PREFACE

The dissertation entitled "Ray analysis of stationary shearing dust and gravitational tidal force in empty space" is devoted to the application of the 'amazingly useful' Newman and Penrose formalism for certain problems of the gravitational fields in matter free spaces and dust filled spaces. It consists of three chapters. While the first chapter is of expository character, the second and the third chapters contain some results which are believed to be new.

Chapter I captioned "Ray analysis of gravitational fields" introduces the prime mathematical tool of the thesis, namely the Newman Penrose formalism (NP formalism) invented in 1962. The Algebraic relations, Completeness relation, Special ray scalars, Weyl scalars, Ricci scalars, Commutative relations, the optical components of Ricci identities, Bianchi identities and Einstein's field equations are enumerated for utilization in the rest of the thesis. The mathematical technique used for the ray analysis is based on the following results in complex vector spaces :

$$A_1 l_a l_b + A_2 l_a m_b + \dots + A_{16} n_a n_b = 0$$

$$\Rightarrow A_1 = 0, \quad A_2 = 0, \dots, A_{16} = 0$$

where A_1, A_2, ..., A_{16} are functions of the special ray scalars and $l_a n_a$, m_a, $\bar{m}_a$ are the basis vectors.

In chapter II entitled 'Ray analysis of stationary shearing dust' we utilize an operator called the Jaumann derivative with respect to the flow vector u^a, viz. J_u which is superior to the other time derivatives (e.g. material derivative, Cotter and Rivlin, Green, Truesdell) in the relativistic continuum mechanics. The concept of stationary shear in classical continuum mechanics has been introduced by Eringen (1962) as $J_u q_{ab} = 0$, where q_{ab} is the shear tensor field. By using Synge's "Cuckoo Process" we extend this concept of stationary nature of a tensor field to the relativistic dynamics. The precise strategy is to derive the tensor version of stationary shearing dust by exploiting Carter and Quintanna's (1962) dynamical equation for the evolution of shear. Then the transcription of this version into ray formalism is facilitated by ray equivalents of the kinematical parameters for the special choice of $u_a = (2)^{-1/2}(l_a + n_a)$. As the dynamical equations are still involved, yet another restriction is imposed on the type of the gravitational field namely <u>Petrov type D</u> ($\Psi_2 \neq 0$) which corresponds to the field of the Kerr-Newman Blackhole. The assumption $Im\Psi_2 = 0$ enables us to obtain the necessary and sufficient conditions for the shear of incoherent matter with Petrov type D gravitational field to be stationary in terms of conditions on the ray scalars in the form $\alpha + \bar{\beta} = 0$, $Im(\rho+\mu)=0$.

'Ray analysis of gravitational tidal force C_{ab} in empty space' is the aim of Chapter III. The necessary and sufficient conditions for the (Lie) invariance of different types of C_{ab}

with respect to the null vector l^a are obtained in this chapter. It is shown that except Petrov type I gravitational fields remaining four types are compatible with the functional form invariance of C_{ab} with respect to Lie drag along l^a.

The following statements are proved to be equivalent

(I) C_{ab} is of Petrov type II and $\pounds_l C_{ab} = 0$

(II) $\mathrm{Re}\Psi_2$ is conserved with respect to shearfree l^a ln - surface exists relative to n, $l\bar{m}$ - surface is orthogonal to n.

The Criterion for Petrov type III C_{ab} to satisfy $\pounds_l C_{ab} = 0$ are found in terms of ray formalism. viz.,

$$D\alpha = D\beta = D\gamma = D\lambda = D\mu = D\Psi_3 = 0 ,$$
$$\rho = \sigma = \gamma + \bar{\gamma} = \tau = 0 .$$

The functional form invariance with respect to l^a of C_{ab} corresponding to the mass of the source (Petrov type D) is ensured by the ray conditions

$$\rho + \bar{\rho} = \sigma = \alpha + \bar{\beta} = \gamma + \bar{\gamma} = \lambda = \tau = 0,$$
$$D(\mathrm{Re}\Psi_2) = \delta\Psi_2 = \bar{\delta}\Psi_2 = D\nu = \delta\rho = 0 .$$

The gravitational tidal force in the transverse radiation zone exhibits the geometrical symmetry of Lie invariance with respect to the propagation vector if and only if the ray scalars ρ, β, λ, γ, α, ν, μ are conserved with respect to shearfree l^a, and the Weyl scalar Ψ_4 is conserved with respect to l^a and also w.r.t. respect to hypersurface orthogonal m^a.