PREFACE

The dissertation titled 'A STUDY OF SPACE-LIKE CONGRUENCES IN GENERAL RELATIVITY' is divided into four chapters. Motivation for this exploration of the space-like congruences stems from the two facts :

1. Research on the space-like congruence has been meagre; for instance the recent text book published in 1982 by STEPHANI entitled 'General Relativity : An Introduction to the Theory of the Gravitational Field' refers to the parameters of time-like congruences and null congruences only. The space-like congruences are ignored.

2. There is a plethora of space-like congruences in relativistic continuum mechanics and they are more numerous than the time-like congruences. The reason for this claim is that for one given time-like congruence (say on the world line) one can always construct three orthogonal space-like congruences.

The first chapter 'Historical Introduction' describes the space-like congruences in relativistic hydrodynamics and magnetohydrodynamics. The illustration of the heat flux congruence in the theory of relativistic thermodynamics is given later. In Section 3, the three parameters of a space-like congruence viz., expansion, shear, rotation subject to

the Greenberg transport laws are introduced. The last section contains the generalized Serret-Frenet formulae amenable for relativistic mechanics. Thus this chapter does not go beyond the setting up of the terminology and formulae essential for application in the rest of the dissertation. The Chapters II, III, IV contain results believed to be new.

The caption for the second chapter is 'The three space-like congruences (SC) on the world line of a particle in relativistic continuum mechanics and the three curvature fields'. We start from the famous time-like congruence representing the velocity vector field of a particle in a continuum. The three unit space-like congruences orthogonal to the velocity vector field are constructed and they are interpreted as the unit acceleration field vector, the boost field vector and the fourth vector field is orthogonal to all the three vector fields such that the generalized Serret-Frenet formulae are satisfied. In Section 2, explicit expressions for the three curvature fields are evaluated. The non-existence of the three space-like congruences in the special case when the gravitating matter is pressure free is established. In the next section a simple illustration of the existence of the three space-like congruences as well as the three curvature scalars is developed for the 'early universe' wherein matter dominates radiation. The last section pertains to a general situation; the Frenet apparatus for the Definite Material Scheme.

In an attempt to find the physical significance of the tetrad of Chapter-II, we examine 'Special space-like congruences on the stream line' in Chapter-III. We find the effect of the vanishing of each of the curvature scalars K_1, K_2, K_3 on the space-like congruences associated with the stream-line of a particle. It is argued that $K_3 = 0 (K_1 \neq 0, K_2 \neq 0)$ corresponds to the path of a classically gravitationally self interacting spin particle, while $K_2 = 0 (K_1 \neq 0)$ implies that the particle moves as a charged particle with radiation reaction but no external electromagnetic field. The case $K_1 = 0$, obviously yields a geodesic path. Section 2, confines itself to the acceleration field P^a and portrays the three kinematical parameters of P^a, after analysing the transport laws governing their definition. Explicit evaluation of the physical components of the shear and the rotation of P^a is accomplished here. The transport laws of P^a are expressed in terms of the Ricci rotation coefficients. The generalized Serret-Frenet.formulae and the physical components of the shear and the rotation look simple when expressed in terms of these coefficients.

We exploit the 'amazingly useful' technique of the spin coefficient formalism invented by Newman and Penrose (1962) for studying the special space-like congruences in the last chapter 'Spin coefficient formalism for the three space-like congruences on the world line of a particle'. Under the

condition that the null vector field m^a is shear free and expansion free, we obtain an elegant criterion for the existence of a time-like helix as the world line of a particle.

If the null congruence m^a, given above is harmonic, we get a simple characterization of time-like circle.