CHAPTER – II

ACTIVE FILTERS

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2.1 INTRODUCTION :-

Filter process signals on a frequency dependent basis. It is a frequency selective circuit that passes a specified band of frequencies and attenuates signals of frequencies outside this band. Depending on the components used in the filter there are two types of filters, active filter and passive filter. Filter with active device like operational amplifier, transistor is called active filter. Active filter offers the following advantages over passive filters.

i) Since active device provides a gain, the input signal is not attenuated as it is attenuated in a passive filter. Active filters are easier to tune or adjust.

ii) Due to high input resistance and low output resistance of operational amplifier the active filter does not cause loading of the source or load.

iii) Active filters are more economical than passive filters. The departure of practical operational amplifier from ideality at high frequencies restricts active filter applications below the MHz range.

The behaviour of filter is uniquely characterised by its transfer function H(S). Practical transfer functions are rational functions of S

$$H(S) = \frac{N(S)}{D(S)} \qquad \text{------------------} \qquad \mathbf{2.11}$$

Where N(S) and D(S) are suitable polynomials of S with real coefficient and order of N(S) never exceeds that of D(S). The order of D (S) is called the order of the filter.

2.2 ORDER OF FILTERS : -

To improve performance of filter, large no. of sections should be connected in series. This increases the order of filter. The RC combinations used in circuit determine the order of filter.

Roll off of gain in the stop band is determined by the order of the filter. Each unit increase in the order increase in roll off by 20 dB/decade. Fig. 2.1 represents the change in the rolloff of the gain for different orders in lowpass filter.

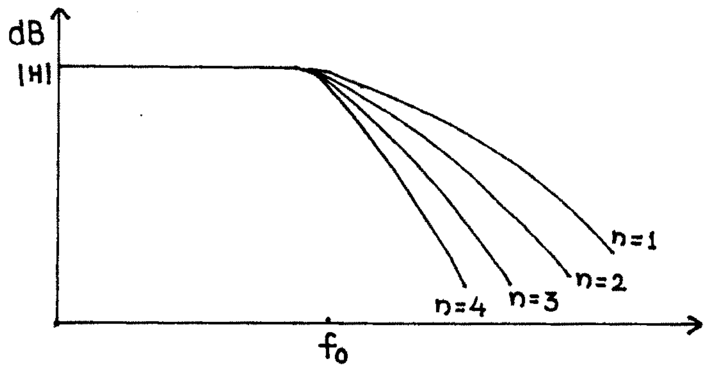

Fig. 2.1 (Low pass filter gain roll-off)

In ideal filters the signal is not attenuated in the pass band and attenuated completely in the stop band. Also the phase shift change is linear with frequency in pass band of the filter. The ideal brickwall response is not possible practically because the synthesis of the sharp edges with continuous function would require complex circuitry. Practical filter can only approximates the ideal model with rational function of jf of the type

$$H(jf) = \frac{N(jf)}{D(jf)} \qquad \text{------- } \mathbf{2.12}$$

Degree of D (jf) determines the order of filter.

Higher the order of filter, the closer the approximation to ideal. The circuit complexity increases with order of filter.

2.3 FIRST ORDER ACTIVE FILTERS :-

The active filters are obtained from the inverting Op.Amp. configuration by replacing one or both resistors with reactive elements. Since the inverting amplifier provides amplification replacing its elements with frequency dependent device will yield frequency dependent amplification.

i) Low pass filter:-

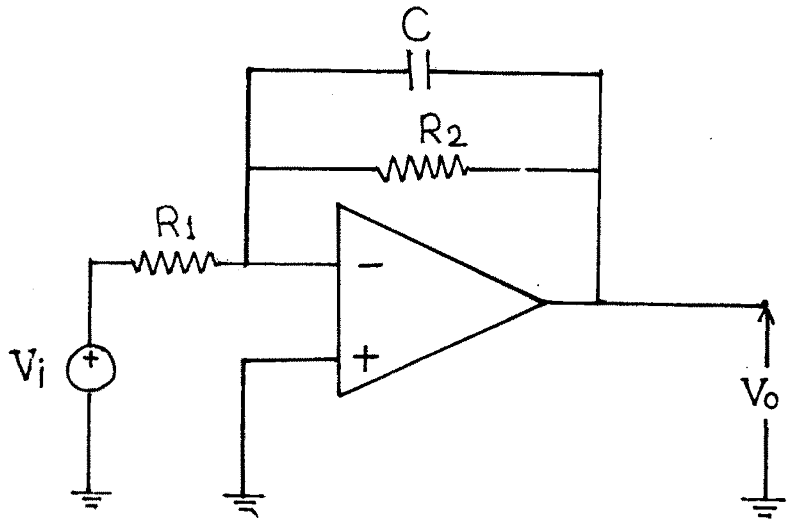

Fig. 2.2 (First order low pass active filter)

In inverting integrator circuit if resistor is placed in parallel with feedback capacitor turns integrator into lowpass filter with gain.

2.3.1 **First order Low pass filter:-**

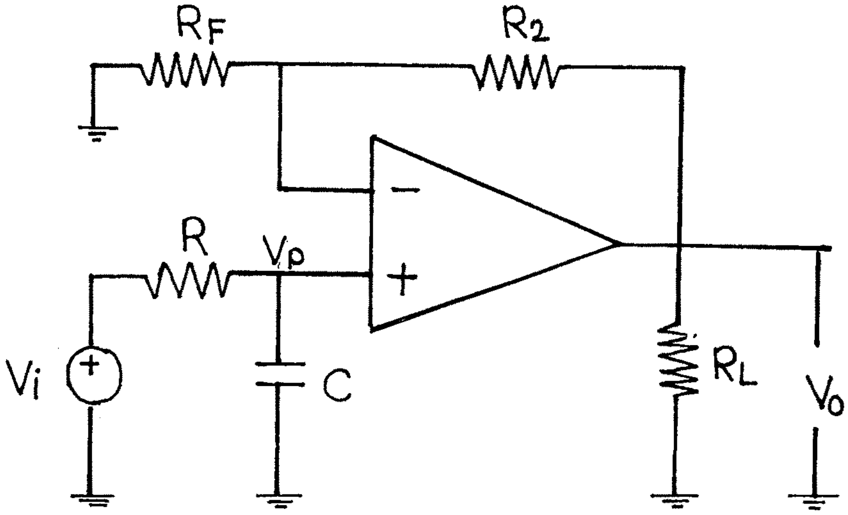

Fig. 2.3 (First order low pass active filter with non inverting configuration)

First order lowpass filter uses RC network for filtering OP.Amp is used in the non-inverting configuration. The gain of the non-inverting amplifier is-

$$H_0 = (1 + R_2/RF)$$

The voltage across the capacitor is-

$$Vp = \frac{\dfrac{1}{J\,2\pi fC} Vi}{\dfrac{1}{J\,2\pi fC} + R} = \frac{Vi}{1 + j2\pi fCR}$$

The output voltage $V0 = (1 + R_2/R_F)\, Vp$

$$H = \frac{V_0}{Vi} = \frac{H_0}{1 + j\,(f/f_0)} \qquad \text{----- 2.22}$$

Where $H0 = (\,1 + R_2/RF)$

$$F0 = \frac{1}{2\pi RC} \qquad \text{------2.23}$$

The magnitude of gain $|H| = |V_0/Vin| = \dfrac{H_0}{\sqrt{(1 + f/f^0)^2}}$ --------2.24

If $f < f_0$ $\left|\frac{V_0}{V_{in}}\right| \cong H_0$

$f = f_0$ $\left|\frac{V_0}{V_{in}}\right| \cong \frac{H_0}{\sqrt{2}}$

$f > f_0$ $\left|\frac{V_0}{V_{in}}\right| \cong H_0$

2.3.2 First order High pass filter

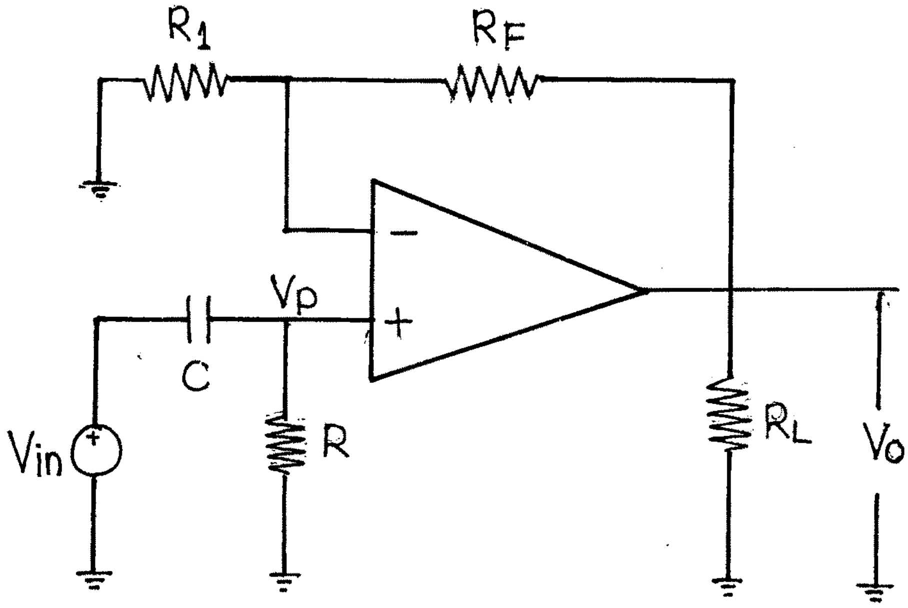

Fig. 2.4 (First order High pass filter)

Interchanging resistor and capacitors in low pass filters forms the high pass filters.

$$V_p = \frac{R}{R + \frac{1}{j2\pi fc}} = \frac{j2\pi fRC}{1 + j2\pi fRC} V_{in} \quad \text{------------- } \mathbf{2.31}$$

The output voltage $V_0 = \left(1 + \frac{R_F}{R_1}\right) V_P$

$$V_0 = \left(1 + \frac{R_F}{R_1}\right) \frac{j2\pi fRC}{1 + j2\pi fRC} V_{in}$$

$$H = \frac{V_0}{V_{in}} = \frac{H_0 j(f/f_0)}{1 + j(f/f_0)} \quad \text{------------- } \mathbf{2.32}$$

$$H_0 = \left(1 + \frac{RF}{R_1}\right)$$

$$f_0 = \frac{1}{2\pi RC}$$

The magnitude of voltage gain is-

$$\left|\frac{V_0}{V_i}\right| = |H| = \frac{H_0\,(f/f0)}{\sqrt{1+(f/f_0)^2}} \qquad \text{------------- 2.33}$$

When $f < f_0 \quad |H| < H_0$

$f = f_0 \quad |H| \cong \frac{H_0}{\sqrt{2}}$

$f > f_0 \quad |H| \cong H_0$

2.3.3 **First order Band pass filter**

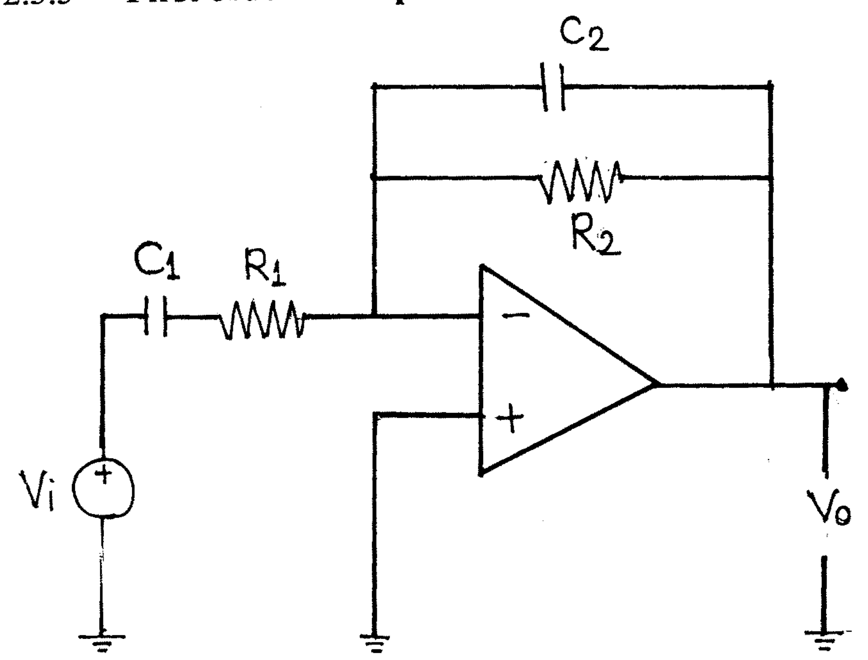

Fig. 2.5 (First order band pass fiiter)

The first order bandpass filter has two feedback paths hence it is called multiple feedback filter. The Op. Amp is used in the inverting mode. The input impedance $Z_1 = R_1 + 1/ jwC_1$ forms a high pass section with cut off frequency f_1

$$f_1 = \frac{1}{2\pi R_1 C_1}$$

The feedback impedance $Z_2 = R_2 \| \frac{1}{JWC2}$ forms a lowpass section with cutoff frequency

$$f_2 = \frac{1}{2\pi R_2 C_2}$$

If $f_1 < f_2$ then the frequencies within the band f_1 to f_2 will succeed in passing through the circuit while those falling outside will be rejected

The transfer function

$$H = \frac{-Z_2}{Z_1}$$

$$H = H_0 \frac{j(f / f1)}{[1+ j(f / f1)]\,[1+ j(f / f_2)]} \qquad \text{---------------2.41}$$

$$H_0 = \frac{-R_2}{R_1} \qquad \text{------------------2.42}$$

$$f_1 = \frac{1}{2\pi R_1 C_1} \qquad \text{------------------2.43}$$

$$f_2 = \frac{1}{2\pi R_2 C_2} \qquad \text{------------------2.44}$$

The magnitude of gain

$$H = \frac{H_0\,(f / f1)}{\sqrt{[1+(f/f1)^2]\,[1+(f/f_2)^2]}} \qquad \text{---------------2.42}$$

2.3.4. **First order Band reject filter:-**

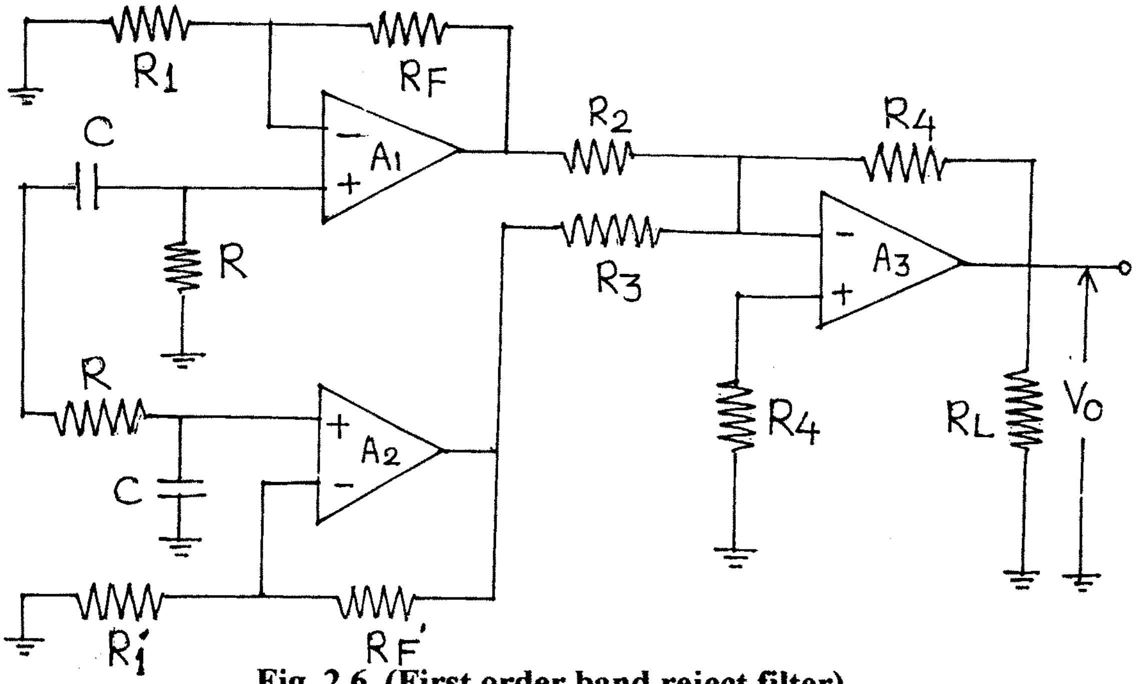

Fig. 2.6 (First order band reject filter)

The wide band reject filter is formed using a lowpass filter, a highpass filter and a summing amplifier.

The lower cufoff frequency f1 is determined by high pass filter and the higher cutoff frequency f2 is determined by lowpass filter. To obtain band reject response $f_1 > f_2$

2.4 SECOND ORDER RESPONSES :-

In all first order responses the denominator term is same $1 + j(f/f_0)$ and the numerator determines the nature of response. The numerator 1 yields the low pass response, numerator of $j(f/f_0)$ yields the high pass and the numerator of $(1 - j\ (f/f_0)$ yields all pass. Multiplying the response by gain constant H_0 does not change the response type.

In second order filter the degree of the denominator is 2. The second order function has a standard form

$$H\ (jf/f_0) = \frac{N\ (jf/f_0)}{1 - (f/f_0)^2 + (v/Q)\ (f/f_0)}$$

When Q is another filter parameter and $N(jf/f_0)$ is a suitable polynomial of degree not greater than 2.

2.4.1) Second order low pass response:-

The second order low pass function have a standard form

$H(jf/f_0) = H_{OLP} H_{LP}(jf/f_0)$

H_{OLP} ----> dc gain

$$H_{LP} = \frac{1}{1 - (f/f_0)^2 + (V/Q)(f/f_0)} \qquad 2.45$$

a) For $(f/f_0) << 1$ $\qquad H_{LP} \to 1$

$|H_{LP}|$ dB = 0

b) For $(f/f_0) >> 1$ $\qquad HLP \to -1/(f/f_0)^2$

$|H_{LP}|$ dB = $20 \log_{10}[1/(f/f_0)^2]$

$|H_{LP}|$ dB = $-40\log_{10}(f/f_0)$

This equation is the equation of straight line with slope of –40 dB/decade

c) For $(f/f_0) = 1$ $\qquad H_{LP} = 1/J/Q = -JQ$

$|H_{LP}|$ dB = Q dB

This indicate that when $f = f_0$ then the value of curve depends on Q.

2.4.2) Second order high pass response :-

The second order high pass function have standard form

$H(jf/f_0) = H_{OHP} H_{HP}(jf/f_0)$

H_{OHP} ----> High frequency gain

$$H_{HP} = \frac{-(f/f_0)^2}{1-(f/f_0)^2 + (j/Q)(f/f_0)} \qquad \mathbf{2.46}$$

a) When $f/f_0 << 1$ $\qquad H_{HP} \longrightarrow -(f/f_0)^2$

$|H_{HP}|\ dB = 40\log_{10}(f/f_0)$

b) For $f/f_0 = 1$ $\qquad H_{HP} \longrightarrow \dfrac{-1}{(j/Q)}$ $\qquad H_{HP} = JQ$

$|H_{HP}|\ dB = Q\ dB$

c) For $f/f_0 >> 1$ $\qquad H_{HP} \longrightarrow \dfrac{(f/f_0)^2}{(f/f_0)^2}$

$|H_{HP}|\ dB = 0dB$

2.4.3) Second order Band pass Response :-

The standard form of second order band pass function is-

$H(if/f_0) = H_{OBP}\, H_{BP}\, (jf/f_0)$

H_{OBP} ----> Resonant gain

$$H_{BP} = \frac{(j/Q)\ (f/f_0)}{1 - (f/f_0)^2 + (j/Q)\ (f/f_0)} \qquad 2.47$$

a) For $(f/f0) << 1$ $\qquad H_{BP}$ --------> $(j/Q)\ (f/f_0)$

$|H_{BP}|\ dB = 20\log_{10}(1/Q)\ (f/f_0)$

$|H_{BP}|\ dB = 20\log_{10}(f/f_0) - Q\ dB$

This is the equation of the type $y = 20x - Q$ dB

That is a straight line with slope 20dB/decade

b) For $(f/f_0) >> 1$

$$HBP \text{ ---> } \frac{-\ j/Q}{(f/f_0)}$$

$|H_{BP}|\ dB = -20\log_{10}\ (f/f_0) - Q\ dB$

This the equation of the type $y = -20x - Q$ dB that is a straight line with slope of –20dB/decade

c) For $(f/f_0) = 1$ $H_{BP} = 1$

$|H_{BP}|$ dB = 0

|HBP| peaks at $(f/f_0) = 1$, regardless of the value of Q. f_0 is called resonant frequency, or center frequency or peak frequency.

2.4.4) Second Order Band Reject response :-

The most common form of notch function is

$$H(jf/f_0) = H_{ON}\, H_N(jf/f_0)$$

Where H_{ON} -----> gain constant

$$H_N = \frac{1 - (f/f_0)^2}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \qquad 2.48$$

When a) $f/f_0 << 1$ $H_N = 1$
$|H_N|$ dB = 0

b) $f/f_0 >> 1$ $H_N = 1$
$|H_N|$ dB = 0

c) $f/f_0 = 1$ $H_N = \infty$

f_0 is called as notch frequency.

H_N can also be written as $H_N = H_{LP} + H_{HP}$

Or $H_N = 1 - H_{BP}$. This relationship indicates alternate ways of achieving a notch response when other responses are available.

2.5) HIGHER ORDER FILTERS :-

Higher order filter design is a multistep process. To design higher order filters at first response type must be chosen and set a specifications generated that will meet the needs of the given application. These specifications are given in terms of fc, fs, Amax and Amin. These data are then used to determine the order of filter n. Once n is known, find individual stage values of f_0 and Q for cascade approach. Finally desired filter circuit can be designed.

In cascade design approach a filter of order n is designed by cascading n/2 second order sections if n is even for odd n, (n-1)/2 second order sections and one first order sections are connected in series. In first order section the corner frequency f_0 is calculated from RC network. For other sections f_0 or Q is calculated from sections to section.

Mathematically the orders in which various sections are cascaded are irrelevant. To avoid the loss of dynamic range and filter accuracy due to signal clipping in the high Q sections, the cascading is done by connecting lower Q section first and high Q section at the last. This connection does not consider the internal noise. To minimise noise, high Q stage should be connected first in cascade. Practically ordering depends on the input frequency spectrum, the filter type and the noise characteristic of the components.

2.6) FILTER APPROXIMATION :-

Ideal filter shows the brickwall response. In practical filter circuit higher the order of the filter, closer the response to brickwall response i.e. closer the approximation to brickwall response. The departure of a practical filter response from its brickwall response can be visualized in terms of band of values as shown in fig. 2.7 for low pass case.

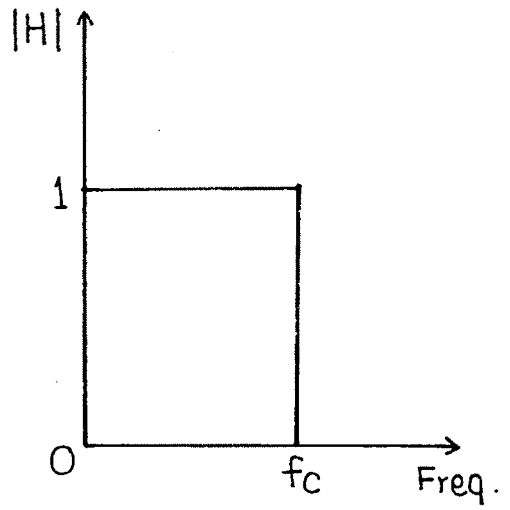

Fig 2.7 (a)

Brick wall low pas response
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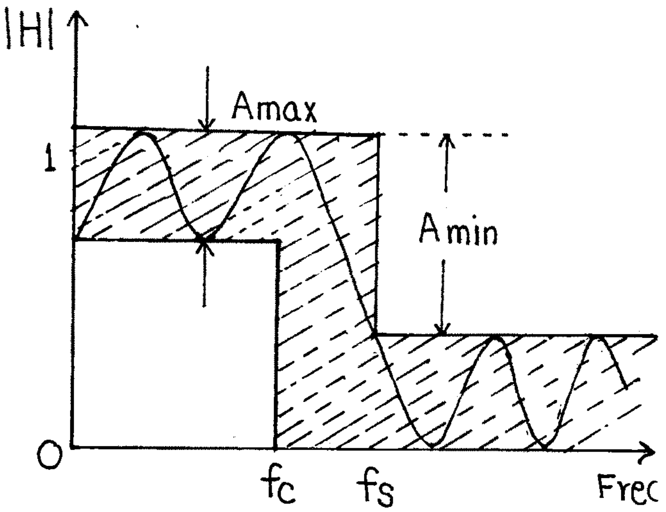

Fig 2.7(b)

Magnitude response limit for Low pass filter approximation

Range of frequency that are passed with little or no attenuation by the filter is called the pass band. Maximum allowable gain change in pass band is Amax is also called minimum pass band ripples.

Above the cutoff frequency there is maximum attenuation. This band is called stopband. The frequency at which stopband begins is fs. The frequency region between fc and fs is called transition band or skirt. Stop band is specified in terms of minimum allowable attenuation Amin.

Choice of approximation is important step in filter design. The approximation is nothing but finding a function whose loss characteristic lies within the permitted region. The function, which is selected, must be realizable using passive or active components. Order of function must be as small as possible to minimize the components required for design.

The steps in filter design are-

i) Filter specification selection

ii) Selection of rational function which satisfy the specifications.

iii) Realization of transfer function and calculations of component values.

iv) Construction and testing of filters.

In practice a suitable function which approximate the gain curve within specified tolerance and which are realizable as physical network is selected for filter design. A method of approximation based on Bode plot is suitable for low order filter design. More complex filter characteristics are approximated by using some well-described rational functions whose roots have been tabulated.

Butterworh, Chebyshev, Bessels and Elliptic are the approximation used in filter theory. These approximations are directly applicable to lowpass filter. Using frequency transformation techniques these approximations are used to design high pass, symmetrical bandpass and bandstop filters.

2.6.1) Butterworth Approximation :-

It is special form of Taylor series approximation in which approximating function t(w) and the specified function f(w) are identical at w=0. For this approximation Kn(w) is selected as-

$$Kn\,(w) = B_0 + B_{1w} + B_2w^2 + \text{-----} + Bnw^n \qquad \textbf{2.61}$$

For Taylor series approximation the function Kn(w) must be maximally flat at the origin (w = 0)

Hence as many derivatives of Kn(w) as possible must vanish at w=0 Therefore for Butterworth approximation.

$$Kn(w) = w^n$$

As order of filter is increased, the pass band is flat over wider intervals and stopband loss is increased.

2.6.2 Chebyshev Approximation :-

For obtaining the best approximation from a polynomial of a given degree the Butterworth function is not useful. Because it concentrate all the approximating ability of the polynomial at w = 0, instead of distributing it over the range $0 < w < 1$. A better result in this regard may be obtained if we look for a rational function that approximate the constant value unity throughout this range in oscillatory manner, Chebyshev approximation does exactly this.

Chebyshev polynomials are defined as linearly independent solutions of the differential equations.

$$(1-w^2)\, y \;\; wy - n^2 y = 0$$

one of the solution is-

$$Y = Tn(w) = \cos (n \cos^{-1} w) \qquad |w| < 1 \qquad \textbf{2.62}$$

$$Y = Tn(w) = \cosh (n\cosh\text{-}1 w) \qquad |w| > 1 \qquad \textbf{2.63}$$

Tn (w) is called Chebyshev polynomial. A power series expansion for Tn(w) can be obtained by rewriting equation

$$Tn(w) = Re\, |e^{jn}\, \phi\, | = Re\, |\text{Cos}\phi + j\sin\phi|^n \qquad \textbf{2.64}$$

Where $\phi = \cos^{-1} w$

i.e. $\cos\phi = w$ $\qquad \sin \phi = \sqrt{1-w^2}$

Binomial expansion of equation yields

$$Tn(w) = Re\, [w + j\, (1-w^2)]^n$$

$$= \frac{w^n - n(n-1)}{2!}\; w^{n-2}\, (1-w^2) \qquad \textbf{2.65}$$

$$+\frac{n(n-1)(n-2)(n-3)}{4!} w^{n-4} (1-w^2)^2$$

From which $T_0(w) = 1$

$T_1(w) = w$

The properties of Chebyshev polynomial are :

(i) The zeros of the polynomial are all located in the interval $|w| < 1$

(ii) The Chebyshev poynomials passes special value at $w = 0$

1 or –1 $\quad T_n(w) = (-1)^{n/2}$ for n = even

$= 0$ for n = odd

(iii) Tn(w) is either even or odd functions depending on whether n is even or odd.

The magnitude function $|T(jw)|^2$ and attenuation function $\alpha(w)$ are given by

$$|T(w)|^2 = \frac{1}{1 + C^2 w^{2n}}$$

$$\alpha(w) = 10\log[1 + c^2 w^{2n}] \qquad \textbf{2.66}$$

C ----> attenuation constant

The w should be interpreted as the frequency normalized with respect to passband edge w_p

$$|T(jw)|^2 = \frac{1}{1 + C^2(w/wp)^{2n}}$$

$$\alpha(w) = 10\log[1 + C^2(w/wp)^{2n}] \quad 2.67$$

The frequency response of Butterworth filter for various values of n is shown in fig.

All the curves passes through the same point at w = wp and this point is determined by αp

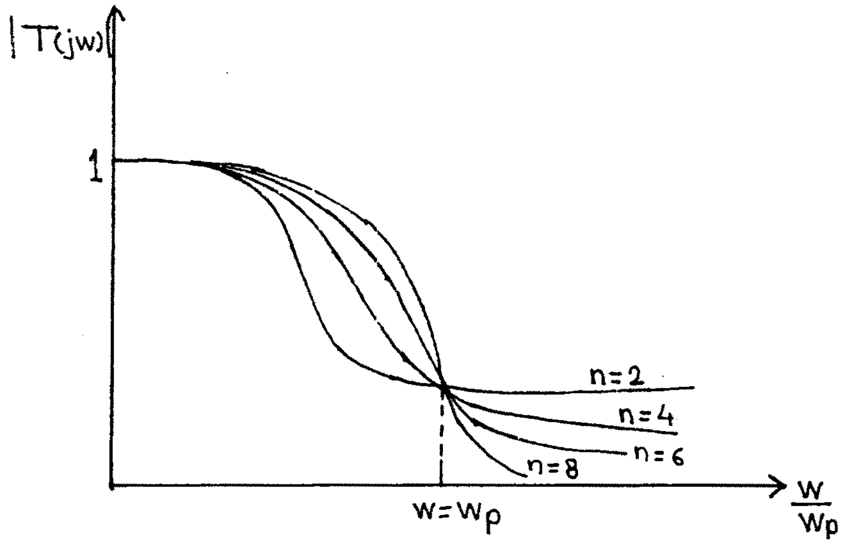

Fig. 2.8 (Frequency response of Butterworth low pass filter for various values of n)

$$\alpha p = 10\log[1 + C^2(wp/wp)^{2n}]$$

$$\alpha s = 10\log[1 + C^2(ws/wp)^{2n}]$$

K = (ws/wp) = sensitivity parameter

$$C^2 = 10^{0.1\alpha p} - 1 \quad ----$$

$$\frac{C^2}{K^{2n}} = 10^{0.1\alpha p} - 1 \text{ -----}$$

$$K^{2n} = \frac{10^{0.1\alpha p} - 1}{10^{0.1\alpha p} - 1}$$

$$K_1 = \left(\frac{10^{0.1\alpha p} - 1}{10^{0.1\alpha p} - 1} \right)^{1/2} \qquad \textbf{2.68}$$

The discrimination parameters

$$n = \frac{\log K_1}{\log K} \qquad \textbf{2.69}$$

The order of the filter n should be selected such that

$$n = \frac{\log K_1}{\log K} \qquad n = \text{integer}$$

If $n = \frac{\log K_1}{\log K}$ the value of C obtained from equation are the same.

If $n = \frac{\log K_1}{\log K}$ the C can be selected to satisfy the passband edge or stopband edge requirement exactly.

iv) Every coefficient of Tn(w) is an integer and the one associated with

Wn is Z^{n-1}

Thus the limit 'w' approaches infinity

$$Tn\ (w) = Z^{n-1}\ w^{2n}$$

v) In the range $-1 < w < 1$ all of the Chebyshev polynomials have the equal ripple property varying between maximum and minimum and outside this range their magnitude increase monotonically as w is increased and approaches infinity.

The even order chebyshev lowpass filter has a zero frequency loss which is equal to the pass band ripple maximum gain. However this implies that source resistance can not be equal to the load impedance. One restriction around this is to use a frequency transformation, which changes the loss at dc.

2.63 Elliptic Filter Approximation :-

To improve the performance of achieved by the Chebyshev filter, the equiriple response in both passband and stopband is allowed. This leads to narrower transition band. Such filters are designed by using Elliptical function and refereed as "Elliptical filters". The approximation is called "Elliptical approximation". The elliptic approximation is rational function with finite number of poles and zeros. In this approximation the location of poles must be chosen to provide the equiripple stopband characteristics. The poles closest to the stopband edge significantly increase the slope in the transition band. The further poles are needed to maintain the required level of stopband attention.

By using the finite poles the elliptic approximation able to provide a considerably higher flat level of stopband less than the Butterworth and Chebyshev approximations. Thus for a given requirement the elliptic approximation will in general requires a lower order than the Butterworth or Chebyshev.

2.64 Bessels'Approximation :-

In all approximation discussed so far concentrated on approximating the magnitude of transfer function. In many signal-processing applications, linearity of the phase or constant phase delay is an important factor. The phase distortion is more in Chebyshev filter than the Butterworth filter. An equiripple filter has greater amount of phase distortion than the maximally flat filter.

For Bessel approximation the stop band has poor response so that Bessel approximation is not useful for practical implementation for most filtering applications. Therefore the alternate solution to this problem is to obtaining a flat delay characteristic by the use of delay equalizers. The Bessel approximation is a polynomial that approximates this ideal characteristic.

In this approximation the delay at the origin is maximally flat i.e. as many derivatives are possible are zero at origin. It is covinient to consider approximation of the normalized function, with the dc delay $T_0 = 1$ that is

$$H(s) = e^{s}$$

The Bessel approximation to this normalized function is

$$H(s) = \frac{Bn(s)}{Bn(o)} \qquad \textbf{2.70}$$

Where Bn(s) is the nth order Bessel polynomial which is defined by following equation.

$$Bu(s) = 1$$

$$Bn(s) = S + 1$$

$$B1(s) = (2n-1)\, Bn(S1) + B_2Bn(S_2) \qquad \textbf{2.71}$$

The polynomial B(s) is called Bessel's polynomial of order n. The delay error of Bessel's approximation are shown in fig.2.9

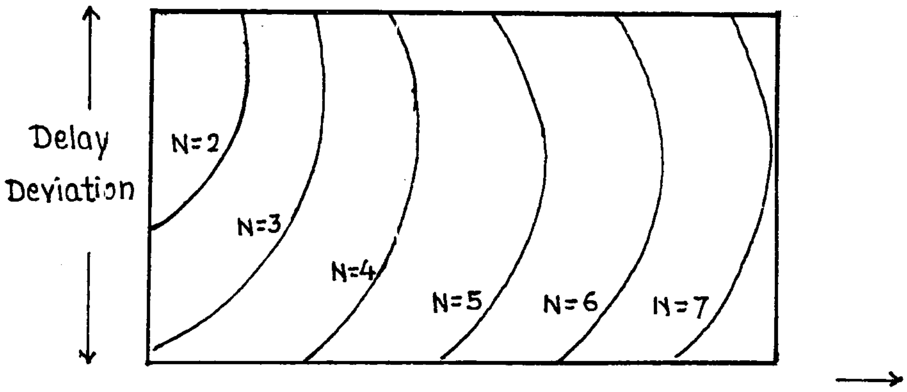

Fig. 2.9 (Delay error in Bessel Approximation)

2.7 COMPARISON OF BUTTERWORTH, CHEBYSHEV AND ELLIPTIC RESPONSE :-

Butterworth response maximizes the flatness of the magnitude response with the passband. The response is extremely flat near dc and somewhat rounded near cutoff frequency f_0 and approaches an ultimate rolloff rate of – 20n dB/decade in the stopband.

There are applications in which sharp cutoff is more important than maximum flatness. Chebyshev filter maximizes the transition band cutoff rate at the prize of introducing passband ripples. Thus compared to the Butterworth response, which exhibits appreciable departure from it ,dc value only at the upper end of the passband. The Chebyshev response improves the transition band characteristics by spreading equal sized ripples throughout the passband. The number of ripples increases with n. Although both response exhibit an ultimate rolloff rate that depends only on n.

The Chebyshev response can achieve a given transition band cutoff rate with a lower order reducing filter complexity and cost. In the limit of O dB passband ripple the Chebyshev response becomes the Butterworth response.

Elliptic filters also called Cauer filters ,carry the Chebyshev approach one step further by accepting ripples in both the passband and the stop band in order to achieve an even sharper characteristics in the transition band. The idea is to follow on already sharp lowpass response

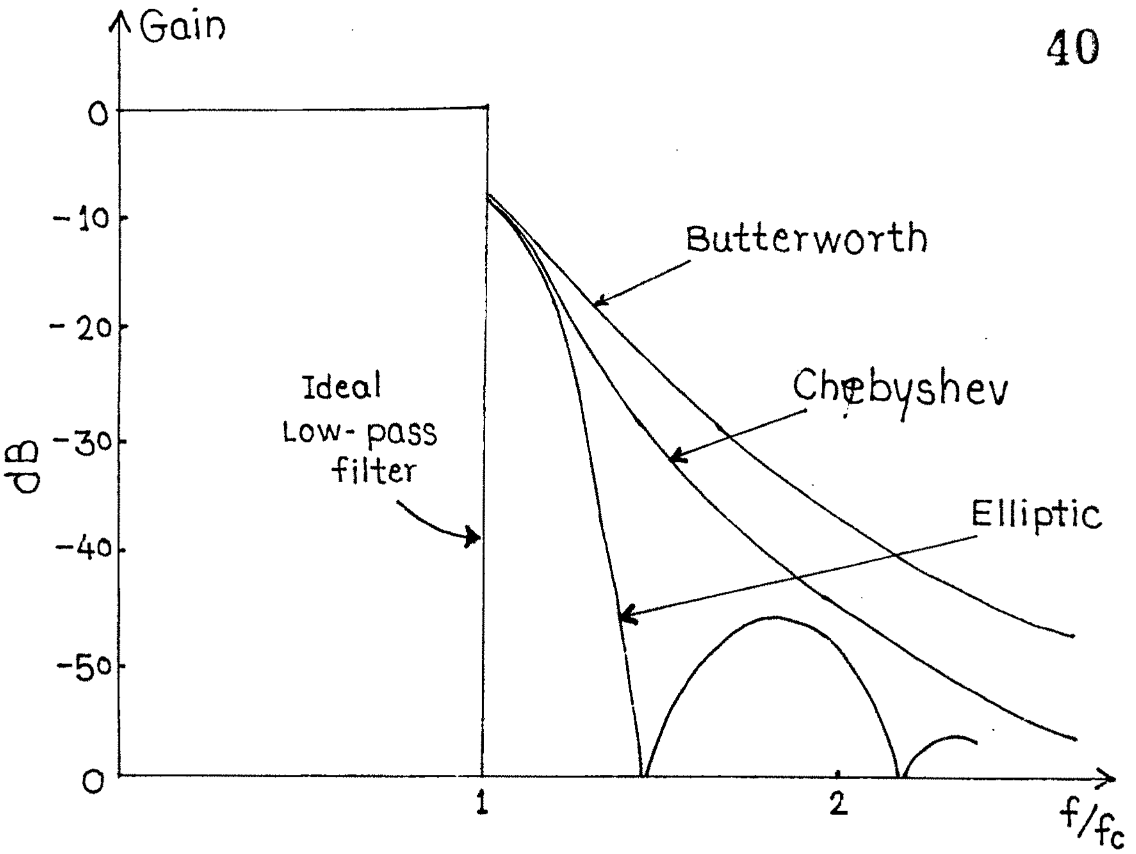

Fig.2.10 Comparison of Butterworth, chebyshev and elliptic responses for n=5

with a notch just above the cutoff frequency to sharpen the response even further. To be effective notch must be narrow, indicating that the curve will come backup just past the notch itself. At this point another notch is created to press the curve back down and the process is repeated until the net profile within the stopband is pushed below the prescribed minimum attenuation level. Amin.

2.8 FILTER SENSITIVITY :-

Practically the filter performance is affected by the component tolerance, drift and aging and nonlinearities of operational Amplifier. The filter parameters like cutoff frequency, Quality factor and gain find departure from theoretical values due to component tolerance drift, aging and nonlinearities of Op.Amp.

The drift parameters are reduced by tuning the parameters. For parameters tuning trimmers are required. The tuning can be time consuming and expensive. The careful designer reduces the need of tuning through suitable choice of circuit topology as well as component quality.

Suppose y is the filter parameter (f_0, Q or gain) and x is the circuit component (resistance or capacitance). It is important to know the fractional parameter change $\Delta y/y$ caused by fractional component change $\Delta x/x$. If fractional changes are multiplied by 100, we obtain percentage change.

Sensitivity is defined as the percentage change in parameter with respect to percentage change in component value and is represented by-

$$ {}_{x}S^{y} = \frac{dy/y}{dx/x} = \frac{x}{y}\,\frac{dy}{dx} \qquad 2.81 $$

Partial derivatives are used because filter parameters usually depends on more than just one component.

If value of sensitivity ${}_{x}S^{y}$ is known, from that percentage parameter change is calculated.

$$ 100\,\frac{\Delta y}{y} = 100\,{}_{x}S^{y}\,\frac{\Delta x}{x} $$

The interdependence of a given parameter y and a given component x is of the type

$$ y = Ax^{k} \qquad 2.82 $$

A is appropriate expression that is independent of x and k is a suitable exponent.

$$ \frac{dy}{dx} = KA\,x^{k-1} = \frac{KAx^{k}}{x} = Ky/x \qquad 2.83 $$

$$ {}_{x}S^{y} = K $$

Sensitivities constitute important factor in weighing different realizations of the some filter function for the purpose of selecting the one best suited to the application. Sensitivities help the designer to specify the component tolerance required to meet the design objectives.