CHAPTER - IV

PROBABILISTIC MODELS.

4.1 DESCRIPTION AND DEVELOMENT OF PROBABILISTIC INVENTORY MODELS.

4.2 SOFTWARE PACKAGE FOR PROBABILISTIC INVENTORY MODELS.

DESCRIPTION AND DEVELOPMENT OF PROBABILISTIC INVENTORY MODELS

4.1 INTRODUCTION :

Many times it is not possible to keep both EOQ and number of periods per cycle fixed and uniform because either demand or lead time or both are random variables.

The EOQ models considered so far assumes that demand for the item under consideration is certain, continuous and constant. In reality, however, the demand is more likely to be uncertain, discontineous and variable. Another assumption in the EOQ models is that all demand is supplied immediately and there is no question of a shortage. However, even when the demand and lead time are known and constant, stock out may be deliberately permitted. When demand and lead time are not certain the EOQ models needs modification and safety stock are required to be kept. The EOQ model is based on the assumption that the unit price is the same. Further uniformity of the unit holding cost and of the cost of placing an order is the other underlying assumption. If either or both of these assumptions are not satisfied. Finally the implicit assumption in the EOQ model that the entire quantity ordered for would be received in singly lot may not hold true sometimes. Therefore probabilities are used to represent them.

The probabilistic models considered here will be concerned with answering the following question. What should be the optimum level at the beginning of a planning period during which uncertain demand is likely to occur.

ASSUMPTION MADE UNDER PROBABILISTIC INVENTORY MODELS :

1. The lead time is zero.
2. Decision regarding replenishment are made at regular equal interval of time.
3. Costs of carrying surpluses and shortages of inventory items are linear quantity.
4. Demand is instanteneous and units replenishment is descrete (Model-1).
5. Demand is instanteneous and units replenishment is contineous (Model-2).
6. Demand is contineous and units replenishment is descrete (Model-3).
7. Demand is instantaneous and units replenishment is also contineous and with set-up cost to be fixed (Model-4).

NOTATIONS USED UNDER PROBABILISTIC INVENTORY MODELS :

t = reorder period, it is fixed and known.

L = Lead time.

D = the quantity required or sold, it is a random variable which can be discrete or contineous.

f(D) = probability density function of D, it could be either discrete or contineous as per the nature of random variable D.

C_1 = cost of carrying (or holding) a surplus of one unit for one interval of time due to over stocking.

C_2 = cost of shortage (or penalty) occur during the period due to under stocking.

I = number of items on hand before an order is placed.

Q = Economic Order Quantity.

EC(S) = the total expected costs associated with an inventory level of S(= I + Q) units.

C = cost per unit of an item.

MODEL-1

PROBABILISTIC INVENTORY MODEL

SINGLE PERIOD PROBABILISTIC MODELS WITHOUT SET-UP COSTS, INSTANTANCEOUS DEMAND AND DESCRETE UNITS REPLENISHMENT.

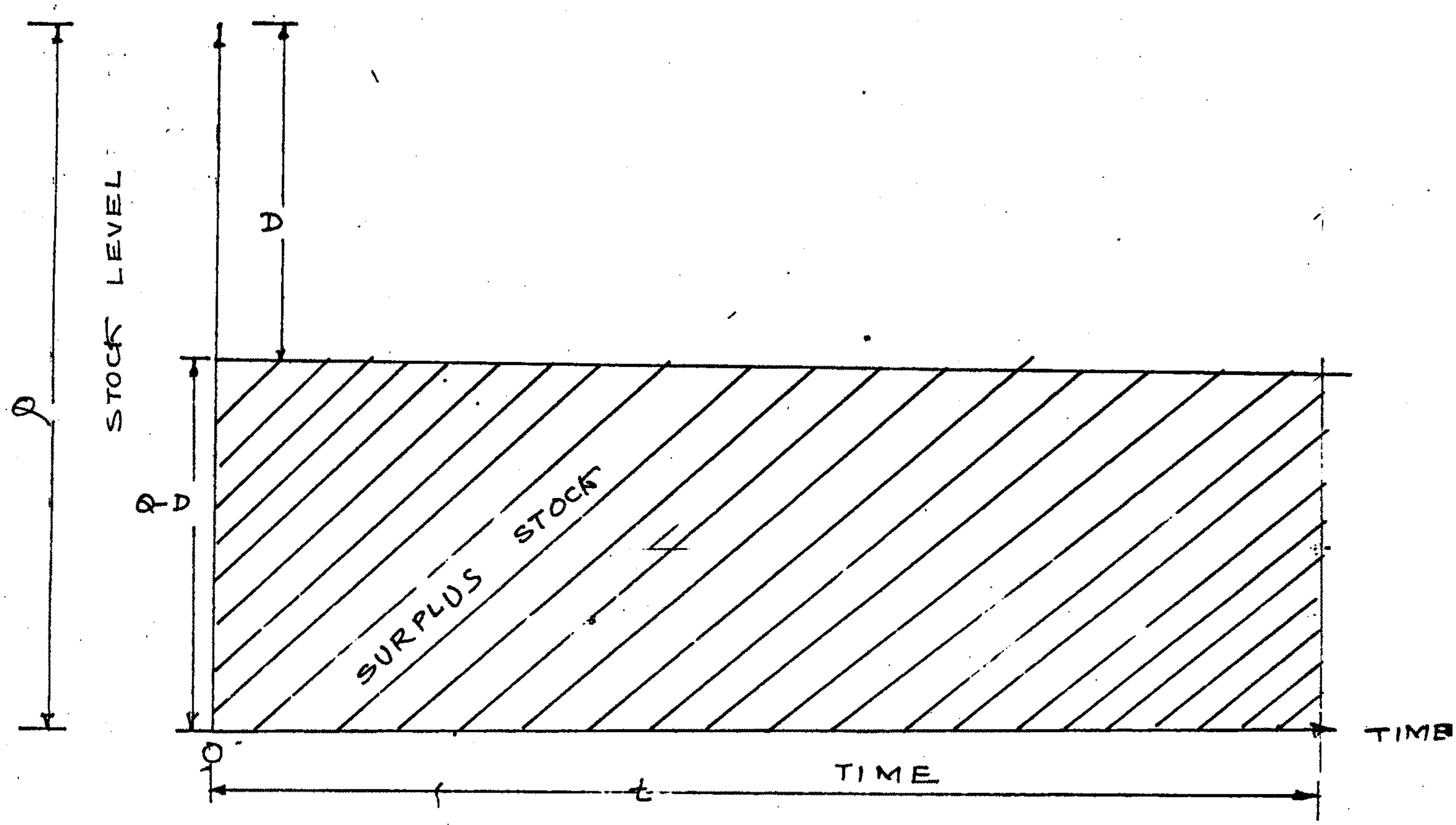
The single period model considers a situation where an item can be ordered only once to satisfy the demand of a specific period of time and no further orders can be made to replenish inventory. After the period is over, there is a cost associated with the stock of left over at the end of the period and the corresponding cost for shortage.

Since demand D is intantaneous, therefore total demand is filled at the beginning of the planning period. Let I be the amount of inventory at the beginning of the said period.

Since demand D is instanteneous, therefore total demand is filled at the beginning of the planning period. Let I be the amount of inventory at the beginning of the said period. Now the question is that, what should be the best level Q^* of inventory with which to start the period so as to satisfy the uncertain demand during the comming period and to minimize the total expected cost associated with surpluses and shortages. There may be two cases depending upon the relation of D and Q.

CASE-1 : DEMAND IS LESS THAN STOCK.

In this case the carrying cost will be $C_1 t(Q - D)$ as shown in the following figure.

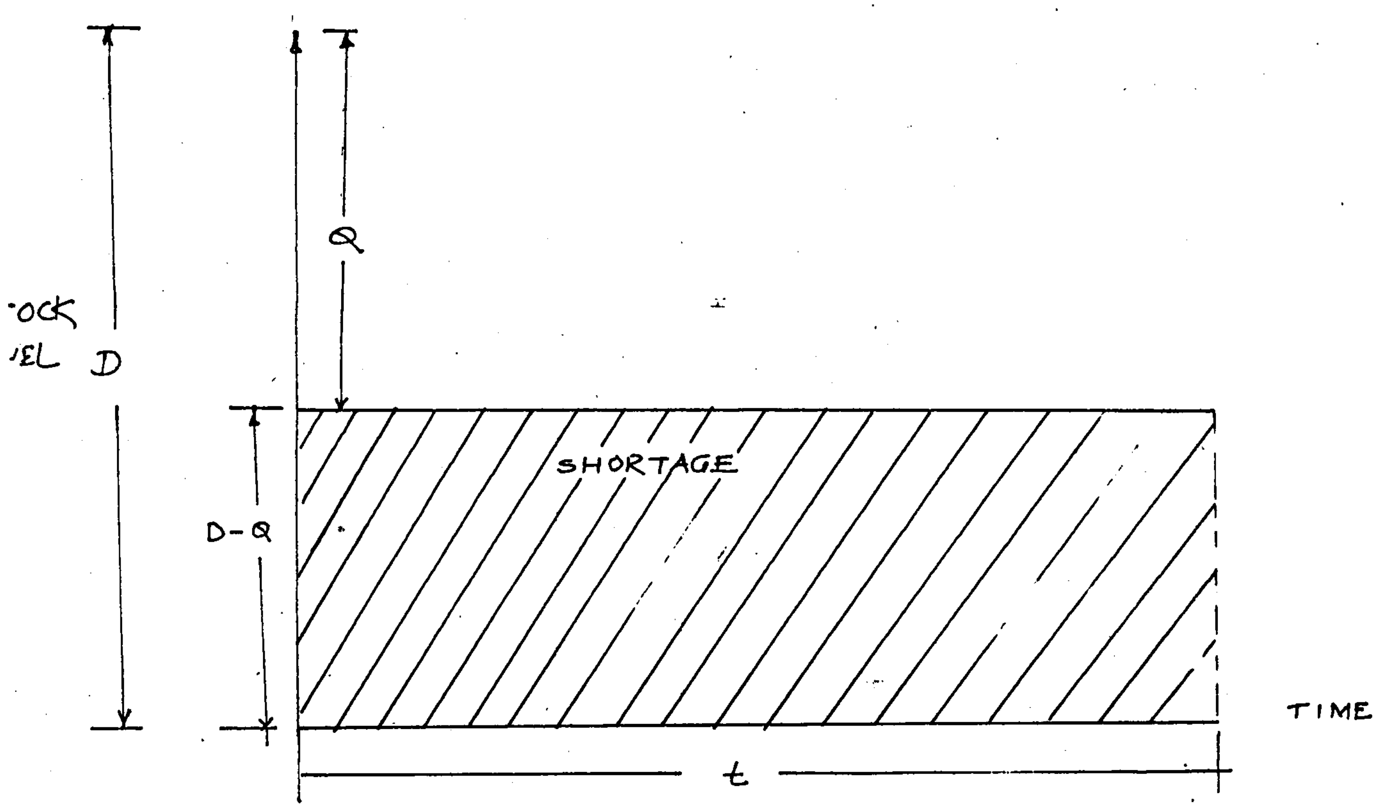


Thus the expected carrying cost if we have Q items on hand at the beginning of the period will be

$$C_1 \sum_{D=0}^{Q} (Q - D)\, t\, f(D) \qquad D \leq 0 \tag{1}$$

<u>CASE-2</u> : DEMAND IS MORE THAN THE STOCK.

In this case the shortage cost will be $C_2 t(D - Q)$ as shown in the following figure.



DEMAND > SUPPLY

Thus the expected shortage cost if we have Q items on hand at the beginning of the period will be

$$C_2 \sum_{D=Q}^{\infty} (D - Q)\ t\ f(D) \qquad D > Q \tag{2}$$

If the initial inventory I is zero, then the total expected single period cost EC(S) associated with an inventory level of S = Q will be

$$EC(Q) = C_1 t \sum_{D=0}^{Q} (Q - D)\ f(D) + C_2 t \sum_{D=Q}^{\infty} (D - Q)\ f(D) \tag{3}$$

In order to find the optimum value Q^* so as to minimise EC(Q) the following condition

$$\Delta C(Q^* - 1) < 0 < \Delta C(Q^*)$$

must hold.

Now replacing Q by (Q + 1) in equation (3) we get

$$EC(Q+1) = \left\{C_1 \sum_{D=0}^{Q+1} (Q + 1 - D)\ f(D) + C_2 \sum_{D=Q+2}^{\infty} (D - Q - 1)\ f(D)\right\} t$$

$$= \left\{C_1 \sum_{D=0}^{Q} (Q - D)\ f(D) + C_1 \sum_{D=0}^{Q} f(D) + C_2 \sum_{D=Q+1}^{\infty} (D - Q)\ f(D) - C_2 \sum_{D=Q+1}^{\infty} f(D)\right\} t$$

$$= EC(Q) + \left\{(C_1 + C_2)\ F(Q) - C_2\right\} t \tag{4}$$

Since the term D = Q + 1 is zero in both the summations

where $F(Q) = \sum_{D=0}^{Q} f(D)$ and $\sum_{D=Q+1}^{\infty} f(D) = 1 - \sum_{D=0}^{Q} f(D)$

because $\sum_{D=0}^{\infty} f(D) = 1$

Similarly

$$EC(Q-1) = EC(Q) - \left\{(C_1 + C_2)\ F(Q-1) - C_2\right\} t \qquad (5)$$

By differential calculus, we know that

$$\left.\begin{aligned} \Delta C(Q^*) &= C(Q^* + 1) - C(Q^*) \\ &= (C_1 + C_2)\ F(Q^*) - C_2 \geq 0 \\ \Delta C(Q^* - 1) &= EC(Q^* - 1) - EC(Q^*) \\ &= -\ (C_1 + C_2)\ F(Q^*) + C_2 \geq 0 \end{aligned}\right\} \qquad (6)$$

For any integer value (Q+1) more than Q and for any integer (Q-1) < Q inequalities (6) would hold because F(Q) is non-decreasing for increasing Q. Hence if (6) hold, then for local minimum and EC(Q) we must have

$$EC(Q^* - 1) \geq EC(Q^*) \qquad (Q^* - 1) < Q^*$$

and $EC(Q^* + 1) \geq EC(Q^*) \qquad (Q^* + 1) > Q^*$

Thus Q^* is the value of Q which minimizes the EC(Q) satisfying (6).

Rearranging inequalities (6) we have

$$F(Q^* - 1) \leq \frac{C_2}{C_1 + C_2} \leq F(Q^*) \qquad (7)$$

Where F(Q) is the cumulative probability distribution for F(D) and is equal to the probability that $D \leq Q$.

$$P(D \leq Q) = F(Q) = \sum_{D=0}^{D} f(D) = \frac{C_2}{C_1 + C_2} \qquad (8)$$

ALGORITHMIC PROCEDURE FOR THE ABOVE MODEL IS AS FOLLOWS :

STEP-1 : Calculate $\frac{C_2 - C}{C_1 + C_2}$ where C is cost/unit of an item, provided it is to be considered.

STEP-2 : Determine cumulative probability distribution F(Q).

STEP-3 : Determine value Q^* and $Q^* - 1$ where the ratio $\frac{C_2 - C}{C_1 + C_2}$ lies in the cumulative distribution F(Q).

STEP-4 : Take the higher value as optimum level (Q^*) to start the period.

STEP-5 : The optimal ordering policy in the presence of $I (< Q^*)$ must have

i) order $Q^* - I$ if $Q^* > I$

ii) do not order if $Q^* \leq I$.

MODEL-2

PROBABILITY INVENTORY MODEL

SINGLE PERIOD PROBABILISTIC MODELS WITHOUT SET-UP COST, INSTANTANEOUS DEMAND AND CONTINEOUS UNITS REPLACEMENT

This model is similar to Model-1 except that the problem is formulated as a contineous problem. If f(D) is defined as the contineous probability distribution of having a demand of exactly D units, then equation (3) under Model-1 becomes

$$EC(Q) = C_1 \int_0^Q (Q-D)t\ f(D)\ dD + C_2 \int_Q^\infty (D-Q)t\ f(D)\ dD \qquad (9).$$

For getting the optimum value of Q so as to mnimize EC(Q), first differentiate (9) w.r. to Q and equate it to zero. That is

$$\frac{d\ EC(Q)}{dQ} = C_1 t \int_0^Q f(D)dD - C_2 t \int_Q^\infty f(D)dD$$

$$= C_1 t \int_0^Q f(D)dD - C_2 t \left\{\int_Q^\infty f(D)dD - \int_0^Q f(D)dD\right\}$$

$$= C_1 t \int_0^Q f(D)dD - C_2 t \left\{1 - \int_0^Q f(D)dD\right\}$$

Since $\int_0^\infty f(D)dD = 1$

$$= C_1 t\ F(Q) - C_2 t + C_2 t\ F(Q)$$

$$= \left[(C_1 + C_2)\ F(Q) - C_2\right]t \qquad \because\ f(Q) = \int_0^Q f(D)dD$$

The EC(Q) will have a relative minimum at Q^* if

$$\left.\frac{d\ EC(Q)}{dQ}\right|_{Q=Q^*} = (C_1 + C_2)\ F(Q^*) - C_2 = 0$$

$$\therefore \quad \Longrightarrow \quad F(Q^*) = \frac{C_2}{C_1 + C_2}. \qquad (10)$$

Since $\left.\frac{d^2 EC(Q)}{dQ^2}\right|_{Q=Q^*} = (C_1 + C_2)\ f(Q^*)t \geq 0$

$$\because\ C_1 \geq 0,\quad C_2 \geq 0,\quad f(Q) > 0.$$

It follows that EC(Q) attain minimum value at $Q = Q^*$ and therefore it also satisfy (10). Thus the condition for optimality which gives the optimum value, Q^* to have quantity on hand at the beginning of period is

$$P(D \le Q) = F(Q^*) = \frac{C_2}{C_1 + C_2} \qquad (11)$$

Hence we must order $Q^* - I$ units, $I < Q^*$

<u>ALGORITHMIC PROCEDURE IS AS FOLLOWS</u> :

D = Demand is assumed to be rectangular between [A, B]
i.e., $D \sim U\,[A, B]$

<u>STEP-1</u> : Read C_1, C_2, I_1, A, B (lower and upper limits in rectangular distribution).

<u>STEP-2</u> : Compute $R = C_2/(C_1 + C_2)$

<u>STEP-3</u> : Assuming distribution of demand is uniform over [A, B], equate

$$\frac{C_2}{C_1 + C_2} = \frac{1}{B-A}\,[Q-A]$$

$$\Longrightarrow \quad Q = Q^* = A + \frac{C_2}{C_1 + C_2} \times (B-A) = A + R(B-A)$$

<u>STEP-4</u> : Find $Q \leq Q^*$

<u>STEP-5</u> : order $Q^* - I1$ if $I1 < Q^*$

do not order if $I1 > Q^*$

MODEL-3

PROBABILISTIC INVENTORY MODEL

SINGLE PERIOD PROBABILISTIC MODEL WITHOUT SET-UP COST, CONTINEOUS DEMAND AND DISCRETE UNIT REPLACEMENT

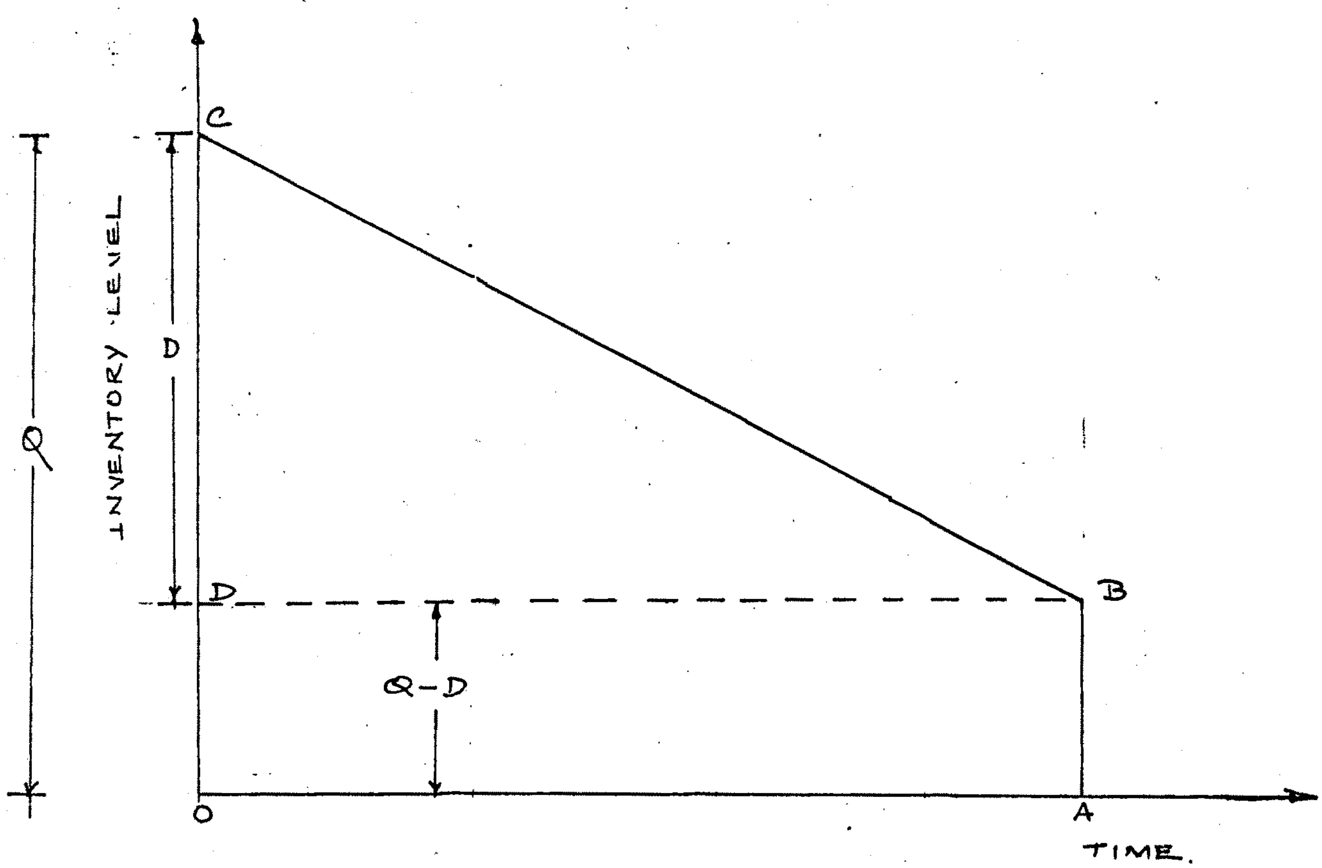
This model is similar to Model-1, except that demand is contineous. Like Model-1 here also two cases arises.

I. Demand is less than stock.

II. Demand is more than stock.

CASE-1 : DEMAND IS LESS THAN STOCK

In this case only carrying cost will incur. This cost is determined with the help of the situation described in following figure.



Average carrying inventory = $\left[Q - \frac{D}{2}\right]$

Average shortage inventory = 0

$$\text{Carrying cost} = C_1 \times \text{Inventory area of DABC}$$

$$= C_1 \times \frac{1}{2} \{AB + DC\} DB$$

$$= C_1 \times \frac{1}{2} \{Q - D + Q\} t$$

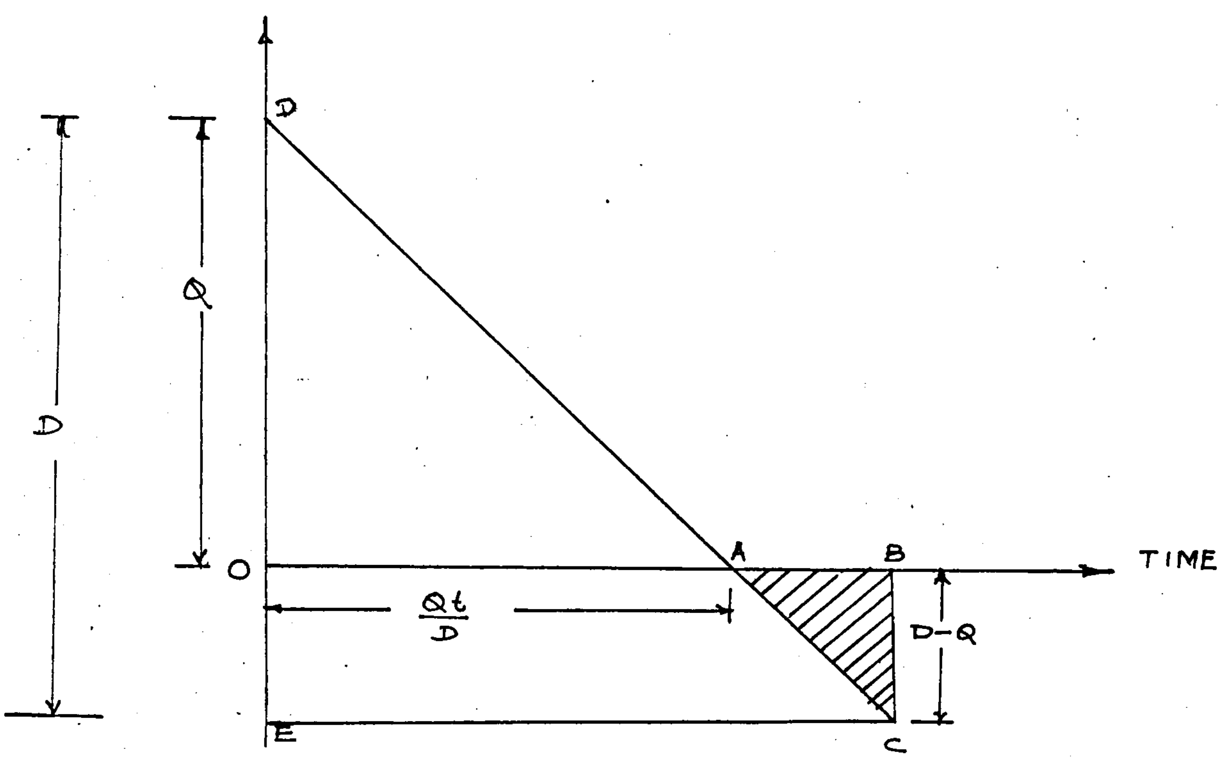
$$= \frac{1}{2} C_1 (2Q - D)$$

Thus expected carrying cost is given by

$$\frac{1}{2} C_1 \, t \sum_{D=0}^{Q} (2Q - D) \, f(D) \quad D \leq Q$$

<u>CASE-2</u> : DEMAND IS MORE THAN STOCK

In this case only the shortage cost will incur. This cost is determined with the help of the solution described in following figure.



Average carrying inventory $= \frac{Q^2}{2D^2}$

Average shortage inventory $= \frac{(D-Q)^2}{2D}$

Inventory area $\triangle OAD = \frac{1}{2}\,(D \times OA)$

$$= \frac{1}{2}\left\{Q \times \frac{Qt}{D}\right\}$$

$$= \frac{1}{2}\,\frac{Q^2\,t}{D}$$

Also by the property of similarity of $\triangle DEC$ and $\triangle OAD$ we have

$$\frac{DO}{DE} = \frac{OA}{EC} \quad \text{or} \quad \frac{Q}{D} = \frac{OA}{t}$$

$$\therefore \quad OA = \frac{Qt}{D}\,.$$

In the above figure shortage is shown by $\triangle ABC$

$$\therefore \quad \text{Area of } \triangle ABC = \frac{1}{2}\,AD \times BC$$

$$= \frac{1}{2}\,(EC-OA) \times BC$$

$$= \frac{1}{2}\,\left(t - \frac{Qt}{D}\right)\,(D-Q)$$

$$= \frac{1}{2}\,(D-Q)^2\,t.$$

The expected shortage cost is then given by

$$\sum_{D=Q+1}^{\infty} \left[C_1\,\frac{Q^2\,t}{2D} + \frac{C_2}{2D}\,(D-Q)^2\,t\right]\,f(D), \quad D > Q$$

Thus the total expected cost is given by

$$EC(Q) = C_1 t \sum_{D=0}^{Q} \left(Q - \frac{D}{2}\right)\,f(D)$$

$$+ \sum_{D=Q+1}^{\infty} \left[\frac{1}{2} C_1 \frac{Q^2 t}{2D} + \frac{C_2}{2D} (D-Q)^2 t\right] f(D) \quad . \qquad (12)$$

In order to calculate the optimum value Q^* of Q which ninimizes EC(Q) the following condition

$$\Delta EC(Q^* - 1) < 0 < \Delta EC(Q^*)$$

must hold.

By differential calculus, we know that

$$\Delta EC(Q) = EC(Q+1) - EC(Q)$$

Thus replacing Q by Q + 1 in equation (12) we get

$$EC(Q+1) = C_1 \sum_{D=0}^{Q+1} (Q+1 - \frac{D}{2}) f(D) + C_1 \sum_{D=Q+2}^{\infty} \frac{(Q+1)^2}{2D} f(D)$$

$$+ C_2 \sum_{D=Q+2}^{\infty} \frac{(D-Q-1)^2}{2D} f(D)\Bigg] t \qquad (13)$$

Now

$$C_1 \sum_{D=0}^{Q+1} (Q+1 - \frac{D}{2}) f(D) = C_1 \sum_{D=0}^{Q} (Q+1 - \frac{D}{2}) f(D)$$

$$+ C_1 (Q+1 - \frac{Q+1}{2}) f(Q+1)$$

$$= C_1 \sum_{D=0}^{Q} (Q - \frac{D}{2}) f(D) + C_1 \sum_{D=0}^{Q} f(D)$$

$$+ C_1 (\frac{Q+1}{2}) f(Q+1)$$

$$C_1 \sum_{D=Q+2}^{\infty} \frac{(Q+1)^2}{2D} f(D) = C_1 \sum_{D=Q+1}^{\infty} \frac{Q^2}{2D} f(D)$$

$$+ C_1 Q \sum_{D=Q+1}^{\infty} \frac{f(D)}{D} + \frac{C_1}{2} \sum_{D=Q+1}^{\infty} \frac{f(D)}{D}$$

$$- C_1 \frac{Q+1}{2} f(Q+1)$$

and $$C_2 \sum_{D=Q+2}^{\infty} \frac{(D-Q-1)^2}{2D} = C_2 \sum_{D=Q+1}^{\infty} \frac{(D-Q)^2}{2D} F(D)$$

$$- C_2 \sum_{D=Q+1}^{\infty} F(D) + C_2 Q \sum_{D=Q+1}^{\infty} \frac{F(D)}{D} + \frac{1}{2} C_2 \sum_{D=Q+1}^{\infty} \frac{F(D)}{D}$$

Thus from equation (12) and (13) we have

$$\Delta EC(Q) = \left[(C_1 + C_2) \left\{ F(Q) + (Q + \frac{1}{2}) \sum_{D=Q+1}^{\infty} \frac{f(D)}{D} \right\} - C_2 \right] t \quad (14)$$

where $$F(Q) = \sum_{D=0}^{Q} f(D)$$

Let $$L(Q) = F(Q) + (Q + \frac{1}{2}) \sum_{D=Q+1}^{\infty} \frac{f(D)}{D}$$

then equation (14) becomes

$$\Delta EC(Q) = EC(Q+1) - EC(Q) = \left[(C_1 + C_2) L(Q) - C_2 \right] t \quad (15)$$

Similarly, substituting (Q-1) for Q in equation (15) we have

$$\Delta EC(Q-1) = EC(Q) - EC(Q-1) = \left[(C_1 + C_2) L(Q-1) - C_2 \right] t \quad (16)$$

But $\Delta EC(Q) > 0$ and $\Delta EC(Q-!) < 0$ for minimum of $EC(Q)$ therefore consider now such a value of Q say Q^* such that

$$\left. \begin{array}{ll} & (C_1 + C_2) L(Q^*) - C_2 \geq 0 \\ \text{and} & (C_1 + C_2) L(Q^* - 1) C_2 \leq 0 \end{array} \right\} \quad (17)$$

For any $(Q^*+1) > Q^*$ and $(Q^*-1) < Q^*$ inequalities (17) holds since $L(Q)$ is non-decreasing for increasing Q. Thus rearranging the terms in (17) we get

$$L(Q^*-1) \le \frac{C_2}{C_1+C_2} \le L(Q^*)$$

where

$$L(Q) = F(Q) + (Q + \frac{1}{2}) \sum_{D=Q+1}^{\infty} \frac{f(D)}{D}$$

ALGORITHMIC PROCEDURE FOR THIS MODEL IS AS FOLLOWS :

STEP-1 : Read C_1, Q, f(D), D = 0, 1, 2,..., 6

STEP-2 : Find lower bound

$$LB = \sum_{D=0}^{Q-1} f(D) + (Q - \frac{1}{2}) \sum_{D=Q}^{N} \frac{f(D)}{D}$$

$$UP = \sum_{D=0}^{Q} f(D) + (Q + \frac{1}{2}) \sum_{D=Q+1}^{N} \frac{f(D)}{D}$$

STEP-3 : Set $\frac{C_2}{C_1+C_2} = LB \Longrightarrow C_2 = (C_1+C_2)\, LB$

$\Longrightarrow$ find $C_2 = \frac{C_1\, LB}{1-LB}$ $(1-LB)\, C_2 = C_1\, LB$

$\Longrightarrow C_2 = C_1\, LB/(1-LB)$

STEP-4 : Set $\frac{C_2}{C_1+C_2} = UP \Longrightarrow C_2 = C_1\, UP/(1-UP)$

STEP-5 : Print C_2 lies in LB and UB

i.e., $LB \le C_2 \le UB$

MODEL-4

PROBABILISTIC INVENTORY MODEL

SINGLE PERIOD PROBABILISTIC MODEL WITHOUT SET-UP CPST, CONTINEOUS DEMAND AND DISCRETE UNIT REPLENISHMENT

This model is identical to Model-2 except that the fixed set-up cost say K is associated with buying or making items in a given time period. Let I be the initial inventory before starting of the period. This implies that an order of size Q-1 items will be placed to bring the on hand inventory of the item up to Q. Thus expected cost will become

$$EC'(Q) = K + C(Q-I) + C_1 \int_{D=0}^{Q} (Q-D)\ f(D)\ dD$$

$$+ C_2 \int_{D=Q}^{\infty} (D-Q)\ f(D)\ dD$$

$$= K + EC(Q) \qquad (18)$$

where optimal value of Q say Q^* that minimizes EC(Q) is given by

$$F(Q^*-1) \le \frac{C_2 - C}{C_1 + C_2} \le F(Q^*)$$

where $$F(Q) = \int_0^D f(D)dD = \frac{C_2 - C}{C_1 + C_2}$$

Since K is constant, therefore minimum value of EC'(Q) must also be given by the same condition as given in (7) and (8) and hence Q^* will also minimize EC'(Q).

Let S = Maximum stock level, and s = reordered level i.e., when stock level fall on s, an order is placed to bring the stock of inventory items up to S. Thus the value of $S = Q^*$ the value of s is determined by the relationship

$$EC(s) = EC'(S) = K + EC(S), \quad s < S$$

As I is the initial inventory before starting the period, then to determine the order size to bring the on hand inventory of items upto Q^* the following 3 cases may be analysed

i) $I < s$ i.e., Initial inventory < Reorder level.

ii) $s \leq I \leq S$ i.e., Initial inventory lies between reorder level and maximum stock level.

iii) $I > S$ i.e., Initial inventory exceeds maximum level.

<u>CASE-1</u> : $(I < s)$

If we start the period with I units of inventory and do not buy or more, then $EC(I)$ is expected cost. But if we intend to buy additional $(Q-I)$ units so as to bring inventory level upto Q^* then $EC'(Q^*)$ will include the set-up cost also. Thus for all $I < s$, the condition for ordering is

$$\min_{Q>I} \left\{EC'(Q)\right\} = EC'(S) < EC(I)$$

i.e., when inventory level reaches $S = Q^*$ order for $Q-I$ units of inventory may be placed.

<u>CASE-2</u> : $S \le I \le Q$

In this case if $I < Q$, the order size is determined by the condition

$$EC(I) \le \min_{Q>I} EC'(Q) = EC'(S)$$

This implies that no ordering is less expensive than ordering.

Hence $\quad Q^* = I$.

<u>CASE-3</u> : If $(Q > I)$, then expected cost for an order upto Q will be more than total expected cost of no order is placed

i.e., $\quad EC(Q) \ge EC(I)$

Hence it is better not to place order and then $Q^* = I$.

```
10 REM ************************************************************************
20 REM *      SOFTWARE FOR PROBABILISTIC INVENTORY MODELS IS DEVELOPED BY       *
30 REM *      Mr. T. M. GARUD UNDER THE GUIDENCE OF Dr. R. V. KULKARNI          *
40 REM ************************************************************************
50 REM
60 REM ========================================================================
70 REM :               SOFTWARE FOR PROBABILISTIC INVENTORY MODELS              :
80 REM ========================================================================
90 REM : DIFFERENT NOTATIONS USED FOR DEVELOPING PORGRAM FOR MODEL Ist ARE AS :
100 REM : FOLLOWS                                                              :
110 REM -----------------------------------------------------------------------
120 REM : C1    :  INVENTORY CARRYING COST(HOLDING) COST PER UNIT              :
130 REM : C2    :  SHORTAGE COST PER UNIT                                      :
140 REM : P(I)  :  PROB. DISTN. OF NO. OF UNITS REQUIRED OR SOLD               :
150 REM : I1    :  QTY ON HAND BEFORE AN ORDER IS PLACED                       :
160 REM : CP(I):   CUMULATIVE PROB. DISTRIBUTION                               :
170 REM : Q     :  OPTIMUM ORDER QUANTITY                                      :
180 REM -----------------------------------------------------------------------
190 REM : DIFFERENT NOTATIONS USED FOR DEVELOPING PORGRAM FOR MODEL IInd ARE   :
200 REM : AS FOLLOWS                                                           :
210 REM : THIS MODEL ASSUMES DISTRIBUTION OF DEMOND TO BE RECTANGULAR          :
220 REM : ORDER [A,B]                                                          :
230 REM -----------------------------------------------------------------------
240 REM : C1    :  LOSS DUE TO SALE OF LEFT OVERS                              :
250 REM : C2    :  PROFIT MADE PER EVERY POUND SOLD                            :
260 REM : I1    :  QUANTITY ON HAND BEFORE AN ORDER IS PLACED                  :
270 REM : A     :  LOWER LIMIT FOR DEMOND AS PER RECTANGULAR DISTRIBUTION      :
280 REM : B     :  UPPER LIMIT FOR DEMOND AS PER RECTANGULAR DISTRIBUTION      :
290 REM -----------------------------------------------------------------------
300 REM : DIFFERENT NOTATIONS USED FOR DEVELOPING PORGRAM FOR MODEL IIIrd ARE  :
310 REM : AS FOLLOWS                                                           :
320 REM -----------------------------------------------------------------------
330 REM : FD(I):   PROBABILISTIC DISTRIBUTION FOR MONTHLY SALES                :
340 REM : D     :  LOWER BOUND                                                 :
350 REM : UB    :  UPPER BOUND                                                 :
360 REM : LBC2 :   LOWER BOUND FOR SHORTAGE COST                               :
370 REM : UPB2 :   UPPER BOUND FOR SHORTAGE COST                               :
380 REM : C1    :  INVENTORY CARRYING COST                                     :
390 REM : Q     :  OPT. QUANTITY AT THE BEGINING OF MONTH                      :
400 REM : N     :  MAXIMUM LIMIT OF MONTHLY SALES                              :
410 REM -----------------------------------------------------------------------
420 REM : DIFFERENT NOTATIONS USED FOR DEVELOPING PORGRAM FOR MODEL IVth ARE   :
430 REM : AS FOLLOWS                                                           :
440 REM : F(D) :   RECTANGULAR DISTRIBUTION                                    :
450 REM : C     :  UNIT PRICE                                                  :
460 REM : C1    :  INVENTORY CARRYING COST PER UNIT                            :
470 REM : C2    :  SHORTAGE COST PER UNIT                                      :
480 REM : I     :  INVENTORY AT THE BEGINNING OF THE PERIOD                    :
490 REM : K     :  SET UP COST IN Rs.                                          :
500 REM : L     :  LOWER LIMIT FOR RECTANGULAR DISTRIBUTION                    :
510 REM : L     :  UPPER LIMIT FOR RECTANGULAR DISTRIBUTION                    :
520 REM : S1    :  MAXIMUM STOCK LEVEL                                         :
530 REM : S2    :  REORDER LEVEL                                               :
540 REM -----------------------------------------------------------------------
```

```
550 REM PROBABILISTIC INVENTORY MODEL SOFTWARE
560 CLS
570 PRINT STRING$(70,"-")
580 PRINT "PROBABILISTIC INVENTORY MODELS "
590 PRINT STRING$(70,"-")
600 PRINT :PRINT
610 PRINT "1. SINGLE PERIOD PROB. MODELS WITHOUT SET UP COST ,INSTANTANEOUS DEMA
ND ,DESCRETE UNITS REPLENISHMENT "
620 PRINT
630 PRINT "2. SPPM WITHOUT SET UP COST ,INSTANTANEOUS DEMAND,CONT. UNITS REPLENI
SHMENT "
640 PRINT
650 PRINT "3. SPPM WITHOUT SET UP COST , CONT. DEMAND AND DESCRETE UNITS REPLENI
SHMENT "
660 PRINT
670 PRINT "4. SPPM WITH SET UP COST , INSTANTANEOUS DEMAND AND CONTINEOUS UNITS
REPLENISHMENT "
680 PRINT "5. STOP EXECUTION "
690 PRINT :PRINT :PRINT
700 INPUT "ENTER YOUR CHOICE (1,5) : ",I
710 ON I GOTO 740,1270,1760,2310,2750
720 PRINT "..... INVALID CHOICE..... TRY AGAIN " :PRINT
730 GOTO 560
740 REM PROGRAM TO FIND OPTIMUM ORDER QTY WHICH MINIMIZES THE TOTAL COST
750 REM C1= INVENTORY CARRYING COST(HOLDING ) COST PER UNIT
760 REM C2= SHORTAGE COST PER UNIT
770 REM P(I)= PROB. DISTN. OF NO. OF  UNITS REQUIRED OR SOLD
780 REM I1= QTY ON HAND BEFORE AN ORDER IS PLACED
790 REM CP(I) = CUMULATIVE PROB. DISTRIBUTION
800 REM Q = OPTIMUM ORDER QTY
810 CLS
820 INPUT "ENTER INVT. CARRYING COST PER  UNIT :",C1
830 PRINT :INPUT "ENTER SHORTAGE COST PER UNIT : ",C2
840 PRINT :INPUT "ENTER QTY ON HAND BEFORE AN ORDER IS PLACED(I1) :",I1
850 R=C2/(C1+C2)
860 INPUT "ENTER LAST VALUE IN THE DISTRIBUTION OF QTY REQUIRED  IS  N : ",N
870 PRINT :PRINT "ENTER PROB. DISTRIBUTION "
880 PRINT :FOR I=0 TO N
890 READ P(I) :NEXT I
900 DATA .01,.14,.20,.30,.25,.10
910 CLS
920 PRINT STRING$(79,"-")
930 PRINT "THE DATA SUPPLIED FOR EMC MODEL EK 5D BREAK  IS AS FOLLOWS"
940 PRINT :PRINT
950 PRINT STRING$(79,"-")
960 PRINT :PRINT "INVT. CARRYING COST PER UNIT C1 = ";C1:PRINT
970 PRINT "SHORTAGE COST PER UNIT C2 = ";C2:PRINT
980 PRINT "QTY ON HAND  BEFORE AN ORDER IS PLACED I1 = ";I1
990 PRINT
1000 CP(0)=P(0)
1010 FOR I=1 TO N:CP(I)=CP(I-1)+P(I):NEXT I
1020 PRINT STRING$(75,"-")
1030 PRINT "DEMAND ","PROBABILITY","CUMM. PROB."
1040 PRINT STRING$(75,"-")
1050 FOR I=0 TO N :PRINT I,P(I),CP(I):NEXT I:PRINT STRING$(75,"-")
1060 FOR I=0 TO N
1070 IF CP(I)<R THEN 1100
1080 Q=I
```

```
1090 GOTO 1120
1100 NEXT I
1110 PRINT
1120 IF Q>I1 THEN 1190
1130 PRINT "PRESS F5 TO SEE THE OUTPUT " :PRINT :PRINT
1140 STOP
1150 CLS:PRINT STRING$(75,"-")
1160 PRINT "THE OUTPUT OF MODEL I FOR SUPPLIED DATA IS AS FOLLOWS ":PRINT STRING
$(75,"-")
1170 PRINT :PRINT "..... DO NOT ORDER .....":PRINT :PRINT STRING$(75,"-")
1180 GOTO 1250
1190 PRINT "PRESS F5 TO SEE THE OUTPUT ":PRINT :PRINT :STOP
1200 CLS:PRINT STRING$(55,"-"):PRINT "THE O/P OF MODEL I IS AS FOLLOWS "
1210 PRINT STRING$(55,"*")
1220 PRINT
1230 PRINT "OPT. ORDERING POLICY IS TO ORDER ";Q-I1;"UNITS"
1240 PRINT STRING$(55,"*")
1250 PRINT "PRESS F5 TO STOP EXECUTION ":STOP:GOTO 2750
1260 REM PROGRAM TO FIX OPTIMUM ORDER QTY WHICH MINIMIZES TOTAL COST
1270 CLS
1280 'PROBABILISTIC INVENTORY MODEL II.
1290 ' IT ASSUMES DISTRIBUTION OF DEMAND TO BE RECTANGULAR OVER [A,B]
1300 INPUT "ENTER LOSS DUE TO SALE OF LEFT OVERS C1",C1
1310 INPUT "ENTER PROFIT MADE PER EVERY POUND SOLD C2",C2
1320 INPUT "QTY ON HAND BEFORE AN ORDER IS PLACED I1",I1
1330 PRINT "QTY ON HAND BEFORE AN ORDER IS PLACED IS = ";I1
1340 R=C2/(C1+C2)
1350 INPUT "ENTER LOWER LIMIT FOR DEMAND AS PER RECTANGULAR DISTRIBUTION",A
1360 INPUT "ENTER UPPER LIMIT FOR DEMAND AS PER RECTANGULAR DISTRIBUTION ",B
1370 PRINT
1380 FD=1/(B-A)
1390 Q=A+R/FD
1400 PRINT STRING$(55,"-")
1410  PRINT "THE INPUT DATA FOR MODEL II IS AS FOLLOWS "
1420 PRINT STRING$(55,"-")
1430 PRINT
1440 PRINT "LOSS DUE TO SALE OF LEFT OVERS IS C1= ";C1
1450 PRINT
1460 PRINT "PROFIT EARNED PER EVERY POUND SOLD IS =";C2
1470 PRINT
1480 PRINT "QTY ON HAND BEFORE AN ORDER IS PLACED I1",I1
1490 PRINT
1500 PRINT "LOWER LIMIT FOR RECTANGULAR DISTRIBUTION IS A=";A
1510 PRINT
1520 PRINT "UPPER LIMIT FOR RECTANGULAR DISTRIBUTION IS B=";B
1530 PRINT
1540 PRINT STRING$(55,"-")
1550 PRINT "PRESS F5 TO SEE THE OUTPUT "
1560 STOP
1570 IF I1 <Q THEN 1670
1580 CLS
1590 PRINT STRING$(55,"-")
1600  PRINT "THE OUTPUT  FOR MODEL II IS AS FOLLOWS "
1610 PRINT STRING$(55,"-")
1620 PRINT
```

```
1630 PRINT STRING$(55,"-")
1640 PRINT "DO NOT ORDER "
1650 PRINT STRING$(55,"-")
1660 GOTO 1750
1670 CLS
1680 PRINT STRING$(55,"-")
1690  PRINT "THE OUTPUT  FOR MODEL II IS AS FOLLOWS "
1700 PRINT STRING$(55,"-")
1710 PRINT
1720 PRINT STRING$(55,"-")
1730 PRINT "OPTIMUM ORDER QTY IS :";Q;"UNITS"
1740 PRINT STRING$(55,"-")
1750 PRINT "PRESS F5 TO STOP EXECUTION ":PRINT :PRINT :STOP :GOTO 2750
1760 REM PROGRAM FOR PROBABILISTIC MODEL III
1770 REM PROGRAM TO FIND SHORTAGE COST WHEN OPT, ORDER QTY (Q) IS KNOWN
1780 CLS
1790 REM FD(I)= PROB. DISTRIBUTION FOR MONTHLY SALES
1800 REM D= LOWER BOUND ,UB =UPPER BOUND, LBC2=LOWER BOUND FOR SHORTAGE COST C2
1810 REM UPB2 = UPPER BOUND FOR SHORTAGE COST C2
1820 INPUT "ENTER INVT. CARRYING COST C1 :";C1
1830 PRINT :INPUT "ENTER OPT. QTY AT THE BEGINNING OF MONTH (Q): ",Q
1840 INPUT "ENTER MAX. LIMIT OF MONTHLY SALES (N) :",N
1850 PRINT "ENTER PROB. DISTN. OF MONTHLY SALES ":PRINT
1860 PRINT
1870 REM ...
1880 FOR I= 0 TO N
1890 READ FD(I)
1900 NEXT I
1910 DATA .02,.05,.30,.27,.20,.10,.06
1920 REM TO FIND LOWER BOUND FOR COST RATIO
1930 FOR I=0 TO Q-1
1940 S1=S1+FD(I)
1950 NEXT I
1960 FOR I=Q TO N
1970 S2=S2+FD(I)/I
1980 NEXT I
1990 LB=S1+(Q-.5)*S2
2000 LBC2=(C1*LB)/(1-LB)
2010 REM COMPUTATION OF UPPER BOUND
2020 FOR I=0 TO Q
2030 S3=S3+FD(I)
2040 NEXT I
2050 FOR I=Q+1 TO N
2060 S4=S4+FD(I)/I
2070 NEXT I
2080 UB=S3+(Q+.5)*S4
2090 UBC2=(C1*UB)/(1-UB)
2100 PRINT STRING$(75,"-")
2110 PRINT "INPUT DATA SUPPLIED FOR IMPLEMENTATION OF MODEL III IS AS FOLLOWS"
2120 PRINT STRING$(75,"-")
2130 PRINT :PRINT "INVT. CARRYING COST C1 = ";C1
2140 PRINT
2150 PRINT "OPT. ORDER QTY AT THE BEGINNING OF EACH MONTH Q =";Q
2160 PRINT
```

```
2170 PRINT STRING$(75,".")
2180 PRINT "SALES","PROB."
2190 FOR I=0 TO N
2200 PRINT I,FD(I)
2210 NEXT I
2220 PRINT STRING$(75,".")
2230 CLS
2240 PRINT STRING$(75,"."):PRINT "THE O/P FOR MODEL III IS AS FOLLOWS ":PRINT ST
RING$(75,".")
2250 PRINT "LOWER BOUND FOR C2 = ";LBC2
2260 PRINT "UPPER BOUND FOR C2 = ";UBC2
2270 PRINT "HENCE VALUE OF C2 MUST LYE IN  THE FOLLOWING INTERVAL "
2280 PRINT LBC2;"<= C2<= ";UBC2
2290 GOTO 2750
2300 REM PROBABILISTIC INVENTORY MODEL IV
2310 REM SPPM WITH SET UP COST
2320 REM F(D) ASSUMES TO BE RECTANGULAR DISTRIBUTION
2330 REM C IS UNIT PRICE
2340 CLS
2350 INPUT "ENTER INVT. CARRYING COST PER UNIT C1 :",C1:PRINT
2360 INPUT "ENTER SHORTAGE COST PER UNIT C2 =";C2:PRINT
2370 INPUT "ENTER INVT. AT THE BEGINNING OF THE PERIOD I = ";I:PRINT
2380 INPUT "ENTER SET UP COST IN Rs. (K) :",K:PRINT
2390 INPUT "ENTER LOWER LIMIT IN RECTANGULAR DISTRIBUTION (L) :",PRINT
2400 INPUT "ENTER UPPER LIMIT IN RECTANGULAR DISTRIBUTION (U) :",U:PRINT
2410 INPUT "ENTER MAX . STOCK LEVEL S1 =";S1:PRINT
2420 INPUT "ENTER REORDER LEVEL S2 =";S2 :PRINT
2430 F=1/(U-L)
2440 Q=(C2-C)/(C1+C2)*(U-L)
2450 CLS :PRINT STRING$(75,".")
2460 PRINT "THE INPUT DATA SUPPLIED FOR THE IMPLEMENTATION OF MODEL IV IS AS FOL
LOWS "
2470 PRINT STRING$(75,"."):PRINT
2480 PRINT "INVT. CARRYING COST PER UNIT C1= ";C1:PRINT
2490 PRINT "ENTER SHORTAGE COST PER UNIT C2 =";C2:PRINT
2500 PRINT "INVT. AT THE BEGINNING OF PERIOD I = ";I:PRINT
2510 PRINT "SET UP COST IN Rs. K= ";K:PRINT
2520 PRINT "LOWER LIMIT IN THE RECTANGULAR DISTRIBUTION L =";L
2530 PRINT
2540 PRINT "UPPER LIMIT IN THE RECTANGULAR DISTRIBUTION U= ";U:PRINT
2550 PRINT "MAX. STOCK LEVEL S1 =";S1:PRINT
2560 PRINT "REORDER LEVEL S2 = ";S2:PRINT
2570 PRINT
2580 PRINT STRING$(75,".")
2590 PRINT "PRESS F5 TO SEE THE OUTPUT"
2600 STOP
2610 PRINT STRING$(70,".")
2620 CLS
2630 PRINT STRING$(75,"."):PRINT "THE O/P OF MODEL IV IS AS FOLLOWS"
2640 PRINT STRING$(75,".")
2650 PRINT
2660 IF I<S1 THEN 2700
2670 IF Q>I THEN 2720
2680 PRINT "PLACE AN ORDER OF ";I;" UNITS"
2690 GOTO 2750
2700 PRINT "PLACE AN ORDER FOR ";Q-I;" UNITS"
```

```
2710 GOTO 2750
2720 PRINT "BETTER NOT TO ORDER "
2730 PRINT :PRINT "OPT. ORDER QTY IS INITIAL QTY ON HAND BEFORE BEGINNING OF THE
 PERIOD "
2740 PRINT STRING$(75,".")
2750 CLS
2760 LOCATE 12,10 :PRINT STRING$(55,"*")
2770 LOCATE 14,15 :PRINT "...... EXECUTION OVER ..... "
2780 LOCATE 16,15 :PRINT " ..... THANK  YOU ....."
2790 LOCATE 18,15 :PRINT "...... HAVE A NICE TIME ....."
2800 LOCATE 20,10 :PRINT STRING$(55,"*")
2810 END
```