2 + \left[\frac{k(2 - k)}{(1 - k)^2 \tau^2} \right] (\mu - T)^2}}$$

which results in C_{pv} with

$$\omega = \frac{k(2 - k)}{(1 - k)^2 \tau^2} \ . \qquad (1.6.6)$$

Now consider

$$k = \frac{|\mu - T|}{d}$$

$$= \frac{2|\mu - T|}{USL - LSL}$$

$$= \frac{2\tau}{\frac{(USL - LSL)}{\sigma}}$$

$$= \frac{\tau}{3C_p}$$

substituting in (1.6.6), we get

$$\omega = \frac{6C_p - \tau}{(3C_p - \tau)^2 \tau} \ .$$

Hence the proof of (iii).

Now We have

$$C_{pmk} = (1 - k)C_{pm}$$

$$= (1 - k)\frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

$$= \frac{USL - LSL}{6\sqrt{\dfrac{\sigma^2 + (\mu - T)^2}{(1 - k)^2}}} \tag{1.6.7}$$

Now,

$$\frac{\sigma^2 + (\mu - T)^2}{(1 - k)^2} = \frac{\sigma^2\left[1 + \left(\dfrac{\mu - T}{\sigma}\right)^2\right]}{(1 - k)^2} - \sigma^2 + \sigma^2$$

$$= \frac{\sigma^2(1 + \tau^2)}{(1 - k)^2} - \sigma^2 + \sigma^2$$

$$= \sigma^2\left[\frac{1 + \tau^2}{(1 - k)^2} - 1\right] + \sigma^2$$

$$= \left(\frac{\sigma}{\mu - T}\right)^2\left[\frac{\tau^2 + 2k - k^2}{(1 - k)^2}\right](\mu - T)^2 + \sigma^2$$

$$= \frac{\tau^2 + 2k - k^2}{(1 - k)^2\tau^2}(\mu - T)^2 + \sigma^2$$

Substituting in (1.6.7), it follows that C_{pmk} results in C_{pv} with

$$\omega = \frac{\tau^2 + 2k - k^2}{(1 - k)^2\tau^2} .$$

Hence the proof of (iv).■

Through the use of weight function a general measure of process capability is promotted that ties together the most common measures. C_{pv} encompasses a large group of capability measures and permits comparison among the measures. For example, the relationship that exists among C_p, C_{pk} and C_{pm} is illustrated in Figure 1.5 for $C_p = 1$. As the process mean deviates from the target (in this case measured by $\tau = |\mu - T|/\sigma$), the value of C_p remains unchanged. Both C_{pm} And C_{pk} reflect the fact that the process is not on target and their magnitudes diminish.

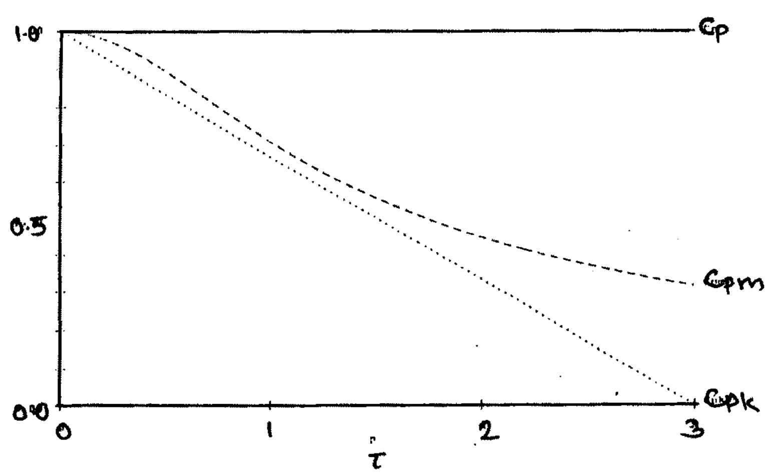


Figure 1.5: *(Source: Spiring (1997), pp 51.)*

Figure 1.5 can be used to compare the equivalency of C_{pk} and C_{pm} and their relationship with C_p. If the process is centered at the target ($\tau = 0$), then the three measures are identical. If the process is centered one standard deviation from the target ($\tau = 1$), $C_p = 1$, $C_{pm} = 0.707$, $C_{pk} = 0.667$. Through the use of weight function and plots similar to Figure 1.5, it is possible to investigate the numerical relationship that exists between C_{pk} and C_{pm}. However, due to the dynamic nature of the relationship between C_{pk} and C_p, plots that compare the relationship between C_{pk} and C_{pm} will be conditional on the magnitude of C_p.

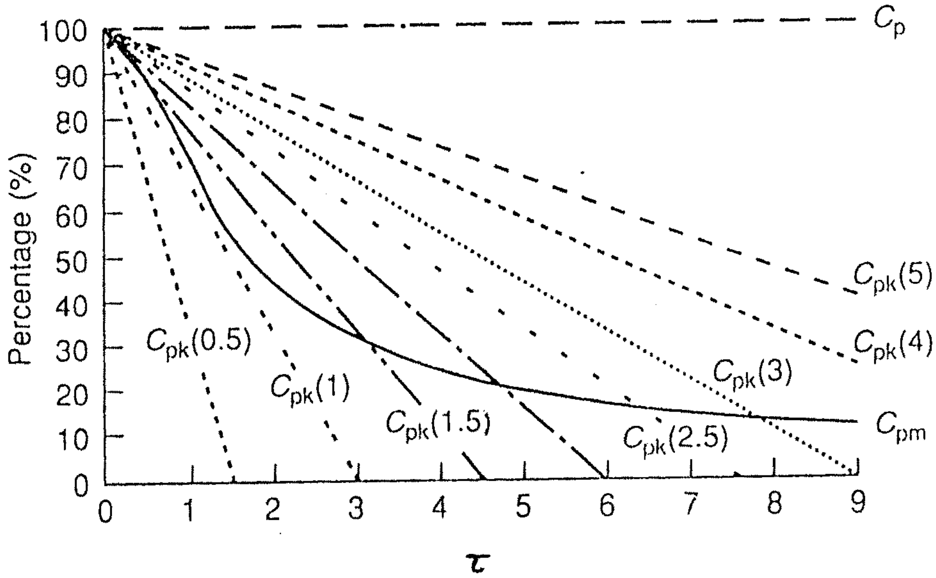


Figure 1.6: *(Source: Spiring (1997), pp 52.)*

The general relationship that exists among the indices C_p, C_{pk} and C_{pm} is illustrated in Figure 1.6. The relationship between C_p and C_{pm} is unaffected by the magnitude of C_p. If the process is centered at a point one standard deviation from the target, then C_{pm} will always be 70.7% of C_p. However, C_{pk} has a relationship that is dependent upon the magnitude of C_p, which is evident from its weight function (see (1.6.3)) and depicted in Figure 1.6. The relationship has been drawn for value of C_p = 0.5, 1, 1.5, 2, 2.5, 3,4 and 5 (i.e.$C_{pk}(0.5)$ $C_{pk}(1)$,..........in Figure 1.6). To illustrate, consider a process centered two standard deviations from the target and where $C_p = 3$. The relationship between C_p and C_{pm} remains unchanged; for $\tau = 2$, C_{pm} will be 44.72% of C_p ($C_{pm} = 1.34$). Using the transect line associated with $C_p = 3$ ($C_{pk}(3)$), it is easy to see that the associated C_{pk} is 2.33 (77.8% of C_p). If C_p was 2 in the example (rather than 3), the relationship between C_p and C_{pk} as well as C_{pm} and C_{pk} would be different. Again for $\tau = 2$ but now with C_p = 2, C_{pm} would continue to be 44.72% of C_p (C_{pm} =0.8944), while the transect $C_{pk}(2)$ indicates C_{pk} would be 66.7% of C_p (C_{pk} = 1.333). Figure 1.6 is helpful in displaying the dynamic relationship that exists between C_p and C_{pk} while allowing insights into numerical equivalencies for C_{pm} and C_{pk}.

Spiring (1997) also suggests some additional weight functions that may be of some interest to practioners. These include

$$\omega = \begin{cases} \tau & ; 0 < \tau \\ 0 & ; \text{otherwise} \end{cases} \tag{1.6.8}$$

$$\omega = \begin{cases} \frac{1}{\tau} & ; 0 < \tau \\ 0 & ; \text{otherwise} \end{cases} \tag{1.6.9}$$

and

$$\omega = \begin{cases} c & ; 0 < c < \infty \\ 0 & ; \text{otherwise} \end{cases} \tag{1.6.10}$$

where c is a constant used to adjust the effect of departure on the index.

The weight function in (1.6.8) would typically be used in those situations where minor departures from the target are of little consequences, but where major departures are considered critical. On the other hand the weight function in (1.6.9) applies greater weight to minor departures, while only a marginal decrease in capability index results once the process deviates from the target by more than one standard deviation.

1.7 CONCLUDING REMARKS

All PCIs discussed above are developed under the assumptions:

(a) The process is in control.

(b) The target is the midpoint of the specification limits;

(c) The process measurements are normally distributed.

and (d) The measurement system is ideal.

If the process is centered at the midpoint of the specification limits, the indices are equivalent. For those processes affected by special or assignable causes, process capability should not be assessed. Similarly, for any process whose target is not the midpoint of the specification limits or that exhibits non-normal characteristics or both, the above indices should not be calculated. Otherwise, misleading results will be obtained. However, Kane(1986) also defines C_p and C_{pk} for the process whose target is not the midpoint of the specification limits as

$$C_p^* = \min\left(\frac{T - LSL}{3\sigma}, \frac{USL - T}{3\sigma}\right)$$

and

$$C_{pk}^* = (1 - K^*)C_p^*$$

where $$k^* = \frac{|\mu - T|}{\min(T - LSL, USL - T)} .$$

Also, Chan et.al.(1988) have modified the index C_{pm} for such processes as

$$C_{pm}^* = \frac{\text{minimum}(USL - T, T - LSL)}{3\sigma'}$$

where σ' is as defined in section 1.4. These indices decreases as the process mean moves away from the target and they have maxima at $\mu = T$. Thus, they indicate that the process is optimal when it is centered at the target, which is a paradox because moving the process mean towards the target reduces the fraction of the distribution which is within the specification limits. We will not investigate these indices in details. Infact the target should not be considered as a process parameter while assessing the process capability. Customary (though not practically) it should be the midpoint of the specification limits.

In Chapter 2 we focus on estimation part of some of the PCI's introduced in this Chapter.