CHAPTER - III

DETERMINISTIC MODELS.

3.1 DESCRIPTION AND DEVELOPMENT OF DETERMINISTIC INVENTORY MODELS.

3.2 INTEGRATED SOFTWARE FOR DETERMINISTIC INVENTORY MODELS.

DESCRIPTION AND DEVELOPMENT OF DETERMINISTIC INVENTORY MODELS

3.1 INTRODUCTION :

An inventory problem can be solved by using several methods starting from trial-and-error methods to mathematical and simulation models. Mathematical models helps in deriving certain rules which may suggest how to minimize the total inventory cost in case of deterministic demand or how to minimize expected cost in case of probabilistic demand. The deterministic model are usually termed as EOQ models. EOQ is the size and shortage cost (if allowed) during given period of time.

NOTATION USED TO DEVELOP INVENTORY MODELS :

The notation used for development of various inventory models are as follows. The brackets indicates the unit of measurement of each.

C_u = Purchase (or manufacturing) cost per unit (Rs./unit).

C_p = Ordering (set-up) cost per order (Rs./order).

C_h = Cost of carrying one unit in the inventory for a given length of time (Rs./unit-time).

r = Cost of carrying one Rs.'s worth of inventory per time period (percent/time).

C_s = Shortage cost per unit per time (Rs./unit-time).

D = Demand rate, units per time (unit/time).

Q = Order quantity i.e., no. of units ordered per order (unit).

R = Reorder point i.e., the level of inventory at which an order is placed (units).

L = Replenishment lead time (time).

n = No. of orders per time period (orders/time).

t = Reorder cycle time i.e., the time interval between successive orders to replenish (time).

t_p = Production period (time).

P = Production rate i.e., the rate of which quantity Q is added to inventory (quantity/time).

TC = Total inventory cost (Rs.).

TIC = Total incremental inventory cost (Rs.)

T = Optimal cycle time

The deterministic inventory models can be classified into the following three categories.

1. Inventory Models with no shortage
2. Inventory Models with shortage
3. Economic Lot size model with quantity discounts

1. INVENTORY MODELS WITH NO SHORTAGE :

MODEL-1

1.1 THE ECONOMIC LOT SIZE MODEL :

The assumption made under this model are as follows :

i) The demand is contineous at a constant rate

ii) Replenishment is instantaneous i.e., the entire order quantity is received all at one time as soon as the order is placed.

iii) No shortages are allowed

iv) Quantity discounts are not available

From the above assumptions, the following relationship can be established.

i.e., $Q = D.t.$

In other words, one can say that the quantity ordered must be sufficient to satisfy the demand rate during reorder cycle time t.

Figure 3.1 shows that the behaviour of an inventory system which operates by the above listed assumptions. At the beginning of the cycle time we start with a maximum amount of inventory equal to the order quantity Q. As this amount is used up, the level of inventory drops at a fixed rate equal to the demand rate D. When it reaches a reorder point R, enough inventory is available to cover expected demand during the lead time L. At this point, an order is placed equal to Q which arrives at the end of lead time, when the inventory level reaches zero. This amount is placed in stock all at once and the inventory level goes up to its maximum value.

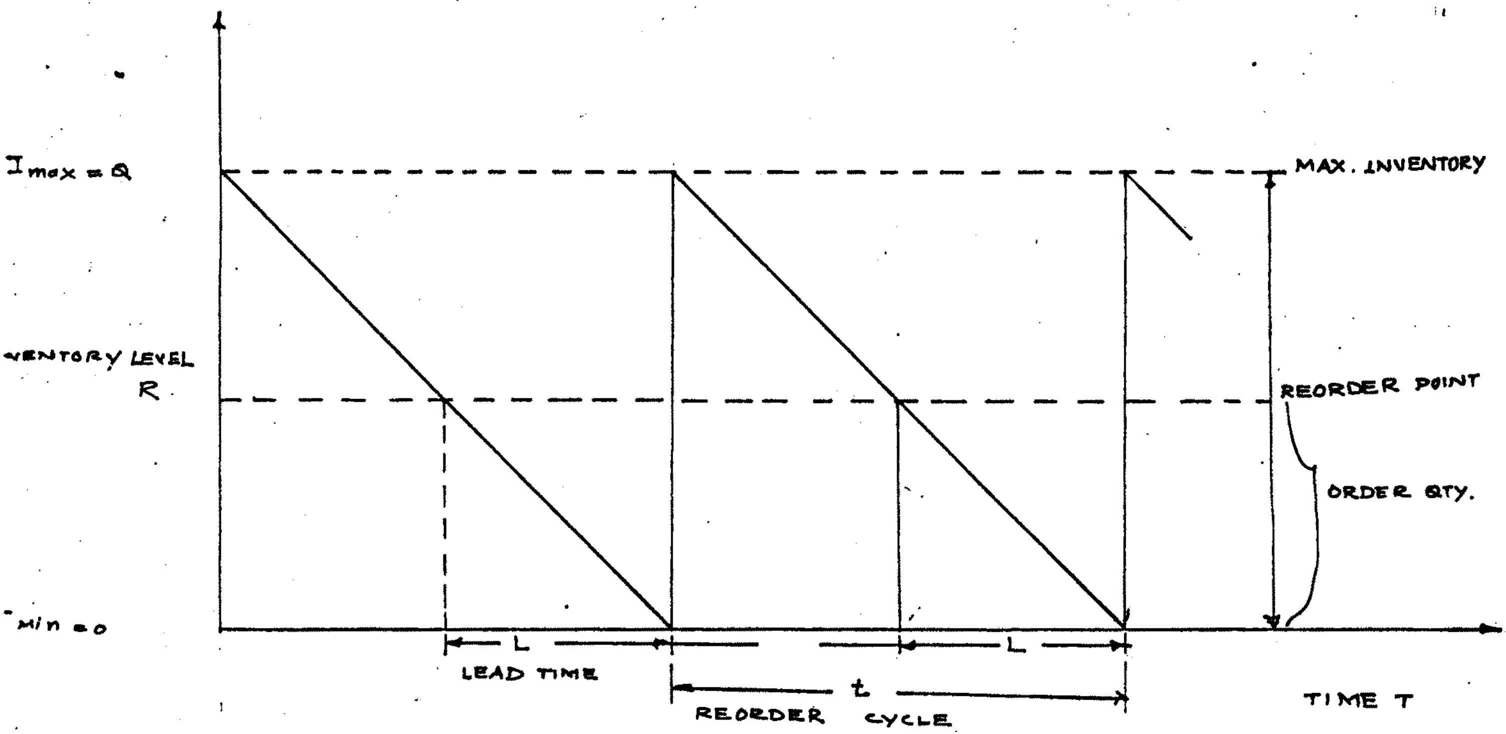


Fig. 3.1 : Inventory level with cont. demand and instanteneous supply

In this case the total incremental inventory cost is given by

Total incremental cost = carrying cost + ordering cost

$$= \left\{ \begin{array}{l} \text{Average investment} \\ \text{level x carrying} \\ \text{cost/unit/year} \end{array} \right\} + \left\{ \begin{array}{l} \text{No. of order placed} \\ \text{per year x ordering} \\ \text{cost/order} \end{array} \right\}$$

$$= \left\{ \frac{Imax + Imin}{2} \right\} . \; C_h + \frac{D}{Q} \; . \; C_p .$$

$$TIC = \frac{Q}{2} C_h + \frac{D}{Q} C_p . \qquad (3.1)$$

Here the shortage cost $C_s = 0$ because shortages are not allowed and purchase cost C is constant because quantity discounts are not available.

The optimal value of Q which minimizes TIC is obtained by equating the first derivative of TIC w.r. to Q to zero.

$$\therefore \quad \frac{d}{dQ}(TIC) = \frac{1}{2} C_h - \frac{D}{Q^2} C_p = 0$$

$$\therefore \quad Q^* = EOQ = \sqrt{\frac{2DC_p}{C_h}} \qquad (3.2)$$

The formula (3.2) is known as Harris or Wilson economic lot size model formula.

Further, to find optimal replenishment cycle time t^* we use

$$Q = D.t.$$

$$t^* = T = \frac{Q^*}{D} = \sqrt{\frac{2C_p}{C_h D}} \qquad (3.3)$$

Now to find number of orders to be placed in given time period can be determined by using

$$N = \frac{D}{Q^*} = \sqrt{\frac{DC_h}{2C_p}} \qquad (3.4)$$

Finally, minimum value of TIC can be determined by

$$TIC^* = \sqrt{2DC_h C_p} \qquad (3.5)$$

GRAPHICAL REPRESENTATION :

Graphical Representation of Economic Lot Size Model is as follows :-

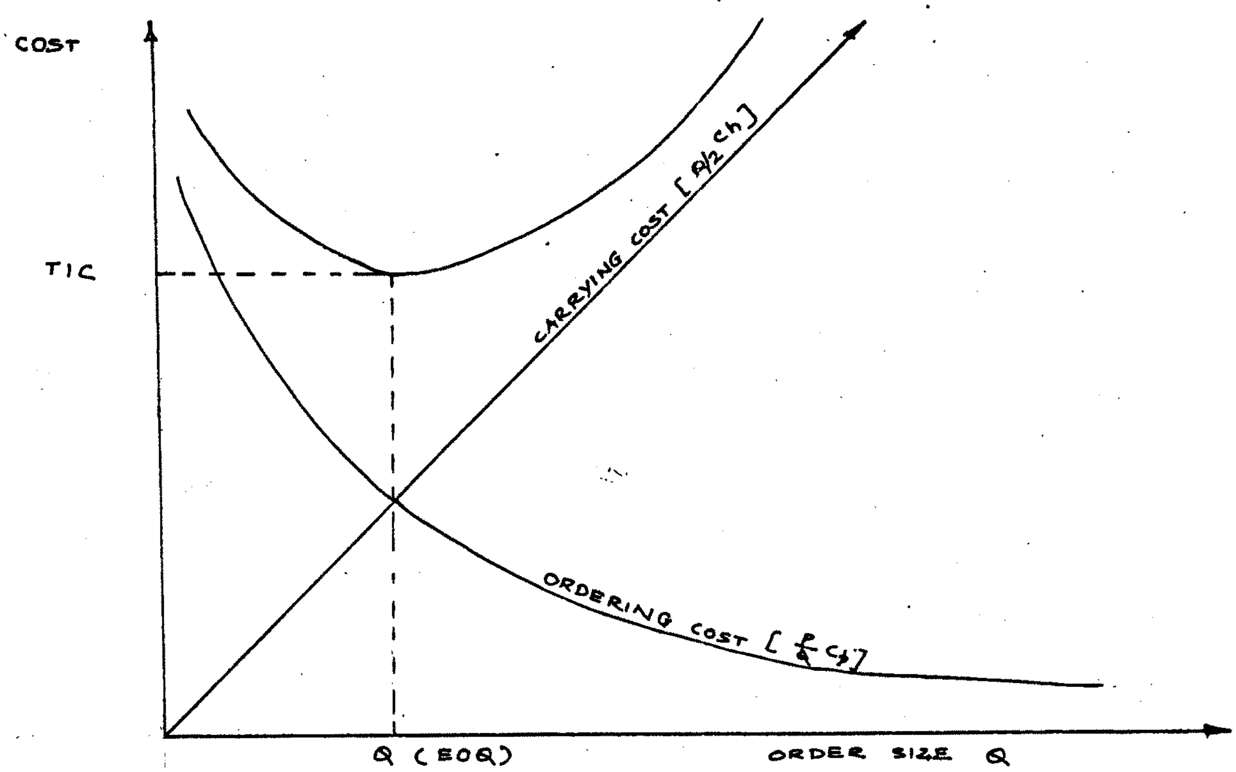


Fig. 3.2 : Relationship between EOQ & TIC

MODEL-2

1.2 THE ECONOMIC LOT SIZE MODEL WITH DEFFERENT RATES OF DEMAND IN DIFFERENT CYCLES :

The inventory system in this case is based on the same assumptions as in model-1 except that the demand is constant with different rates in different cycles i.e., Demand is dynamic.

If $t_1, t_2, \cdots, t_n$ are the times of successive cycles and $D_1, D_2, \cdots D_n$ are the demand rates at these cycles respectively then the total period T is given by

$$T = t_1 + t_2 + \cdots + t_n.$$

The behaviour of an inventory system which operates by the above assumptions is shown in the following figure.

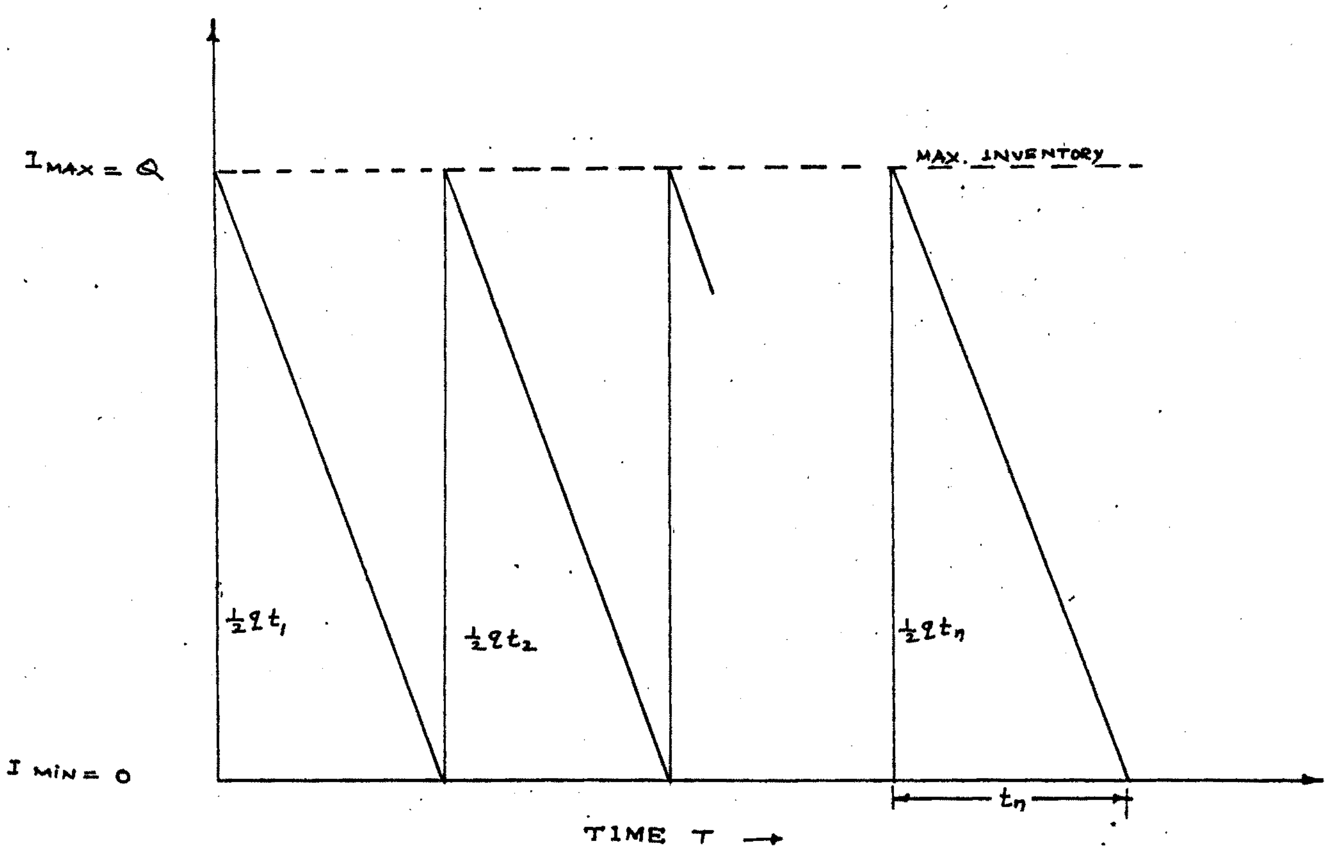


Fig. 3.3 : Inventory level with different rates of demand in different cycles.

Suppose each time a fixed quantity say q is produced then the number of production cycles in total time period T will be

$$n = \frac{D}{q}$$

where D is the total demand in time period T. Thus, the inventory carrying cost and set-up cost for the time period T will be

$$\text{carrying cost} = \frac{1}{2}\, q\, t_1\, C_h + \frac{1}{2}\, q\, t_2\, C_h + \cdots + \frac{1}{2}\, q\, t_n\, C_h$$

$$\text{carrying cost} = \frac{1}{2}\, q\, C_h\, (t_1 + \cdots + t_n) = \frac{1}{2}\, q\, C_h\, T$$

$$\text{set-up cost} = \left[\frac{D}{q}\right] \times C_p$$

Hence the total yearly incremental inventory cost is given by

$$TIC = \frac{1}{2}\, q\, C_h\, T + \left[\frac{D}{q}\right] C_p \qquad (*)$$

For optimum value of Q which minimizes TIC, we have

$$\frac{d}{dQ}(TIC) = \frac{1}{2}\, C_h\, T - \frac{DC_p}{q^2} = 0$$

$$\Longrightarrow \quad \frac{DC_p}{q^2} = \frac{1}{2}\, C_h\, T$$

$$\Longrightarrow \quad q = Q = EOQ = \sqrt{\frac{2DC_p}{C_h T}}$$

Also

$$\frac{d^2}{dQ^2}(TIC) = \frac{2DC_p}{q^3} > 0$$

Thus, the minimum value of TIC can be obtained by substituting the value of q in equation (*)

$$TIC = \sqrt{2C_p \; C_h (D/T)} = \text{optimum cost} \qquad (3.6)$$

From above equation it can be noted that there is a uniform rate of demand D if Model-1 is replaced by the average rate of demand (D/T).

MODEL-3

1.3 THE ECONOMIC LOT SIZE MODEL WITH FINITE REPLENISHMENT (SUPPLY) RATE :

This model is also based on the assumptions in Model-1 except that of instantaneous replenishment. The additional assumptions made here are as follows :

i) Supply is contineous and constant until Q units are supplied to stock then it stops.

ii) The production rate per unit of time is greater than demand rate i.e., P > D.

iii) Production begins immediately after production set-up.

The inventory system is shown in the following diagram.

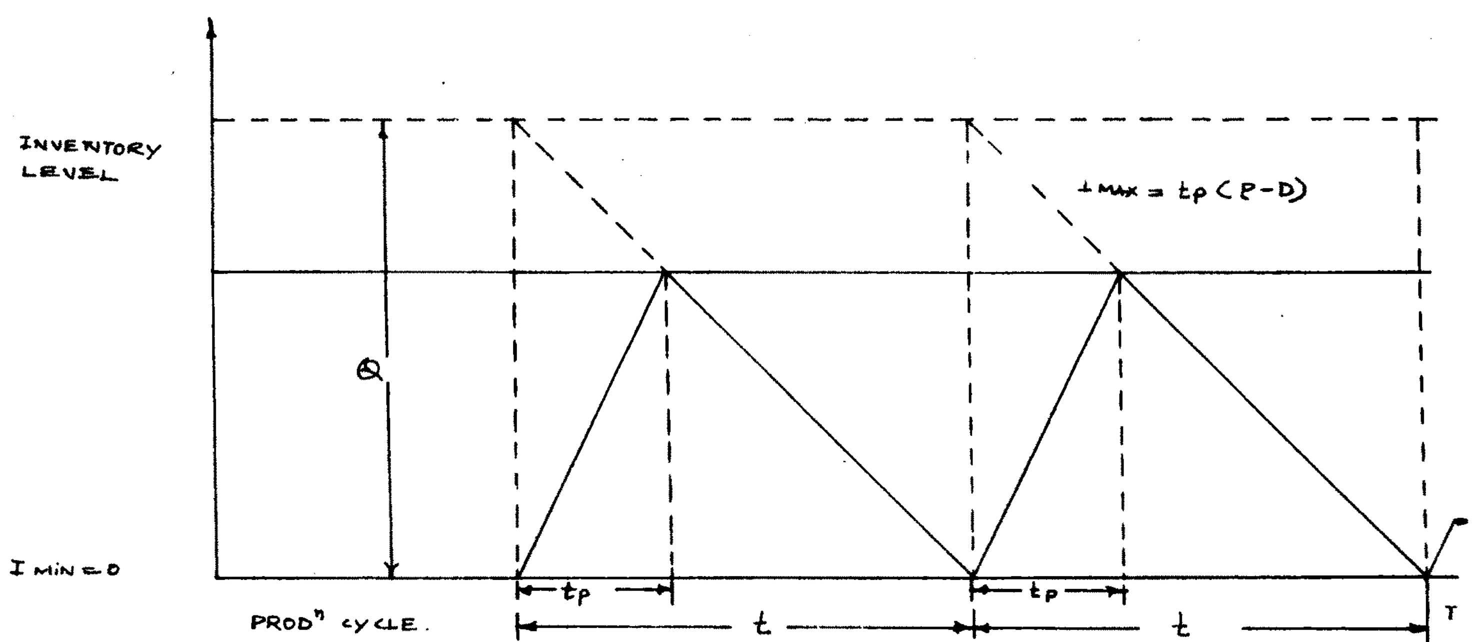


Fig. 3.4 : Inventory level with finite replenishment rate.

If t_p is the time required to produce the amount Q at a rate P and simultaneously decreased at the rate of D. Thus inventory accumulates at the rate of (P-D) units. Therefore, the maximum inventory level reached at the end of t_p will be

$$I_{max} = (P-D)\, t_p$$

$$= (P-D)\, \frac{Q}{P} = \left(1 - \frac{D}{P}\right) Q.$$

Since $I_{min} = 0$ therefore the average inventory will be $\frac{Q}{2}\left(1 - \frac{D}{P}\right)$.

Thus the carrying cost per unit of time is given by

$$\text{carrying cost} = \frac{Q}{2}\left(1 - \frac{D}{P}\right) C_h$$

$$\& \qquad \text{set-up cost} = \frac{D}{Q} \cdot C_p.$$

Adding the above two costs we have

$$TIC = \frac{Q}{2}\left(1 - \frac{D}{P}\right) C_h + \frac{D}{Q} C_p \qquad (**)$$

For optimum value of Q which minimizes TIC, we must have

$$\frac{d}{dQ} TIC = \frac{1}{2}\left(1 - \frac{D}{P}\right) C_h - \frac{D}{Q^2} C_p = 0.$$

$$\therefore \qquad Q = EOQ = \sqrt{\frac{2D\, C_p}{\left(1 - \frac{D}{P}\right) C_h}}$$

$$= \sqrt{\frac{2DC_p}{C_h}\left(\frac{P}{P-D}\right)} \qquad (3.7)$$

and

$$\frac{d^2}{dQ^2}(TIC) = \frac{2D}{Q^3} C_p > 0.$$

Thus, the minimum value of TIC can be obtained by substituting the value of Q in (**)

$$TIC = \sqrt{2DC_p\, C_h\left(1 - \frac{D}{P}\right)} \qquad (3.8)$$

Finally, optimum production cycle time

$$\frac{Q}{D} = \sqrt{\frac{2C_p\, P}{C_h D\, (P-D)}} \qquad (3.9)$$

2. <u>INVENTORY MODELS WITH SHORTAGES</u> :

All models discussed so far were based on the assumption that shortage cost was not allowed. As a result, all EOQ models presented in earlier section involved a trade-off between ordering cost and carrying cost. However, there could be situations in which an economic advantage may be gained by allowing shortages to occur.

The advantages of allowing shortages to occur are as follows :

1. Increase in cycle time results in spreading the ordering (set-up) costs over a longer period.
2. The reduced ventory carrying cost in a planning period is less tha ie increase in total inventory cost due to shortage condition.

MODEL-4

2.1 ECONOMIC LOT SIZE MODEL WITH CONSTANT DEMAND AND VARIABLE ORDER CYCLE TIME :

This model is based on all assumptions of Model-1 except that shortages (stock outs) are allowed. The figure 3.5 describes the changes in the inventory level with time. Every time the quantity Q is received, all back orders equal to an amount R are taken care of first and the remaining quantity M is placed in inventory as the surplus from which demand during next cycle will be satisfied. Here it may be noted that R units of Q are always in the back order list, i.e., these are never carried in stock. Thus it yields savings on the inventory carrying cost.

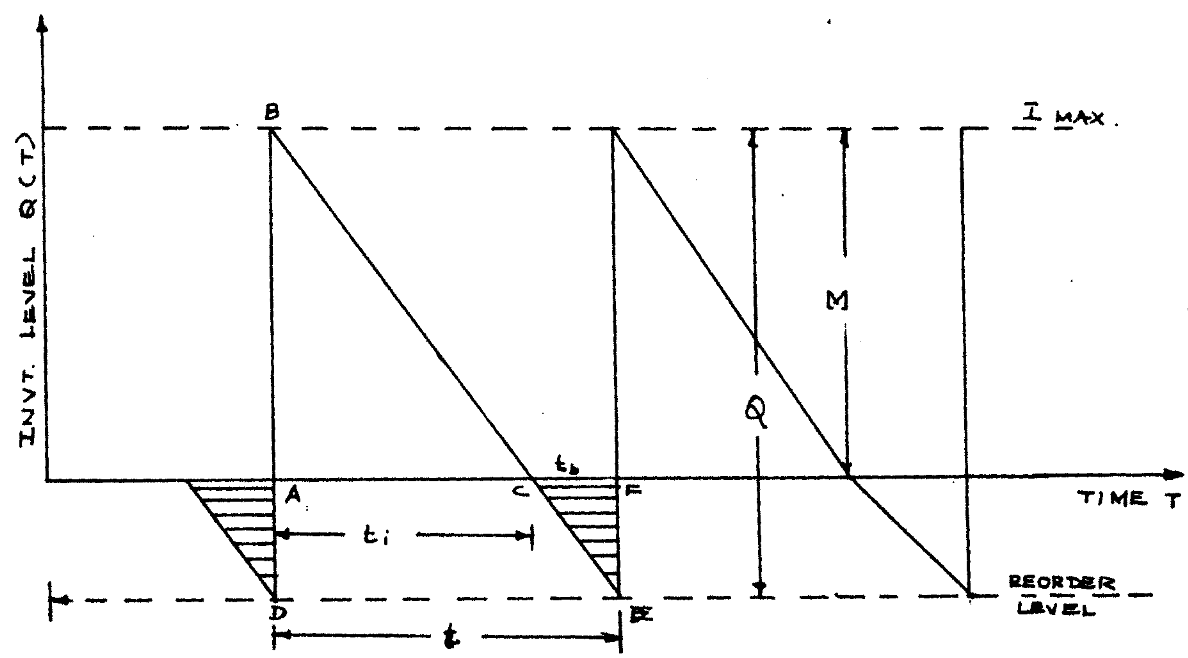


Fig. 3.5 : Inventory level with shortages

In this system, except for the purchase cost C which will be fixed, all other types of costs will be affected by the decisions concerning Q and M. Thus, in this case, we seek to minimize the total incremental cost.

TIC = ordering cost + carrying cost + shortage cost.

Suppose that shortage cost incurred is proportional to the number of units short and the duration t_b of the shortage. Now the time t_i when a positive inventory is available is given by

$$t_i = \frac{M}{D}.$$

Also the total cycle time t is given by

$$t = \frac{Q}{D}.$$

Thus, the average inventory on hand can be determined by dividing the area of the triangle ABC by the cycle time.

$$\therefore \quad \text{Average inventory} = \frac{1}{t}\left(\frac{M}{2}\right).\ t_i$$

$$= \frac{M^2}{2Q}$$

for $$t_i = \frac{M}{D}, \qquad t = \frac{Q}{D} \quad \text{and}$$

$$\therefore \quad \text{carrying cost} = \frac{M^2}{2Q}.\ C_h.$$

In this way, the time during which back orders incurred is given by

$$t_b = \frac{Q-M}{D}$$

Therefore, the average back order level can be determined by dividing the area under the triangle CFE by the cycle time t.

i.e., Average inventory $\frac{1}{t}\ (\frac{Q-M}{2}).\ t_b$

for $$t_b = \frac{Q-M}{D},$$

$\therefore$ shortage cost $= \frac{(Q-M)^2}{2Q}\ .\ C_s.$

Hence, the total yearly incremental cost is given by

$$TIC(Q,M) = \frac{D}{Q}.\ C_p + \frac{M^2}{2Q}C_h + \frac{(Q-M)^2}{2Q}.\ C_s \qquad (3.10)$$

Since TIC is the function of two variables Q and M

$\therefore$ for its minimum we must have

$$\frac{\partial^2(TIC)}{\partial Q^2}.\ \frac{\partial^2(TIC)}{\partial M^2} - \frac{\partial^2(TIC)}{\partial Q\cdot\partial M} > 0.$$

and $$\frac{\partial(TIC)}{\partial Q} = 0 \quad \text{and} \quad \frac{\partial(TIC)}{\partial M} = 0.$$

$\therefore$ The resultant optimum values are

$$Q^* = \sqrt{\frac{2DC_p}{C_h}.\ (\frac{C_h+C_s}{C_s})} \quad \text{Economic order quantity} \qquad (3.11)$$

and $$M^* = \sqrt{\frac{2DC_p}{C_h}.\ (\frac{C_h+C_s}{C_s})} \quad \text{Optimal stock level} \qquad (3.12)$$

In view of (3.11) and (3.12), (3.10) gives

$$TIC^* = \sqrt{2DC_p\ C_h(\frac{C_s}{C_h+C_s})} \quad \text{Optimal cost}$$

Further, optimal amount back ordered in units $R = Q^* - M^*$.

Finally,

Total cycle time $t = \frac{Q^*}{D}$.

$$= \sqrt{\frac{2C_p}{DC_h} \cdot \left(\frac{C_h+C_s}{C_s}\right)} \qquad (3.13)$$

MODEL-5

2.2 ECONOMIC LOT SIZE MODEL WITH CONSTANT DEMAND AND REORDER CYCLE TIME :

Let the reorder cycle time t be fixed i.e., inventory is to be supplied after every time period t. Also let $Q = D.t$ where D is the demand rate per unit time, Q is the fixed lot size to meet the demand for the period t. As shown in figure 3.5 the amount $M(< Q)$ is planned to meet the demand during time $t_i = M/D$. Since the ordering (set-up) cost and time t are constant, therefore the total incremental cost (TIC) is given by

TIC(M) = carrying cost + shortage cost

$$= \frac{M^2}{2Q} \cdot C_h + \frac{1}{2Q} (Q-M)^2 \cdot C_s \qquad (3.14)$$

Since TIC is the function of M only, therefore for optimal value of M and for minimum value of TIC, we must have

$$\frac{d}{dM}(TIC) = \frac{M}{Q} \cdot C_h + \frac{1}{Q} (Q-M) C_s (-1) = 0.$$

$$\Longrightarrow \qquad M = \left(\frac{C_s}{C_h+C_s}\right) Q = \left(\frac{C_s}{C_h+C_s}\right). Dt.$$

Since the condition

$$\frac{d^2}{dM^2}(TIC) > 0.$$

for minimum cost is also satisfied

$\therefore$ optimal value of M is given by

$$M^* = \left(\frac{C_s}{C_h+C_s}\right) . D.t. \quad \text{(optimal inventory level).}$$

By substituting this value of M in TIC equation, the minimum cost so obtained is as follows :

$$TIC = \left(\frac{C_h . C_s}{C_h . C_s}\right) . D.t. \quad \text{(optimal cost).} \qquad (3.15)$$

MODEL-6

2.3 ECONOMIC LOT SIZE MODEL WITH FINITE REPLENISHMENT RATE AND SHORTAGE ALLOWED :

This model is based on the same assumptions as in model-3 except that shortages are allowed. The inventory system can graphically be presented as follows.

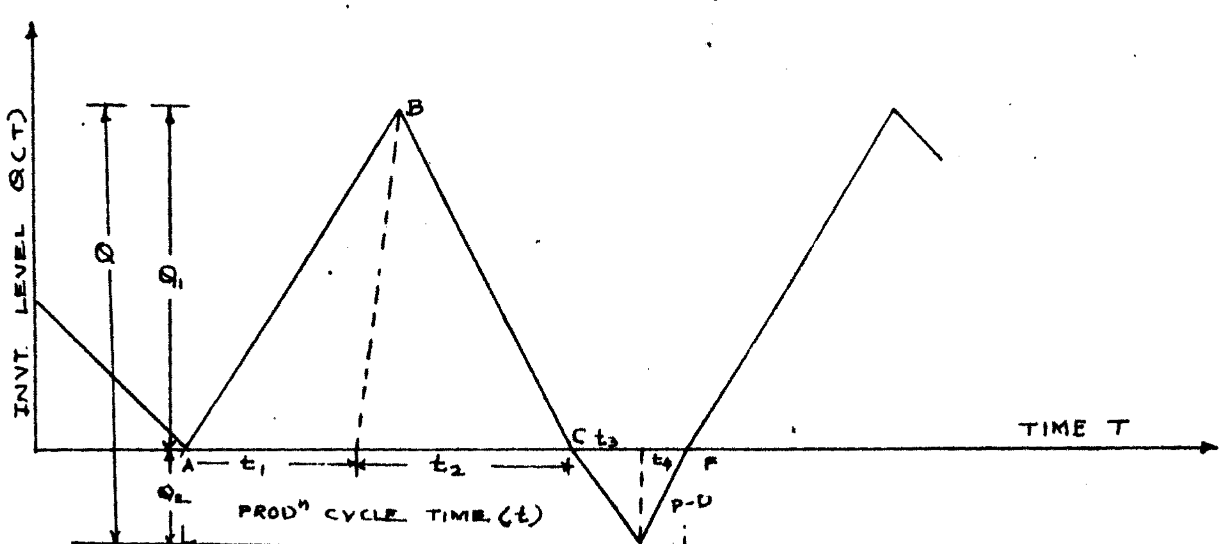


Fig. 3.6 : Inventory level with finite replenishment and shortages allowed.

In this case we are interested to minimise the total yearly incremental cost

TIC = set-up cost + carrying cost + shortage cost. *

As in case of model-3, the maximum inventory level say Q_1 reached at the end of time t_1 is given by

$$Q_1 = (P-D)\, t_1 \qquad (a)$$

After time t_1 the stock Q_1 is used up during t_2 thus we have

$$Q_1 = Dt_2 \qquad (b)$$

During time t_3 shortage occured at the rate of D. Thus the maximum shortage occured is given by

$$Q_2 = Dt_3 \qquad (c)$$

After time t_3, the production starts and therefore shortage starts reducing at the rate of $(P-D)\, t_4$ i.e.,

$$Q_2 = (P-D)\, t_4 \qquad (d)$$

The average inventory and amount of shortage during the production cycle time t is given by

$$\text{Average inventory} = \frac{1}{2}\,\frac{Q_1(t_1+t_2)}{t}$$

and $$\text{Average shortage} = \frac{1}{2}\,\frac{Q_2(t_3+t_4)}{t}$$

But production cycle time $t = t_1 + t_2 + t_3 + t_4$

$$= \frac{Q_1}{P-D} + \frac{Q_1}{D} + \frac{Q_2}{D} + \frac{Q_2}{P-D}$$

$$= Q_1\left(\frac{1}{P-D} + \frac{1}{D}\right) + Q_2\left(\frac{1}{D} + \frac{1}{P-D}\right)$$

$$t = \frac{P}{D(P-D}\,(Q_1 + Q_2) \tag{3.16}$$

Now if Q is the lot size then

$$Q_1 = Q - Q_2 - Dt_1 - Dt_4$$

$$= Q - Q_2 - D\left(\frac{Q_2}{P-D} + \frac{Q_2}{P-D}\right)$$

$$= \left(\frac{P-D}{P}\right) Q - Q_2; \quad Q = D.t.$$

$$Q_1 + Q_2 = \left(\frac{P-D}{P}\right).\ Q.$$

Substituting value of $Q_1 + Q_2$ in equation (3.16) we get

$$t = \frac{P}{D(P-D} \times \left(\frac{P-D}{P}\right).\quad Q = \frac{Q}{D} \tag{3.17}$$

Hence the expression for TIC can be written as

$$TIC(Q,Q_1,Q_2) = \frac{D}{Q}\,C_p + \frac{1}{2}\,\frac{Q_1(t_1 + t_2)}{t}\,.\ C_h + \frac{1}{2}\,\frac{Q_2(t_3 + t_4)}{t}\,C_s$$

$$= \frac{D}{Q}.\ C_p + \frac{1}{2Q}\,.\ \frac{P}{P-Q}\,(Q_1^2\ .\ C_h + Q_2^2\ .\ C_s)$$

or $$TIC(Q,Q_2) = \frac{D}{Q}.\ C_p + \frac{1}{2Q}\left(\frac{P}{P-Q}\right)\left[C_h\left\{\frac{P-D}{P}\,Q - Q_2\right\}^2 + Q_2^2\,C_s\right]$$

The necessary conditions for the optimal value of TIC are

$$\frac{\partial(TIC)}{\partial Q_2} = \frac{1}{2Q}\left(\frac{P}{P-D}\right)\left[2C_h\left\{\frac{P-D}{P}\,Q - Q_2\right\} \times (-1) + 2Q_2\,C_s\right] = 0$$

$$C_h\left\{\frac{Q_2}{Q(1 - \frac{D}{P})} - 1\right\} + \frac{Q_2\,C_s}{Q(1 - \frac{D}{P})} = 0$$

$$Q_2^* = \frac{C_h\ Q}{(C_h + C_s)}\ (1 - \frac{D}{P}) \qquad (3.18)$$

$$\frac{\partial(TIC)}{\partial Q} = -\ \frac{D}{Q^2}\ C_p - C_h \left[\frac{1}{2}\ (\frac{P-D}{P}) - \frac{Q_2^2}{2Q^2(\frac{P-D}{P})}\right]$$

$$= -\ \frac{D}{Q^2}\ C_p + C_h \left[\frac{1}{2}\ (1 - \frac{D}{P}) - \frac{Q_2^2}{2Q^2(1 - \frac{D}{P})}\right]$$

$$-\ \frac{Q_2^2\ C_s}{Q^2(1 - \frac{D}{P})} = 0.$$

$$\Longrightarrow \qquad -\ \frac{D}{Q^2}\ C_p + \frac{C_h}{2}\ (1 - \frac{D}{P}) - \frac{(C_h + C_s)Q_2^2}{2Q^2(1 - \frac{D}{P})} = 0 \qquad (3.19)$$

Now putting the value Q_2^* from (3.18) in (3.19) we get

$$-\ \frac{D}{Q^2}\ C_p + \frac{C_h}{2}(1 - \frac{D}{P}) - \frac{C_h^2}{2(C_h + C_s)}\ (1 - \frac{D}{P}) = 0$$

$$\frac{D}{Q^2}\ C_p = \frac{C_h}{2}(1 - \frac{D}{P})\ (1 - \frac{C_h}{C_h + C_s})$$

$$\Longrightarrow \qquad Q^* = \sqrt{\frac{2DC_p(C_h + C_s)}{C_h\ C_s(1 - \frac{D}{P})}}$$

$$= \sqrt{\frac{2C_p(C_h + C_s)}{C_h\ C_s}\ (\frac{PD}{P-D})} \qquad (3.20)$$

Since $\frac{\partial^2(TIC)}{\partial Q^2} > 0$ and $\frac{\partial^2(TIC)}{\partial Q^2} > 0$

$\therefore$ values of Q and Q_2 so obtained are optimum values and which minimizes values of TIC

$$Q^* = \sqrt{\frac{2C_p(C_h + C_s)}{C_h\ C_s} \left(\frac{PD}{P-D}\right)}$$

$$Q_2^* = \sqrt{\frac{2C_p\ C_h}{C_s(C_h + C_s)} \cdot D.\left(1 - \frac{D}{P}\right)} \qquad (3.21)$$

Further

$$\text{production cycle time} \quad t = \frac{Q^*}{D} = \sqrt{\frac{2C_p(C_h + C_s)}{DC_h\ C_s\left(1 - \frac{D}{P}\right)}}$$

$$= \sqrt{\frac{2C_p(C_h + C_s)}{C_h\ C_s} \left(\frac{P}{D(P-D)}\right)} \qquad (3.22)$$

$$Q_1^* = \left(\frac{P-D}{P}\right) Q^* - Q_2$$

$$= \sqrt{\frac{2DC_s\ C_p}{C_h + C_s} \left(1 - \frac{D}{P}\right)} \qquad (3.23)$$

Lastly $\quad TIC = \sqrt{\frac{2DC_p\ C_h\ C_s}{C_h + C_s} \left(1 - \frac{D}{P}\right)}$ optimal cost (3.24)

3. <u>THE ECONOMIC LOT SIZE MODELS WITH QUANTITY DISCOUNTS</u> :

At the time of purchasing a quantity discount should be taken into consideration. The discounts are normally offered to encourage buyers to purchase more units of an item. They are usually offered in one of the following two ways.

1. All units discounts.
2. Incremental quantity discounts.

ASSUMPTIONS :

i) Demand is uniform and deterministic.

ii) Shortages are not allowed.

iii) Replenishment is instantaneous.

The models in this case will be different from those discussed earlier because the purchasing price of inventory items is also included in the total cost which is to be minimized.

MODEL-7

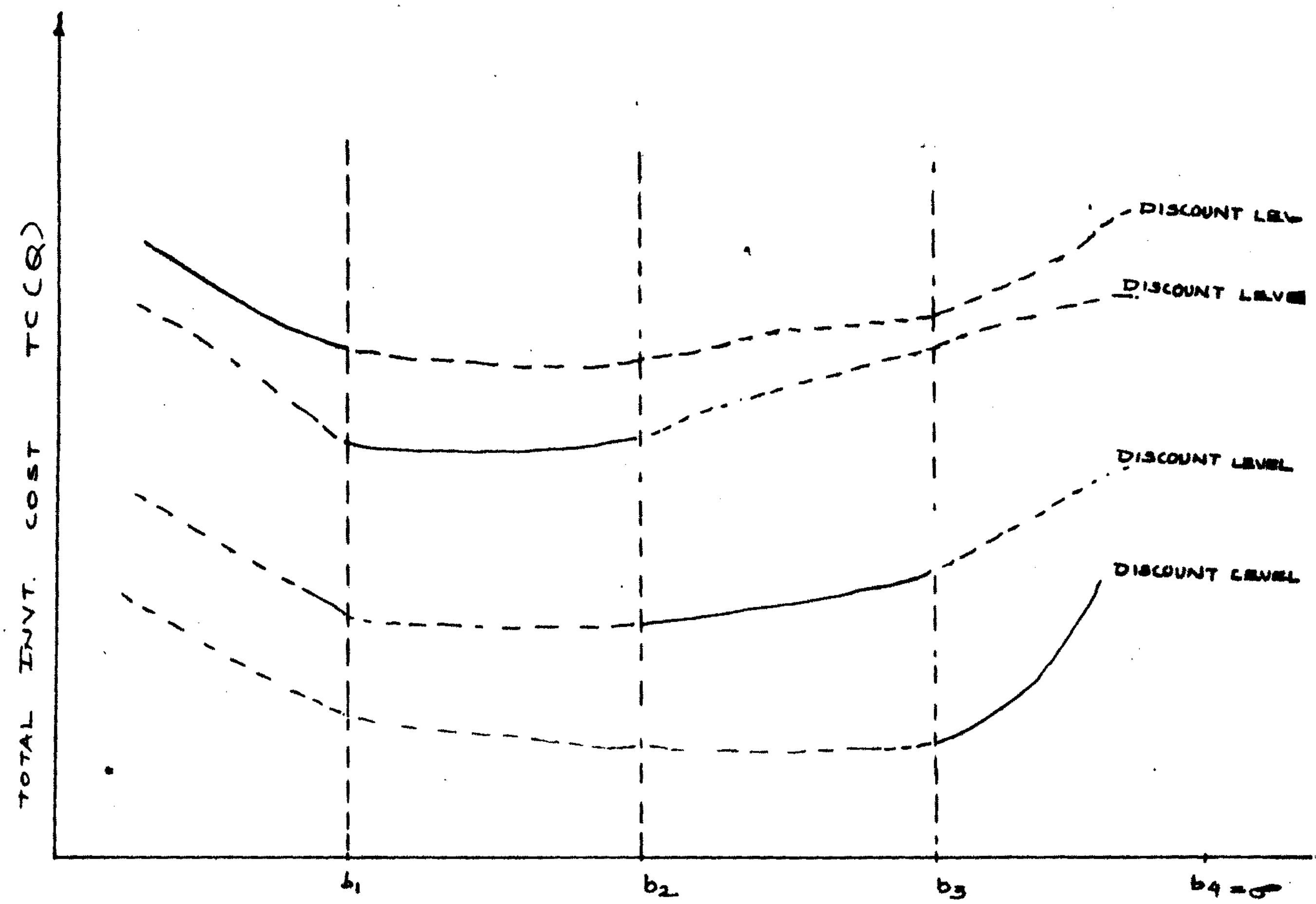
3.1 LOT SIZE MODEL WITH ALL UNITS DISCOUNT AVAILABLE :

In this case discount price is applicable to all units purchased, if there are several price breaks say $0, b_1, b_2, \cdots, b_i$ and the ordered quantity Q lies in the discount interval, say $b_i - 1 \leq Q \leq b_i$, then prices per unit for Q units is C_i where $C_i < C_i - 1$. Therefore, the total cost for a discount level is given by

$$TC_i = DC_i + \frac{D}{Q} C_p + \frac{Q}{2} C_h$$

$$= DC_i + \frac{D}{Q} C_p + \frac{Q}{2} (C_i \times r) \qquad (3.25)$$

For each value of i, a separate cost function for each unit price or each discount range can be defined.



Order quantity, Q.

Fig. 3.7 : Inventory level with price breaks.

Figure 3.7 represents the total annual cost function TC(Q) for any purchase quantity Q, in the range $0 \leq Q < \infty$. The broken line segment indicates value of TC_i beyond their prescribed range for Q and hence have no physical significance. From figure 3.7 the following interesting characteristics of the curves at different levels can be observed.

1. The cost curves do not interested each other because $C_i < C_i - 1$ and $TC_i(Q) < TC_{i-1}(Q)$ for all Q.

2. If $\bar{Q}^{(1)}, Q^{(2)}, \cdots, Q^{(n)}$ are economic order quantities

for the cost function $TC_i(Q)$, $i = 1, 2, \cdots, n$, then

$$Q^{(1)} < Q^{(2)} < \cdots < Q^{(n)}$$

3. The cost function $TC(Q)$ has $(n-1)$ points discounting at $Q = b_1, b_2, \cdots, b_{n-1}$.

To find the optimal order quantity Q^* which gives the minimum total annual cost $TC(Q)$, take the first derivatives of $TC_i(Q)$ w.r. to Q and setting the resulting expression equal to zero.

$$\frac{d}{dQ}(TC_i(Q)) = -\frac{D}{Q^2} \cdot C_p + \frac{(C_i \times r)}{2} = 0.$$

$$\Longrightarrow \qquad Q^* = \sqrt{\frac{2DC_p}{C_i \times r}} \tag{3.26}$$

Substituting value of Q^* from (3.26) in (3.25) the minimum value of $TC_i(Q)$ is

$$TC_i(Q) = DC_i + \frac{DC_p}{\sqrt{\frac{2DC_p}{C_i \times r}}} + \left(\frac{C_i \times r}{2}\right) \cdot \sqrt{\frac{2DC_p}{C_i \times r}}$$

$$= DC_i + \sqrt{2DC_p\, C_i\, r}\ . \tag{3.27}$$

<u>MODEL-7</u>

3.2 LOT SIZE MODEL WITH ONE PRICE BREAK :

Suppose the following price schedule is quoted by a supplier in which a price break occurs at quantity b_1. This means

Quantity	Price per unit (Rs.)
$0 \leq Q < b_1$	C_1
$b_1 \leq Q < b_2$	C_2

Notice that $C_1 > C_2$

The algorithmic procedure for determining optimal purchase quantity is as follows.

STEP-1 : Determine Q_2^* based on price C_2 by using

$$Q_2^* = \sqrt{\frac{2DC_p}{C_2 \times r}} \; .$$

If $b_1 \leq Q \leq b_2$ then Q_2^* is the EOQ for the given problems i.e., $Q = Q_2$ and the optimal cost TC^* is the total cost associated with the Q_2^*

i.e., $$TC^* = TC_2 = DC_2 + \frac{D}{b_2} C_p + \frac{b_2}{2} (C_2 \times r).$$

STEP-2 : If $Q_2^* < b_1$ then price per unit will be C_1 for Q_2^* units and not C_2. Thus the minimum cost associated with buying at the rate C_2 per unit will be observed at $Q_2 = b_2$, because the cost function TC_2 is monotonically increasing in the range b_1 to b_2.

Now calculate Q_1^*, the EOQ with price C_1 and the corresponding minimum total cost TC_1 Compare $TC_2(b_2)$ and $TC_1(b_1)$.

If $TC_2(b_2) > TC_1(Q_1)$ then EOQ is $Q^* = Q_1^*$ otherwise $Q^* = b_2$ in the required EOQ.

MODEL-8

MODEL WITH TWO PRICE BREAKS :

Suppose that the following price schedule is quoted by a supplier, in which a price break occurs at quantity b_1 and b_2. This means

Quantity	Price per unit (Rs.)
$0 \leq Q < b_1$	C_1
$b_1 \leq Q < b_2$	C_2
$b_2 \leq Q$	C_3

Notice that $C_3 < C_2 < C_1$.

The Algorithmic procedure for determining optimal quantity is as follows.

STEP-1 : Determine Q_3 based on price C_3. If $Q_3^* \geq b_2$ then Q_3^* is the EOQ for the given problem. i.e, $Q^* = Q_3^*$ and optimal cost $TC(Q_3)$ is the cost associated with Q_3^*. If $Q_3^* < b_2$ then go to step-2.

STEP-2 : Calculate Q_2^* based on price C_2. If $b_1 \leq Q_2^* \leq b_2$ then compare the costs $TC(Q_3)$ and $TC(b_2)$ to obtain EOQ.

If $TC(Q_2^*) < TC(b_2)$, the EOQ is $Q^* = Q_2^*$, otherwise

If $TC_2(b_2) > TC_1(Q_1)$ then EOQ is $Q^* = Q_1^*$ otherwise $Q^* = b_2$ in the required EOQ.

MODEL-8

MODEL WITH TWO PRICE BREAKS :

Suppose that the following price schedule is quoted by a supplier, in which a price break occurs at quantity b_1 and b_2. This means

Quantity	Price per unit (Rs.)
$0 \le Q < b_1$	C_1
$b_1 \le Q < b_2$	C_2
$b_2 \le Q$	C_3

Notice that $C_3 < C_2 < C_1$.

The Algorithmic procedure for determining optimal quantity is as follows.

STEP-1 : Determine Q_3 based on price C_3. If $Q_3^* \ge b_2$ then Q_3^* is the EOQ for the given problem. i.e, $Q^* = Q_3^*$ and optimal cost $TC(Q_3)$ is the cost associated with Q_3^*. If $Q_3^* < b_2$ then go to step-2.

STEP-2 : Calculate Q_2^* based on price C_2. If $b_1 \le Q_2^* \le b_2$ then compare the costs $TC(Q_3)$ and $TC(b_2)$ to obtain EOQ.

If $TC(Q_2^*) < TC(b_2)$, the EOQ is $Q^* = Q_2^*$, otherwise is given by

$$TC(Q) = \frac{D}{Q}.\ C_p + \frac{D}{Q}\sum_{k=1}^{i-1} C_k(b_k - b_{k-1})$$

$$+ \frac{D}{Q}\ C_i(Q - b_{i-1}) + \frac{C_h}{2}\sum_{k=1}^{i-1} C_k(b_k - b_{k-1})$$

$$+ \frac{C_i \times r}{2}(Q - b_{i-1}).$$

$$= \left\{C_p + \sum_{k=1}^{i-1} C_k(b_k - b_{k-1}) - C_i\ b_{i-1}\right\}\frac{D}{Q} + \frac{Q}{2}(C_i \times r)$$

$$+ \left\{DC_i + \frac{r}{2}\sum_{k=1}^{i-1} C_k(b_k - b_{k-1}) - \frac{C_i \times r}{2}\ b_{i-1}\right\}.$$

To find the minimum of TC(Q) differentiate it w.r. to Q in the interval $b_{i-1} \le Q \le b_i$ and $b_i \le Q \le b_{i-1}$ respectively as shown below.

$$\frac{d}{dQ}\left\{TC_i(Q)\right\} = -\left\{C_p + \sum_{k=1}^{i-1} C_k(b_k - b_{k-1}) - C_i\ b_{i-1}\right\}\frac{D}{Q^2}$$

$$+ \frac{r\ C_i}{2} \quad \text{for} \quad b_{i-1} \le Q \le b_i$$

and $$\frac{d}{dQ}\left\{TC_{i+1}(Q)\right\} = -\left\{C_p + \sum_{k=1}^{i-1} C_k(b_k - b_{k-1}) - C_{i+1}\ b_i\right\}\frac{D}{Q^2}$$

$$+ \frac{r\ C_{i+1}}{2} \quad \text{for} \quad b_i \le Q \le b_{i+1}.$$

Now for $Q = b_i$, we have

$$\frac{d}{dQ}\left\{TC_i(Q = b_i)\right\} = -\left\{C_p + \sum_{k=1}^{i-1} C_k(b_k - b_{k-1})\right\}\frac{D}{b_i^2}$$

$$+ \frac{D}{b_i^2}\ C_i\ b_{i-1} + \frac{r\ C_i}{2}$$

and

$$\frac{d}{dQ}\left\{TC_{i+1}(Q = b_i)\right\} = -\left\{C_p + \sum_{k=1}^{i-1} C_k(b_k - b_{k-1})\right\}\frac{D}{b_i^2}$$

$$+ \frac{D}{b_i^2}\, C_i(b_i - b_{i-1}) + \frac{D}{b_i}\, C_{i+1} + \frac{r\, C_{i+1}}{2}$$

Since $\quad \frac{1}{2}\, r\, C_i > \frac{1}{2}\, r\, C_{i+1}$

$$\frac{d}{b_i^2}\, C_i\, b_{i-1} > -\frac{D}{b_i^2}\, C_i\,(b_i - b_{i-1}) + \frac{D}{b_i}\, C_{i+1}$$

$$= \frac{D}{b_i^2}\, C_i\, b_{i-1} - (C_i - C_{i+1})\, \frac{D}{b_i}$$

Therefore

$$\frac{d}{dQ}\left\{TC_i(Q = b_i)\right\} > \frac{d}{dQ}\left\{TC_{i+1}(Q = b_i)\right\}$$

This shows that minimum of TC(Q) cannot occur at a price break point b_i as shown in following figure (3.8).

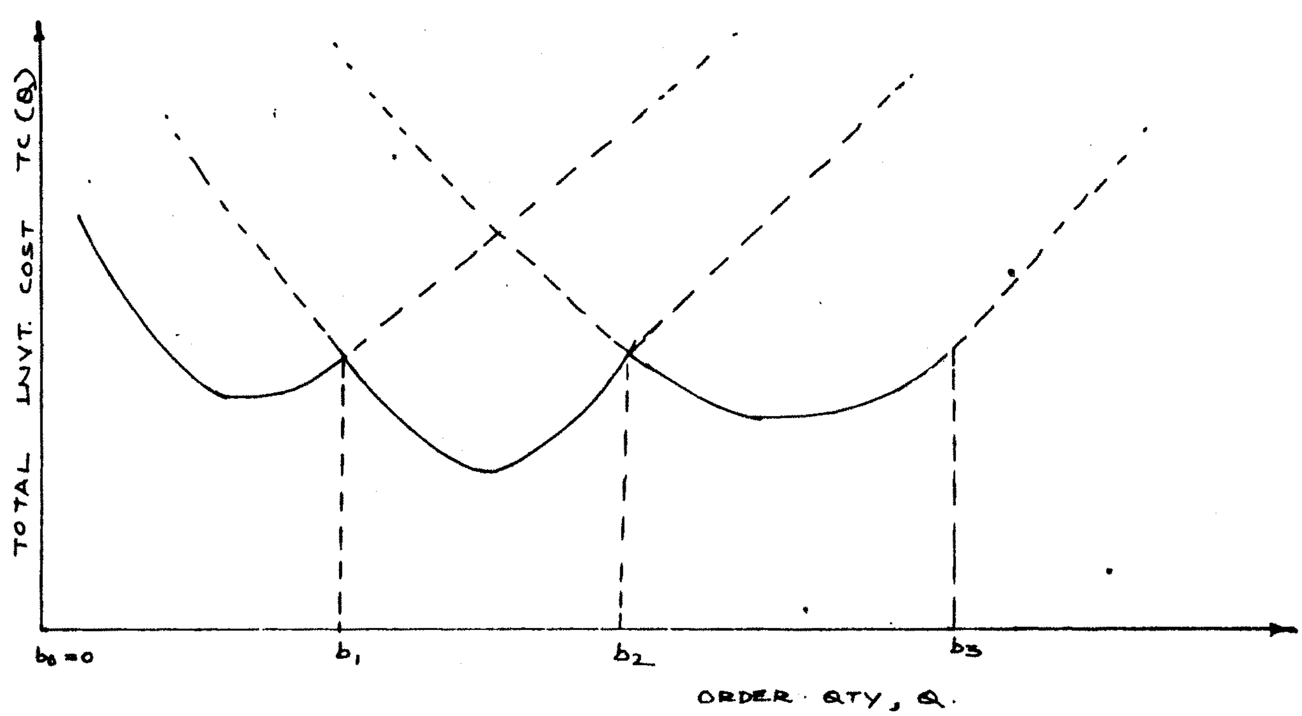


Fig. 3.8 : Inventory level with incremental quantity discount.

The above discussion can be summarized in the following steps to calculate the minimum of TC(Q). i.e.,

The algorithmic procedure is as follows :

STEP-1 : Calculate EOQ for every discount level by using the formula

$$Q_i^* = \left[\frac{2D \left\{ C_p + \sum_{k=1}^{i-1} C_k \, (b_k - b_{k-1}) - C_i \, b_{i-1} \right\}}{r \; C_i} \right]^{1/2}$$

STEP-2 : If EOQ lies in the interval i.e., $b_{i-1} \leq Q_i \leq b_i$, then calculate the associated total yearly inventory cost $TC_i(Q_i)$. Otherwise overall value of Q^*(EOQ) would be equal to Q_i^* for which $TC_i(Q_i)$ is minimum.

```
10 REM ***********************************************************************
20 REM *  INTEGRATED SOFTWARE FOR DETERMINISTIC INVENTORY MODEL IS DEVELOPED  *
30 REM *  BY Mr. T. M. GARUD UNDER THE GUIDENCE OF Dr. R. V. KULKARNI        *
40 REM ***********************************************************************
50 REM
60 REM ======================================================================
70 REM :                SOFTWARE FOR DETERMINISTIC INVENTORY MODELS          :
80 REM ======================================================================
90 REM :THIS SOFTWARE CONSIDERS INVENTORY MODELS IN THREE DIFFERENT CATEGORIES:
100 REM : 1. MODELS WITHOUT SHORTAGE COST                                    :
110 REM : 2. MODELS WITH SHORTAGE COST                                       :
120 REM : 3. MODELS WITH QUANTITY DISCOUNTS                                  :
130 REM =====================================================================:
140 REM : DIFFERENT NOTATIONS USED FOR DEVELOPING THIS SOFTWARE ARE AS FOLLOWS :
150 REM =====================================================================:
160 REM
170 REM : CP   :  ORDERING (SET UP) COST PER ORDER (RS/ORDER)                 :
180 REM
190 REM : CU   :  PURCHASE (MANUFACTURING ) COST PER UNIT (RS/UNIT)           :
200 REM
210 REM : CH   :  COST OF CARRYING ONE UNIT IN THE INVENTORY FOR A GIVEN      :
220 REM
230 REM :      :  LENTH OF TIME                                               :
240 REM
250 REM :      :  (HOLDING COST) IN RS/UNIT TIME PERIOD                       :
260 REM
270 REM : I    :  COST OF CARRYING ONE RS. WORTH OF INVENTORY PER TIME PERIOD :
280 REM
290 REM : CS   :  SHORTAGE COST PER UNIT PER TIME (RS/UNIT TIME)              :
300 REM
310 REM : D    :  DEMAND (CONSUMPTION) RATE IN UNITS PER TIME                 :
320 REM
330 REM : Q    :  ECONOMIC ORDER QTY                                          :
340 REM
350 REM : R    :  REORDER POINT                                               :
360 REM :      :  i.e.THE LEVEL AT WHICH ORDER  IS PLACED                     :
370 REM
380 REM : L    :  REPLENISHMENT LEAD TIME                                     :
390 REM
400 REM : N    :  NO. OF ORDERS PER TIME PERIOD                               :
410 REM
420 REM : T    :  REORDER CYCLE TIME TIME i.e.THE TIME INTERVAL BETWEEN       :
430 REM
440 REM :      :  SUCCESSIVE ORDERS                                           :
450 REM
460 REM : TP   :  PRODUCTION PERIOD                                           :
470 REM
480 REM : P    :  PRODUCTION RATE(YEARLY)                                     :
490 REM
500 REM : TC   :  TOTAL INVENTORY COST                                        :
510 REM
520 REM : TIC  :  MINIMUM TOTAL(YEARLY) INCREMENTAL COST                      :
530 REM
540 REM ======================================================================
```

```
550 CLS
560 PRINT STRING$(70,"*")
570 COLOR 23,0,7
580 PRINT
590 PRINT ".......... S O F T W A R E   F O R .......... "
600 PRINT :PRINT :PRINT
610 PRINT ".....DETERMINISTIC   INVENTORY   MODELS ....."
620 PRINT
630 PRINT "...DEVELOPED BY  Mr. T. M. GARUD ..."
640 PRINT
650 PRINT "...UNDER THE GUIDENCE OF Dr. R. V. KULKARNI..."
660 PRINT
670 COLOR 7,0
680 PRINT STRING$(70,"*")
690 PRINT
700 PRINT
710 PRINT "DO YOU WANT TO USE THIS SOFTWARE ? TYPE(Y/N)"
720 PRINT
730 PRINT
740 B$=INKEY$: IF B$="" THEN 740 ELSE IF B$="n" OR B$="N" THEN PRINT "THANKS":EN
D
750 'IF A1$="N" THEN PRINT "THANK YOU":END
760 PRINT "ENTER THE PASSWARD"
770 PRINT
780 PRINT
790 PSW$ =INPUT$(3)
800 IF PSW$="TMG" OR PSW$="RVK" THEN 830
810 PRINT "INVALID PASSWARD ...ACCESS DENIED... TRY AGAIN..."
820 GOTO 760
830 CLS
840 PRINT STRING$(70,"-")
850 PRINT
860 'COLOR 23,0,7
870 PRINT "..... DETERMINISTIC INVENTORY MODELS ....."
880 'COLOR 7,0
890 PRINT
900 PRINT STRING$(70,"-")
910 PRINT
920 COLOR 7,0
930 'PRINT STRING$(75,"."):PRINT
940 PRINT "1. INVENTORY MODELS WITHOUT SHORTAGE COST ":PRINT
950 PRINT "2. INVENTORY MODELS WITH SHORTAGE COST ":PRINT
960 PRINT "3. INVENTORY MODELS WITH QUANTITY DISCOUNTS ":PRINT
970 PRINT
980 PRINT "4. TO STOP WORKING "
990 PRINT
1000 'PRINT STRING$(75,".")
1010 'PRINT
1020 LOCATE 20,1:PRINT STRING$(75,"-")
1030 LOCATE 22,1:PRINT STRING$(75,"-")
1040 COLOR 23,0,7
1050 LOCATE 21,1:INPUT "ENTER YOUR CHOICE (1,4) " ,C1
1060 COLOR 7,0
1070 ON C1 GOTO 1100,2780,4570,6560
1080 PRINT "INVALID CHOICE ,TRY AGAIN"
```

```
1090 GOTO 1050
1100 CLS
1110 PRINT STRING$(70,"-")
1120 COLOR 23,0,7
1130 PRINT "INVENTORY MODELS WITHOUT SHORTAGE COST "
1140 COLOR 7,0
1150 PRINT STRING$(70,"-")
1160 PRINT
1170 PRINT "1. ECONOMIC LOT SIZE MODEL FOR PURCHASED ITEMS "
1180 PRINT
1190 PRINT "2. ECONOMIC LOT SIZE MODEL WITH DIFFERENT DEMANDS IN DIFF. CYCLES"
1200 PRINT
1210 PRINT "3. ECONOMIC LOT SIZE MODEL WITH FINITE RATE OF REPLENISHMENT "
1220 PRINT
1230 PRINT "4. RETURN TO MAIN OPTIONS "
1240 PRINT
1250 PRINT STRING$(70,"-")
1260 PRINT
1270 LOCATE 19,1 :PRINT STRING$(70,"*")
1280 LOCATE 21,1 :PRINT STRING$(70,"*")
1290 LOCATE 20,1 :INPUT "ENTER YOUR CHOICE (1,4) : ",C2
1300 ON C2 GOTO 1330,1790,2270 ,2770
1310 PRINT "INVALID CHOICE ,TRY AGAIN "
1320 GOTO 1100
1330 REM SUBROUTINE TO COMPUTE EOQ,TIC,T FOR MODEL - I
1340 CLS
1350 PRINT STRING$(70,"*")
1360 PRINT "ECONOMIC LOT SIZE MODEL FOR PURCHASED ITEMS"
1370 PRINT STRING$(70,"*")
1380 PRINT
1390 INPUT "ENTER PROCUREMENT (ORDERING OR SET UP)COST CP ",CP
1400 INPUT "ENTER INVENTORY CARRYING COST EXPRESSED AS % OF AVG.INV.INVSMT.",I
1410 INPUT "ENTER UNIT PRICE IN Rs. CU :",CU
1420 INPUT "ENTER DEMAND IN UNITS (IN UNITS):",D
1430 Q=SQR(2*D*CP/CU/I)
1440 T=Q/D
1450 TIC=SQR(2*D*CP*CU*I)
1460 CLS
1470 LPRINT STRING$(70,"*")
1480 LPRINT "THE INPUT SUPPLIED FOR THE MODEL I IS AS FOLLOWS "
1490 LPRINT STRING$(70,"*")
1500 LPRINT
1510 LPRINT "PROCUREMENT COST PER ORDER CP = ";CP
1520 LPRINT
1530 LPRINT "INVENTORY CARRYING COST  I = ";I
1540 PRINT
1550 LPRINT "UNIT PRICE CP =";CP
1560 LPRINT
1570 LPRINT "DEMAND OR CONSUMPTION  D= ";D
1580 PRINT
1590 LPRINT STRING$(70,"*")
1600 COLOR 23,0,7
1610 PRINT "PRESS F5 TO SEE THE OUTPUT "
1620 COLOR 7,0
```

```
1630 STOP
1640 CLS
1650 LPRINT STRING$(70,"*")
1660 LPRINT "THE OUTPUT OF MODEL I IS AS FOLLOWS "
1670 LPRINT STRING$(70,"*")
1680 LPRINT
1690 LPRINT "ECONOMIC ORDER QUANTITY (EOQ) Q  = ";Q
1700 LPRINT
1710 PRINT "THE OPTIMAL ORDER CYCLE TIME  T = ";T
1720 PRINT
1730 PRINT "THE MINIMUM YEARLY TOTAL COST  TIC = ";TIC
1740 PRINT
1750 PRINT STRING$(70,"*")
1760 PRINT "PRESS F5 TO CONTINUE "
1770 STOP
1780 GOTO  1100
1790 REM SUBROUTINE FOR MODEL II
1800 CLS
1810 PRINT STRING$(70,"*")
1820 PRINT "INV. MODEL WITH DIFFERENT RATE OF DEMAND IN DIFFERENT CYCLES "
1830 PRINT STRING$(70,"*")
1840 PRINT
1850 INPUT "ENTER TOTAL DEMAND IN TIME PERIOD D = ",D
1860 INPUT "ENTER HOLDING COST CU*I, CH =",CH
1870 INPUT "ENTER PROCUREMENT COST CP =",CP
1880 INPUT "ENTER TOTAL TIME PERIOD T=t1+t2+...+tn ",T
1890 Q=SQR(2*D*CP/T*CH)
1900 TIC=SQR(2*CP*CH*D/T)
1910 N1=D/Q1
1920 CLS
1930 PRINT STRING$(70,"*")
1940 PRINT "THE INPUT SUPPLIED FOR MODEL II IS AS FOLLOWS "
1950 PRINT STRING$(70,"*")
1960 PRINT
1970 PRINT "TOTAL DEMAND (D1+D2+...+DN=D) D IN TIME PERIOD T=";D
1980 PRINT
1990 PRINT "PROCUREMENT COST IN RS.  CP =";CP
2000 PRINT
2010 PRINT "HOLDING COST CH=CU*I IS CH =";CH
2020 PRINT
2030 PRINT STRING$(70,"*")
2040 PRINT
2050 COLOR 23,0,7
2060 PRINT "PRESS F5 TO SEE THE OUTPUT "
2070 COLOR 7,0
2080 STOP
2090 CLS
2100 PRINT STRING$(70,"*")
2110 PRINT "THE OUTPUT OF THE MDEL II IS AS FOLLOWS"
2120 PRINT STRING$(70,"*")
2130 PRINT
2140 PRINT "ECONOMIC ORDER QUANTITY(EOQ) Q=";Q
2150 PRINT
2160 PRINT "NO. OF PRODUCTION CYCLES IN TOTAL TIME T = ";N1
```

```
2170 PRINT
2180 PRINT "THE MINIMUM YEARLY INCREMENTAL COST TIC = ";TIC
2190 PRINT
2200 PRINT STRING$(70,"*")
2210 PRINT
2220 COLOR 23,0,7
2230 PRINT "PRESS F5 TO CONTINUE "
2240 COLOR 7,0
2250 STOP
2260 GOTO 1100
2270 REM PROGRAM FOR MODEL III CALLED FROM LN. 910
2280 CLS
2290 PRINT STRING$(70,"*")
2300 PRINT "INVENTORY MODEL WITH FINITE RATE OF REPLENISHMENT "
2310 PRINT STRING$(70,"*")
2320 PRINT
2330 INPUT "ENTER ANNUAL DEMAND IN UNITS D =",D
2340 INPUT "ENTER SET UP COST FOR EACH PRODUCTION RUN CP =",CP
2350 INPUT "ENTER INV. CARRYING /HOLDING(CU*I)",CH
2360 INPUT "ENETR DAILY RATE OF PRODUCTION ",P
2370 INPUT "ENTER NO. OF WORKING DAYS  FOR PRODN. EACH YEAR ,WDAYS : ",WDAYS
2380 Q=SQR(2*D*CP/(1-D/P/WDAYS)/CH)
2390 T=Q/D*WDAYS
2400 TP=Q/P
2410 TIC=SQR(2*D*CP*CH*(1-D/P/WDAYS))
2420 PRINT STRING$(70,"*")
2430 PRINT "THE INPUT SUPPLIED FOR MODEL NO. III IS AS FOLLOWS"
2440 PRINT
2450 PRINT "ANNUAL DEMAND IN UNITS D=";D
2460 PRINT
2470 PRINT "SET UP COST FOR EACH PRODUCTION RUN CP =";CP
2480 PRINT
2490 PRINT "DAILY RATE OF PRODUCTION P= ";P
2500 PRINT
2510 PRINT "NO. OF WORKING DAYS PER YEAR = ";WDAYS:PRINT
2520 PRINT STRING$(70,"*")
2530 PRINT
2540 PRINT "PRESS F5 TO SEE THE OUTPUT"
2550 STOP
2560 CLS
2570 PRINT STRING$(70,"*")
2580 PRINT "THE OUTPUT FOR THE MODEL III IS AS FOLLOWS "
2590 PRINT STRING$(70,"*")
2600 PRINT
2610 PRINT "ECONOMIC ORDER QTY (EOQ) Q=";Q
2620 PRINT
2630 PRINT "PRODUCTION CYCLE TIME T =";T
2640 PRINT
2650 PRINT "PRODUCTION TIME FOR EACH RUN TP =";TP
2660 PRINT
2670 PRINT "THE MINIMUM YEARLY INCREMENTAL COST TIC = ";TIC
2680 PRINT
2690 PRINT STRING$(70,"*")
2700 PRINT
```

```
2710 COLOR 23,0,7
2720 PRINT "PRESS F5 TO CONTINUE "
2730 COLOR 7,0
2740 STOP
2750 GOTO 1100
2760 REM GOT TO MAIN MENU
2770 GOTO 830
2780 REM PROGRAM TO FIND EOQ WHEN SHORTAGES ARE ALLOWED
2790 REM ELSM=ECONOMIC LOT SIZE MODEL
2800 CLS
2810 PRINT STRING$(70,"*")
2820 COLOR 23,0,7
2830 PRINT "INVENTORY MODELS WITH SHORTAGE COST "
2840 COLOR 7,0
2850 PRINT STRING$(70,"*")
2860 PRINT
2870 PRINT STRING$(70,"-")
2880 PRINT
2890 PRINT "1.ELSM WITH CONST. DEMAND AND VARIABLE ORDER CYCLE TIME
2900 PRINT
2910 PRINT "2.ELSM WITH CONSTANT DEMAND AND REORDER CYCLE TIME "
2920 PRINT
2930 PRINT "3.ELSM WITH FINITE REPLENISHMENT RATE & SHORTAGE ALLOWED"
2940 PRINT
2950 PRINT "4. RETURN TO MAIN OPTIONS "
2960 PRINT
2970 PRINT STRING$(70,"-")
2980 PRINT
2990 COLOR 23,0,7
3000 INPUT "ENTER YOUR CHOICE (1,4) :",C2
3010 COLOR 7,0
3020 ON C2 GOTO 3050,3500,3960,4560
3030 PRINT "INVALID CHOICE, TRY AGAIN "
3040 GOTO 2800
3050 CLS
3060 REM INVENTORY MODEL IV
3070 PRINT STRING$(70,"*")
3080 COLOR 23,0,7
3090 PRINT "ELSM WITH CONSTANT DEMAND AND VARIABLE ORDER CYCLE TIME"
3100 COLOR 7,0
3110 PRINT STRING$(70,"*")
3120 PRINT
3130 INPUT "ENTER DEMAND IN UNITS  D :",D
3140 INPUT "ENTER HOLDING COST CH =RS.",CH
3150 INPUT "ENTER PROCUREMENT COST PER TIME PERIOD ,CP=RS.",CP
3160 INPUT "ENTER SHORTAGE COST PER UNIT TIME,CS = Rs.",CS
3170 Q=SQR(2*D*CP/CH*(CH+CS)/CS)
3180 T=Q/D
3190 CLS
3200 PRINT STRING$(70,"*")
3210 COLOR 23,0,7
3220 PRINT "THE INPUT SUPPLIED FOR MODEL IV IS AS FOLLOWS "
3230 COLOR 7,0
3240 PRINT STRING$(70,"*")
```

```
3250 PRINT
3260 PRINT "DEMAND IN UNITS PER TIME PERIOD D = ";D:PRINT
3270 PRINT "HOLDING COST PER UNIT PER TIME PERIOD CP = Rs.";CP:PRINT
3280 PRINT "PROCUREMENT COST PER UNIT TIME CS = ";CS:PRINT
3290 PRINT "SHORTAGE COST PER UNIT TIME PERIOD CS = Rs.";CS
3300 PRINT
3310 PRINT "PRESS F5 TO SEE THE OUTPUT "
3320 STOP
3330 CLS
3340 PRINT STRING$(70,"*")
3350 COLOR 23,0,7
3360 PRINT "THE OUTPUT OF THE MODEL IV IS AS FOLLOWS "
3370 PRINT
3380 COLOR 7,0
3390 PRINT  STRING$(70,"*")
3400 PRINT
3410 PRINT "ECONOMIC ORDER QUANTITY (EOQ), Q= ";Q
3420 PRINT
3430 PRINT "REORDER CYCLE TIME T = ";T
3440 PRINT
3450 PRINT "AN OPTIMUM ORDER QUANTITY";Q;"UNITS MUST BE SUPPLIED AFTER EVERY";T;
"DAYS/MONTH/YEAR/"
3460 PRINT
3470 PRINT "PRESS F5 TO CONTINUE "
3480 STOP
3490 GOTO 2800
3500 CLS
3510 REM INVENTORY MODEL V
3520 REM ELSM WITH CONSANT DEMAND AND REORDER CYCLE TIME
3530 PRINT STRING$(70,"*")
3540 COLOR 23,0,7
3550 PRINT "ELSM WITH CONSTANT DEMAND AND REORDER CYCLE TIME "
3560 COLOR 7,0
3570 PRINT STRING$(70,"*")
3580 PRINT
3590 INPUT "ENTER DEMAND RATE IN UNITS PER TIME PERIOD D= ",D
3600 INPUT "ENETR REORDER CYCLE TIME T = ",T
3610 INPUT "ENTER HOLDING COST PER UNIT PER TIME PERIOD ",CH
3620 INPUT "ENTER SHORTAGE COST PER UNIT PER TIME PERIOD ",CS
3630 M=(CS*D*T)/(CH+CS)
3640 TIC=(CH*CS*D*T)/(CH+CS)
3650 CLS
3660 PRINT STRING$(70,"*")
3670 COLOR 23,0,7
3680 PRINT "THE INPUT SUPPLIED FOR MODEL V IS AS FOLLOWS "
3690 COLOR 7,0
3700 PRINT STRING$(70,"*")
3710 PRINT
3720 PRINT "DEMAND RATE IN UNITS PER TIME PERIOD D =";D
3730 PRINT
3740 PRINT "REORDER CYCLE TIME T= ";T
3750 PRINT
3760 PRINT "HOLDING COST PER UNIT PER TIME PERIOD CH = Rs. ";CH
3770 PRINT
3780 PRINT "SHORTAGE COST PER UNIT PER TIME PERIOD CS = Rs. ";CS
```

```
3790 PRINT
3800 PRINT "PRESS F5 TO SEE THE OUTPUT "
3810 STOP
3820 CLS
3830 PRINT STRING$(70,"*")
3840 COLOR 23,0,7
3850 PRINT "THE OUTPUT OF THE MODEL V IS AS FOLLOWS "
3860 COLOR 7,0
3870 PRINT STRING$(70,"*")
3880 PRINT
3890 PRINT "THE OPTIMUM INVENTORY LEVEL  M=";M
3900 PRINT
3910 PRINT "THE MINIMUM YEARLY INCREMENTAL INVENTORY COST TIC =";TIC
3920 PRINT
3930 PRINT "PRESS F5 TO CONTINUE"
3940 STOP
3950 GOTO 2800
3960 REM INVENTORY MODEL VI
3970 REM ELSM WITH FINITE RATE OF REPLENISHMENT & SHORTAGE ALLOWED
3980 CLS
3990 REM Q2=OPTIMUM NO. OF SHORTAGES
4000 PRINT STRING$(70,"*")
4010 COLOR 23,0,7
4020 PRINT "ELSM WITH FINITE RATE OF REPLENISHMENT & SHORTAGES ALLOWED "
4030 COLOR 7,0
4040 PRINT
4050 INPUT "ENTER DEMAND IN UNITS PER TIME PERIOD D  = ",D
4060 INPUT "ENETR  RATE OF PRODUCTION PER TIME PERIOD P= ",P
4070 INPUT "ENTER HOLDING COST PER UNIT PER TIME PERIOD CH =",CH
4080 INPUT "ENTER SHORTAGE COST PER UNIT PER TIME PERIOD CS=";CS
4090 INPUT "ENTER SET UP COST CP = Rs. ",CP
4100 T1=(CH+CS)/CH/CS
4110 T2=P*D/(P-D)
4120 T3=D*(1-D/P)
4130 T4=CH/(CH+CS)
4140 Q=SQR(2*CP*T1*T2)
4150 Q2=T4*Q*(1-D/P)
4160 T=Q/D
4170 Q1=(1-D/P)*Q-Q2
4180 T1=Q/P
4190 TIC=SQR(2*D*CH*(1-D/P)/T1)
4200 CLS
4210 PRINT STRING$(70,"*")
4220 COLOR 23,0,7
4230 PRINT "THE INPUT SUPPLIED FOR THIS MODEL IS AS FOLLOWS "
4240 COLOR 7,0
4250 PRINT
4260 PRINT "DEMAND IN UNITS PER TIME PERIOD D =";D
4270 PRINT
4280 PRINT "RATE OF PRODUCTION PER TIME PERIOD P =";P
4290 PRINT
4300 PRINT "HOLDING COST PER UNIT PER TIME PERIOD CH =";CH
4310 PRINT
4320 PRINT "SET UP (PROCUREMENT ) COST PER UNIT IS CP =";CP
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4330 PRINT
4340 PRINT "PRESS F5 TO SEE THE OUTPUT "
4350 STOP
4360 CLS
4370 PRINT STRING$(70,"*")
4380 COLOR 23,0,7
4390 PRINT "THE OUTPUT OF THE MODEL VI IS AS FOLLOWS "
4400 COLOR 7,0
4410 PRINT
4420 PRINT "OPTIMAL LOT SIZE Q =";Q
4430 PRINT
4440 PRINT "OPTIMAL NO. OF SHORTAGES Q2= ";Q2
4450 PRINT
4460 PRINT "PRODUCTION TIME T1 = ";T1
4470 PRINT
4480 PRINT "MINIMUM TOTAL INCREMENTAL INVENTORY COST TIC =";TIC
4490 PRINT
4500 PRINT STRING$(70,"*")
4510 PRINT
4520 COLOR 23,0,7
4530 PRINT "PRESS F5 TO CONTINUE "
4540  COLOR 7,0
4550 STOP
4560 GOTO 830
4570 REM DETERMINISTIC MODELS WITH QTY DISCOUNTS
4580 CLS
4590 PRINT STRING$(70,"*")
4600 COLOR 23,0,7
4610 PRINT "INVENTORY MODELS WITH QUANTITY DISCOUNTS"
4620 COLOR 7,0
4630 PRINT STRING$(70,"*")
4640 PRINT
4650 PRINT "1.LOT SIZE MODEL WITH ALL UNITS DISCOUNTS ,ONE PRICE BREAK "
4660 PRINT
4670 PRINT "2.LOT SIZE MODEL WITH ALL UNITS DISCOUNTS,TWO PRICE BREAKS "
4680 PRINT
4690 PRINT "3.LOT SIZE MODEL WITH INCREMENTAL QUANTITY DISCOUNT "
4700 PRINT
4710 PRINT "4. RETURN TO MAIN OPTIONS "
4720 PRINT
4730 PRINT STRING$(70,"*")
4740 COLOR 23,0,7
4750 INPUT "ENTER YOUR CHOICE (1,4) ",C3
4760 COLOR 7,0
4770 ON C3 GOTO 4800 ,5310,6060,830
4780 PRINT "INVALID CHOICE , TRY AGAIN "
4790 GOTO 4580
4800 REM INVENTORY MODEL NO. VII
4810 REM LOT SIZE MODEL WITH ALL UNITS DISCOUNTS WITH ONE PRICE BREAK
4820 CLS
4830 INPUT "ENTER ANNUAL DEMAND IN UNITS D =",D
4840 INPUT "ENTER PROCUREMENT (ORDERING) COST CP ",CP
4850 INPUT "ENTER QTY AT WHICH PRICE BREAK OCCURS ,B",B
4860 INPUT "ENTER PRICE BELOW PRICE BREAK QTY C1 :",C1
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4870 INPUT "ENTER PRICE (<C1)ABOVE & EQUAL TO PRICE BREAK QTY C2 :",C2
4880 INPUT "COST OF CARRYING 1 UNIT OF INVENTORY PER TIME PERIOD :",R1
4890 Q2=SQR(2*D*CP/C2/R1)
4900 PRINT "Q2=";Q2
4910 TC2=D*C2+D*CP/B+B*C2*R1/2
4920 PRINT "TC2 =";TC2
4930 IF Q2 >= B THEN Q=Q2 :TC=TC2:GOTO 5000
4940 Q1=SQR(2*D*CP/C1/R1)
4950 IF Q1 >=B THEN Q=Q1:TC=D*C1+D*CP/Q1+Q1*C1*R1/2 :GOTO 5000
4960 TC1Q1=D*C1+D*CP/Q1+Q1*C1*R1/2
4970 TC2B=D*C2+D*CP/B+B*C2*R1/2
4980 IF TC1Q1 < TC2B THEN Q=Q1:TC=TCQ1: GOTO 5000
4990 Q=B :TCB=D*C2+D/B*CP+B*C2*R1/2
5000 CLS
5010 PRINT STRING$(70,"*")
5020 COLOR 23,0,7
5030 PRINT "THE INPUT DATA SUPPLIED IS AS FOLLOWS "
5040 COLOR 7,0
5050 PRINT STRING$(70,"*")
5060 PRINT
5070 PRINT "ANNUAL DEMAND IN UNITS , D=";D
5080 PRINT
5090 PRINT "PROCUREMENT COST CP =";CP:PRINT
5100 PRINT "QTY AT WHICH PRICE BREAK OCCURS ,B=";B:PRINT
5110 PRINT "PRICE BELOW PRICE BREAK QTY C1 =";C1
5120 PRINT
5130 PRINT "PRICE ABOVE & EQUAL TO PRICE BREAK QTY C2 =";C2:PRINT
5140 PRINT "COST OF CARRYING 1 UNIT OF INVT. PER TIME PERIOD R1 =";R1
5150 PRINT
5160 PRINT "PRESS F5 TO SEE THE OUTPUT "
5170 STOP
5180 CLS
5190 PRINT STRING$(70,"*")
5200 COLOR 23,0,7
5210 PRINT "THE OUTPUT OF THE MODEL VII IS AS FOLLOWS"
5220 COLOR 7,0
5230 PRINT STRING$(70,"*")
5240 PRINT
5250 PRINT "ECONOMIC ORDER QTY (EOQ) Q =";Q
5260 PRINT
5270 PRINT "OPTIMUM TOTAL COST TC2 = ";TC
5280 PRINT "PREES F5 TO CONTINUE "
5290 STOP
5300 GOTO 4580
5310 REM INVENTORY MODEL VIII
5320 REM INVENTORY MODEL WITH QTY DISCOUNT WITH TWO PRICE BREAK
5330 CLS
5340 PRINT STRING$(70,"*")
5350 COLOR 23,0,7
5360 PRINT "INVENTORY MODEL WITH QTY DISOUNT FOR TWO PRICE BREAKS"
5370 COLOR 7,0
5380 PRINT STRING$(70,"*")
5390 PRINT
5400 INPUT "ENTER ANNUAL DEMAND IN UNITS D =",D
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5950 PRINT
5960 PRINT "THE TOTAL OPTIMUM COST TC= ";TC
5970 PRINT
5980 PRINT STRING$(70,"*")
5990 PRINT
6000 PRINT
6010 COLOR 23,0,7
6020 PRINT "PRESS F5 TO CONTINUE "
6030 COLOR 7,0
6040 STOP
6050 GOTO 4580
6060 REM LOT SIZE MODEL WITH INCREMENTAL QUANTITY DISCOUNT
6070 CLS
6080 INPUT "ENTER DEMAND IN UNITS (YEARLY) D :",D
6090 PRINT
6100 INPUT "ENTER PROCUREMENT COST PER ORDER CP : ",CP
6110 PRINT
6120 INPUT "COST OF CARRYING ONE UNIT OF INVT. PER TIME PERIOD R1 :",R1
6130 PRINT
6140 INPUT "ENTER COST UPTO Ist DISCOUNT LEVEL C(1) ",C(1)
6150 B(0)=0
6160 INPUT "ENTER LAST VALUE OF DISCOUNT LEVEL :",N
6170 FOR I=2 TO N
6180 INPUT "ENTER COST FOR EVERY DISCOUNT LEVEL C(I) :",C(I)
6190 NEXT I
6200 FOR K=1 TO N :INPUT "ENTER PRICE BREAK QTY",B(K):NEXT K
6210 FOR I=2 TO N
6220 S=0
6230 FOR K=1 TO I-1
6240 S=S+C(K)*(B(K)-B(K-1))
6250 NEXT K
6260 Q(I)=SQR(2*D*(CP+S-C(I)*B(I-1))/R1/C(I))
6270 NEXT I
6280 FOR I=1 TO N
6290 IF B(I-1)<=Q(I) AND Q(I)<=B(I) THEN Q=Q(I):GOTO 6450
6300 TCIQ(I)=(CP+S-C(I)*B(I-1))*D/Q(I)+Q(I)/2*(C(I)*R1)+(D*C(I)+R1/2*S-C(I)*R1*B
(I-1)/2)
6310 PRINT TCIQ(I);
6320 NEXT I
6330 MIN=TCIQ(1)
6340 FOR I=2 TO N
6350 IF MIN <=TCIQ(I) THEN 6380
6360 MIN=TCIQ(I)
6370 P=I
6380 NEXT I
6390 PRINT "THE OUTPUT OF ELSM WITH INCREMENTAL QTY DISCOUNT IS AS FOLLOWS "
6400 PRINT
6410 PRINT "EOQ = ";Q(P)
6420 PRINT
6430 PRINT "MINIMUM TOTAL COST TCIQI =";TCIQ(P)
6440 PRINT :PRINT "PRESS F5 TO CINTINUE ":STOP:GOTO 6440
6450 CLS
6460 PRINT "THE OUTPUT FOR ELSM WITH INCREMENTAL QTY DISCOUNT IS"
6470 PRINT
6480 PRINT "EOQ = ";Q
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6490 TCIQ=(CP+S-C(I)*B(I-1))*D/Q+Q/2*(C(I)*R1)+(D*C(I)+R1/2*S-C(I)*R1*B(I-1)/2))
6500 PRINT
6510 PRINT "MIN YEARLY TOTAL COST TIC = ";TCIQ
6520 PRINT
6530 PRINT "PRESS F5 TO CONTINUE "
6540 STOP
6550 GOTO 4580
6560 CLS
6570 REM .......................................................................
6580 REM
6590 PRINT :PRINT :PRINT STRING$(70,"-")
6600 PRINT :PRINT "..... E X E C U T I O N    O V E R ....."
6610 PRINT :PRINT
6620 PRINT :PRINT "..... H A V E     A     N I C E     T I M E ....."
6630 PRINT :PRINT STRING$(70,"-")
6640 REM
6650 REM .......................................................................
6660 END
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5410 INPUT " PROCUREMENT (ORDER ) COST PER ORDER  CP :",CP
5420 PRINT
5430 INPUT "ENTER HOLDING COST (CU*I),CH :",CH
5440 PRINT
5450 INPUT "ENTER QTY AT WHICH FIRST BREAK OCCURS(B1) : ",B1:PRINT
5460 INPUT "ENTER QTY AT WHICH SECOND PRICE BREAK OCCURS B2 :",B2:PRINT
5470 INPUT "ENTER PRICE UPTO FIRST QTY BREAK ,C1 : ",C1  :PRINT
5480 INPUT "ENTER PRICE UPTO SECOND QTY BREAK ,C2 :",C2:PRINT
5490 INPUT "ENTER PRICE ABOVE & EQUAL TO SECOND QTY BREAK , C3  :",C3
5500 PRINT
5510 INPUT "ENTER INVENTORY CARRYING COST EXPRESSED AS % OF STOCK VALUE  R1",R1
5520 Q3=SQR(2*D*CP/C3/R1)
5530 IF Q3 >= B2 THEN Q=Q3:TCQ3=D*C3+D/Q*CP+Q*C3*R1/2:TC=TCQ3:GOTO 5720
5540 Q2=SQR(2*D*CP/C2/R1)
5550 IF Q2 >=B1 AND Q2 <=B2 THEN 5570
5560 GOTO 5620
5570 TCQ2=D*C2+D/B1*CP+B1*C2*R1/2
5580  TCB2=D*C3+D/B2*CP+B2*C3*R1/2
5590 IF TCQ2<TCB2 THEN Q=Q2:TC=TCQ2:GOTO 5690
5600 Q=B2
5610 GOTO 5690
5620 Q1=SQR(2*D*CP/C1/R1)
5630 TCB1=D*C2+D/B1*CP+B1*C2*R1/2
5640 TCQ1=D*C1+D/Q1*CP+Q1*C1*R1/2
5650 TCB2=D*C3+D*CP/B2+B2*C3*R1/2
5660 IF TCB1 <=TCQ1 AND TCB1<=TCB2 THEN Q=B1 :TC=TCB1:GOTO 5690
5670 IF TCQ1<=TCB1 AND TCQ1<=TCB2 THEN Q=Q1:TC=TCQ1:GOTO 5690
5680 Q=B2:TC=TCB2
5690 CLS
5700 PRINT STRING$(70,"*")
5710 COLOR 23,0,7
5720 PRINT "THE INPUT DATA SUPPLIED FOR MODEL VIII IS AS FOLLOWS"
5730 COLOR 7,0
5740 PRINT STRING$(70,"*")
5750 PRINT
5760 PRINT "ANNUAL DEMAND IN UNITS D =";D:PRINT
5770 PRINT "PROCUREMENT COST PER ORDER CP =";CP
5780 PRINT
5790 PRINT "HOLDING COST (CU*I) ,CH=";CH
5800 PRINT
5810 PRINT "QTY AT WHICH FIRST PRICE BREAK OCCURS  B1= ";B1:PRINT
5820 PRINT "QTY AT WHICH SECOND PRICE BREAK OCCURS B2 =";B2:PRINT
5830 PRINT "PRICE UPTO FIRST QTY BREAK C1= ";C1:PRINT
5840 PRINT "PRICE UPTO SECOND QTY BREAK C2 =";C2:PRINT
5850 PRINT "PRICE ABOVE OR EQUAL TO SECOND QTY BREAK C3 =";C3:PRINT
5860 PRINT "INVENTORY CARRYING COST EXPRESSED AS % OF STOCK VALUE R1 =";R1:PRINT
5870 PRINT STRING$(70,"*")
5880 CLS
5890 PRINT STRING$(70,"*")
5900 COLOR 23,0,7
5910 PRINT "THE OUTPUT OF MODEL VIII IS AS FOLLOWS "
5920 COLOR 7,0
5930 PRINT
5940 PRINT "THE ECONOMOC ORDER QTY (EOQ) Q=";Q
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