CHAPTER - I

INTRODUCTION

Brief Survey Of The Development Of Boundary Layer Theory

(1) Boundary Layer concept.

(2) Two Dimensional Boundary Layer.

(3) Axially Symmetrical Boundary Layer.

(4) Three Dimensional Boundary Layer

(5) Basic concepts required for our problems to be discussed in Chapter II.

Introduction :

The chapter consists of five sections. In Section 1, we explained the boundary layer concept which was due to Ludwing Prandtl (1904) who was German Mathematician. The credit goes to this mathematician for the further developments in this branch of Mathematics. Section 2 consists of the recent major developments of the two dimensional boundary layer problems. In Section 3, the survey of recent problems for axial symmetrical flow is carried out. In Section 4, we listed some three dimensional boundary layer problems of immense use.

Last Section covers the definitions of some basic concepts which are applicable for the discussion of our problems for dissertation.

1. Boundary Layer Concept :

The concept of boundary layer is one of the corner stones of modern fluid dynamics introduced by a German engineer Ludwing Prandtl [8] is an attempt to account for the considerable inconsistancy between the prediction of the classical inviscid fluid dynamics and the results of the experimental observations. He gave mathematical formulation of boundary layer equations. The complete Navier-Stokes equations are elliptic differential equations while the boundary layer equations are parabolic. The transition is from elliptic type Navier-Stokes equations to parabolic type boundary layer equations. Also Prandtl analysed the flow in the boundary layer subsequently. The boundary layer equations have been well investigated for many engineering problems by various research workers and the results play a very important role in the practical treatment of fluid.

The boundary layer theory is the foundation of all modern developments in fluid mechanics, and in aerodynamics which have been classified by the study of boundary layer flow and its effects on the general flow around the body such as the study of air craft response to atmospheric dust, in further phenomenon involving wing etc. Although more than half a century the subject of boundary layer theory is still receiving considerable interest and three are still a number of unsolved problems affling the investigators.

At first the boundary layer theory was developed mainly for the case of laminar flow in an incompressible fluid. Later the theory was extended to include turbulent incompressible boundary layers which are more important from the point of view of practical applications. Modern investigations in the field of boundary layer research are characterized by a very close relation between theory and experiment.

The starting point of great physical concept was the well known D'Alemberts paradox. In the late 19th century D'Alembert observed that when a solid body moved through a fluid the flow pattern based on the inviscid theory agreed with the experimental results almost every where in the flow field, but strangely enough the resistance experiments by the body was found to be zero. Prandtl made an attempt to resolve the dilema and suggested that the resistance to the body was caused by the viscosity of the fluid and that the flow fields near and away from the body were different in character.

In the two dimensional cases for which the velocity components depend only on two space co-ordinates. At the same time the velocity component in the direction of the third space co-ordinate does not exist. The general three dimensional cases of the boundary layer in which the three velocity components depend on all three co-ordinates has to far, been hardly elaborated because of the enormous mathematical difficulties associated with this problem. The mathematical difficulties encountered in the study of axially symmetrical boundary layer are considerably smaller and hardly exceed those in the two dimensional case.

The boundary layer equations of motion be transformed to in special form so that the computation of numerical results will be simplified or will be easier for special devices. The boundary layer equation of motion may be integrated across the boundary layer so that the momentum integral equation is obtained. The boundary layer equations may be transformed into the generalized heat conduction equation by Von-Mise's transformation. L.L. Moore [9] transformed the boundary layer equations in to an integral forms that is particularly suitable for a differentiation.

Prandtl (1904), in his original paper entitled "On the Motion of Fluid with very small Viscosity" considered the problem for an incompressible fluid and Blasius [10] investigated the same problem in detail in 1908. Blasius [11] studied the boundary layer flow over a flat plate and obtained explicit solution of the Prandtl boundary layer equations.

2. Two Dimensional Boundary Layer :

Due to the application of theory of parabolic differential inequalities to the Prandtl's boundary layer equation, all the problems of existance, uniqueness etc. had been solved by considering the case of two dimensional steady flow of an incompressible medium from the suggestion of Görtler (1950), Nickel (1958) solved many other problems, "who was the first man to use this new method - Then by using the theory of differential inequalities lot of problems were solved. Later on this new method is known as the "Well rounded theory".

The two dimensional boundary layer flow over a flat plate of a compressible fluid studied by E. Pohlhausen [12] in (1921) for a thermally insulated plate for a flows small velocity and its small temperature difference, with constant density and viscosity. Boltz [13] submitted two papers on boundary layer under Prandtls guidance at Göltingen. H. Blasius [14] obtained series solution of the Prandtl boundary layer in the case of steady flow past a flat plate. K. Himenz [15] and L. Howarth 1 developed the work further by H. Blasius [14] .

V.M.Falkner and S.W.Skan [2] obtained the approximate solution for the boundary layer equations. D.R. Hurtree [3] had studied a particular case of Falkner and Skan nonlinear differential equation was investigated by him and later on the work was covered out by other co-workers. The problem of similar solutions of boundary layer equation was first studied by S. Goldstein [4] and later on by W. Mangler [5].

K. Pohlhausen [6] studied the boundary layer flow along a wall of convergent channel and obtained the solution of this problem. H. Gortler [7] introduced a new series method for calculation of steady laminar boundary layer flows. This series shows better convergence than the Blasius type [14] expansion and also it is applicable to the bodies with a sharp leading edge where $U(x) \neq 0$ in addition to that with a forward stagnation point flow. A.N.Valiullin and V.N. Iganter [27] studied the steady flow of an incompressible fluid past a body of revolution r(x). Non-dimensionalization was carried out and the resulting differential equation was solved numerically.

The computed values are compared with values given by Schlichting [28]. Hsu, Chen Chi [29] described a method for the approximate solution to the boundary layer equation by Galerkin technique. The method seems to be unable to approximate the known singular behaviour of the solution near the separation of boundary layer.

Bussemann [30] first studied boundary layer flow for an incompressible fluid. Bussemann [31] and Wada [32] obtained the solution for the flow on a flat plate by keeping Prandtl number (Pr) constant. Hawarth [33] studied the compressible and incompressible boundary layer at zero pressure gradient. Illingworth [34] investigated the transformation of both normal and streamwise co-ordinate and obtained the relation between them at nonzero pressure gradient for incompressible flow. Tani [16] extended the solution for the compressible flow by taking Prandtl number different from unity. Cebeci [17] studied the unsteady laminar and turbulent boundary layer with fluctuations in external velocity. Goldstein [18] constructed a singular solution containing an arbitrary constant in the neighbourhood of separation. Stewartson [19] obtained the general solution involving an infinite number of arbitrary constants. Landu and Lifeschitz [20] made a discussion on flow near separation by postulating that the normal component of velocity tends to infinity at the separation point of the boundary layer. Hartree [21] and Stewartson [22] obtained the series solution for a linearly retarded free stream. Prandtl [22] and Blasius [23] introduced the form of similarity solution, for flow on flat plate. Falkner and Skan [24] extended this form in the case of free stream velocity proportional to x^m representing irrotational flow around a corner found by the plano boundaries meeting at the angle $\pi(m + 1)$. Goldstein [25] applied the boundary layer approximations for a flow in a jet. Van Pyke [48] obtained the series solution for a flow past parabolic cylinder.

Prandtl [47] applied the boundary layer concept to the heat transfer problem. Hiemenz [15] carried out the boundary layer calculation of pressure distribution on circular cylinder. Prandtl [47] explained the change in the flow pattern around a sphere on passing through critical Reynolds number. Tofer [44] refined the numerical computations of Blasius. Eiffel [43] observed the transition of flow in the boundary layer from laminar to turbulent.

During the ten years after Prandtls paper three were seven papers on boundary layer which were published at Göttingon. All these papers were written on the basis of Prandtls original paper. Zhukovsheii assumed that the fluid velocity is zero at the wall and rapidly increases until it becomes equal to the thoeretical velocity of irrotational motion. Then he found the thickness of the layer inversely proportional to theoretical velocity. Gilmbel calculated the frictional resistance of ship.

3. Axially Symmetrical Boundary Layer :

Axially symmetrical flow of boundary layer equations are having two different kinds. One is for the flow in jet or in the wake behind a body of revolution where the axis of revolution of a large radius in comparison with the thickness of the boundary layer. The mathematical difficulties encountered in the study of symmetrical boundary layer were considerably smaller and hardly exceed those in the two dimensional case. Axial symmetrical boundary layer occurs e.g. in the flow past axially symmetrical bodies the axial symmetrical jet.

U.T.Boedewadt [42] studied the problem in which the fluid at large distance from the stationary wall rotates like a rigid body with constant angular velocity. H. Schlichting [26] obtained solution for the laminar circular jet which is analogous to the one of two dimensional jet.

The process of boundary layer formation about an axially symmetrical body accelerated impulsively was investigated by E. Botz [40] in Gottingen thesis. The process of formation of the boundary layer on a rotating disk was studied by K.H. Thiriot [39] in his thesis presented to the University of Gottingen. He considered the case of disk accelerated impulsively in a fluid at rest to a uniform angular velocity as well as the case of disk rotating with fluid and suddenly arrested in its motion. S.D. Nigam [38] computed the growth of boundary layer on a disk started impulsively.

4. Three Dimensional Boundary Layer :

The study of three dimensional boundary layer will be separated to in two main parts. The first of these will be concerned with the boundary layers which possesses a common geometrical property, namely that of rotational symmetry, an example which is the flow along a surface of revolution when the main stream is parallel to the axis of symmetry. Second part of these deals with the boundary layer on a surface of general type. It is necessary to distinguish here between the case where the singular geometrical behaviour of the surfaces. (corners and edges, for example) and that where the surface is smoothly curved.

The problems of the boundary layer of two velocity components, one velocity component being much smaller than the other. In some important practical problems the flow in which two of the velocity components are of the same order of magnitude whereas the third is much smaller than the other two. The simplest three dimensional boundary layer over a yawed infinite flat plate at zero angle of attack. The boundary layer over a finite swept wing is also of three dimensional boundary layer.

In the case of three dimensional boundary layer the external potential flow depends on two co-ordinates in the surface and the flow within the boundary layer possesses all three velocity components which moreover, depend on all three space co-ordinates in the geometrical case.

The boundary layer equations for three dimensional flow, particularly for swept wings and rotating body of revolution. Hawarth [37] showed that in three dimensional flow the curvature effects are present, they do not disappear as they do in two dimensional flow.

The equations of motion for boundary layer flow over a general three dimensional body, subject only to the restriction that geometrical singularities on the surface of the body are to be excluded such equations have been derived by various methods by several authors including Levicivita (1929), Lin (1947), Timman (1955), Howarth (1951) and Hayes (1951).

Another important practical type of three dimensional boundary layer flow is the flow over rotating blades occurring in turbines, helicepter's and propellers. In this case the centrifugal and conolis forces due to rotation combined with pressure gradients and viscous forces,cause the flow to three-dimensional even in the absence of the effects of finite span.

W. Wuest [36] obtained solutions for three dimensional non-steady boundary layers on bodies which perform non-steady motions at right angles to the main flow. W. Dienmann [35] worked out an approximate method for yawed cylinders. The extension of these approximate methods to the general case of three-dimensional flows is vitiated by a very great difficulties, because now the variety of possible velocity distributions has become much larger than was the case with the two-dimensional flow. The three-dimensional boundary layer flow can be considerably simplified if the streamlines and lines of constant potential of the external, irrotational flow are chosen for a curvilinear system of co-ordinates.

5. Basic Concepts Required Four Our Problems :

(I) Fluid : Fluids are usually classified as liquids or gases. A liquid has intermolecular forces which hold it together so that it possesses volume but no definite shape. When it is poured into a container will fill the container upto the volume of the liquid regardless of the shape of the container. Liquids have but slight compressibility. For most purposes it is, however, sufficient to regard liquids as "incompressible fluids". A gas on the other hand, consists of molecules in motion which colloid with each other tending to disperse it so that a gas has no set volume or shape. The intermolecular forces are extremely small in gases. A gas will fill any container into which it is placed and is, therefore, known as a (highly) "Compressible Fluid." It is proper to remark here that for speeds which are not comparable with that of sound etc. if the Mach number (the ratio of velocity of flow to the velocity of sound) is small compared with unity the effect of compressibility on atmospheric air can be neglected and it may be considered to be a liquid and in this sense it is called incompressible air.

2. Viscosity : Viscosity of a fluid is that characteristic of real fluids which exhibits a certain resistance to alterations of form. Viscosity is also known as internal friction. All known fluids (gases or liquids) possess the property of viscosity in varying degrees. The nature of

viscosity can be best illustrated with the aid of the following experiment of Newton. Consider the motion of fluid between two very long parallel plates at a distance say "d" apart, one of which is at rest and other moving with a constant velocity V parallel to itself. Because of viscosity, the fluid will also be in motion. Experiment teaches us that the fluid adheres to both walls (no slip condition), so that its velocity at lower plate, which is stationary is zero and at the upper plate which is moving, is equal to the velocity of the plate U.

The coefficient of viscosity of fluid may be defined as the tangential force required per unit area to maintain a unit velocity gradient, i.e. to maintain unit relative velocity gradient, i.e. to maintain unit relative velocity between two layers unit distance apart.

3. <u>Reynold Number</u> :

The dimensionless quantity Re defined

$$Re = \frac{UL\rho}{\mu} = \frac{U\,L}{\nu}$$

where U, L, e and μ are some characteristic values of the velocity, length, density and viscosity of fluid respectively is known as the Reynolds number in honour of the British Scientist Osborne Reynolds.

4. <u>Prandtl Number</u> : The ratio of kinematic viscosity to the thermal diffusivty of the fluid i.e.

$$\frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\nu}{a} = \frac{\mu/e}{k/eC_p}$$

$$= \frac{\mu C_p}{k} = Pr.$$

is designated as the Prandtl number named after the German scientist Ludwing Prandtl. It is a measure of relative importance of heat conduction and viscosity of the fluid.

5. <u>Eckert Number</u> : The dimensionless quantity "Ec" defined as

$$Ec = (U^2/C_p)T$$

where U, C_p and T are some reference values of the velocity, specific heat at constant pressure and temperature is known as the Eckert number named after the German scientist E.R.G. Eckert. In compressible fluids it determines the relative rise in temperature of the fluid due to adiabatic compression.

6. <u>Grashoff Number</u> : The dimensionless quantity Gr, which characterizes the free convection is known as the Gradshoff number and is defined as

$$Gr = \frac{gL^3\ (T_w - T_\infty)}{\nu^2 * T_\infty}$$

where g is the acceleration due to gravity and T_w and T_∞ are two representative temperatures.

7. <u>Dimensionless Coefficient of Heat Transfer (Nusselt Number)</u>:

In the dynamics of Viscous fluids one is not much interested to know all the details of the velocity and temperature fields but would certainly like to know quantity of heat exchanged between the body and the fluid. If q(x) is the quantity of heat exchanged between the wall and the fluid, per unit area per unit time, at a point x, then

$$q(x) = \alpha(x)\ (T_w - T_\infty)\ \text{(Newton's Law of Cooling).}$$

where $(T_w - T_\infty)$ is the difference between the temperature of the wall and that of the fluid. Since at the boundary the heat exchanged between the fluid and the body is only due to conduction, according to Fourier's law, we have

$$q(x) = -k \left(\frac{\partial T}{\partial n} \right)_{n=0}$$

where n is the direction of normal to the surface of the body. From these two laws we can define a dimensionless coefficient of heat transfer which is generally known as Nusselt Number as follows,

$$Nu = \frac{\alpha(x)\, L}{k} = \frac{-L}{(T_w - T_\infty)} \left(\frac{\partial T}{\partial n} \right)_{n=0}$$

where L is some characteristic length in the problem.

8. <u>Newtonian Fluid</u> : Newton observed that in a simple rectilinear motion of fluid two neighbouring fluid layers, one is moving over the other with the same relative velocity between the tow layers and inversely proportional to the distance between the layers.

If the two neighbouring fluid layers are moving with velocities u and u + δu and at a distance δy the shearing stress is given by

$$\alpha \frac{\delta u}{\delta y} \quad \text{or} \quad \tau = \mu \frac{du}{dy}$$

This is called as "the Newtonian hypothesis" and a fluid satisfying this hypothesis is called Newtonian fluid. The constant of proportionality μ is called the coefficient of viscosity and du/dy is the strian rate of the fluid.

9. <u>Suction</u> : The retarded fluid in the boundary layer is sucked in to the body. The point of suction is near the point of separation either slightly ahead or behind so that no back flow will occur.

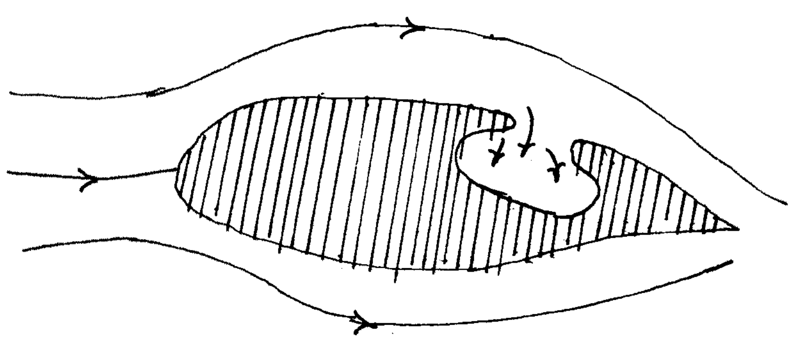

Fig. 1 suction

Fundamental Equations of Boundary Layer Suction :

It is the simplest to begin the mathematical study of laminar boundary layer with suction by first considering the case with the continuous suction which may be imagined realized with the aid of co-ordinates will be adopted, the X-axis being along the wall and the y-axis being at right angles to it. Fig. will be accounted for by prescribing a non-zero normal velocity component $V_o(x)$ at the wall, in the case of suction we shall put V_o ? 0, making V_o ? 0 for discharge. The ratio of suction velocity $V_o(x)$ to free stream velocity V_∞ is very small

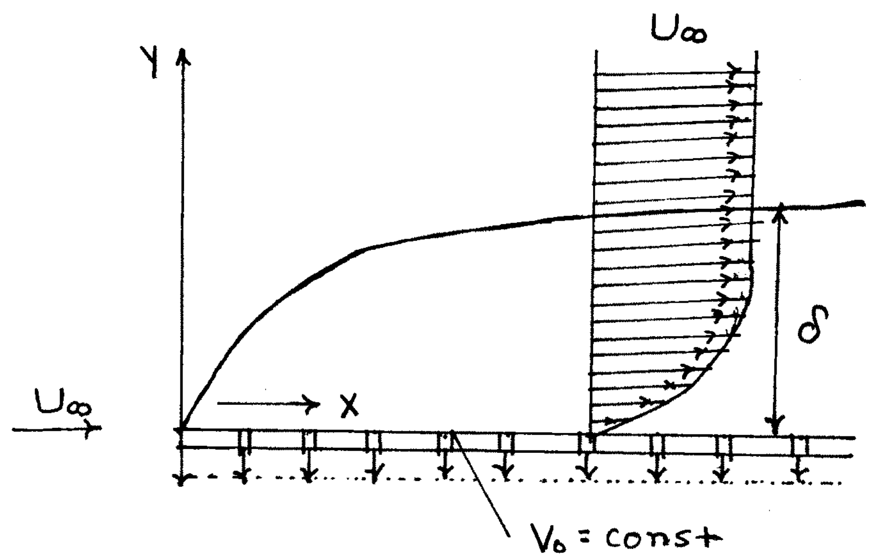

Fig.2. Flat plate with homogeneous suction at zero incidence.

The quantity of fluid removed, Q will be expressed through a dimensionless volume coefficient by putting

$$Q = C_Q A U_\infty \qquad \ldots \text{ i}$$

where A denotes the wetted area.

For flat plate, $Q = b \int_0^l [-v_o(x)] \, dx$

and

$A = b \cdot l$ so that consequently

$$C_Q = \frac{1}{l U_\infty} \int_0^l [-v_o(x)] \, dx \qquad \ldots \text{ ii}$$

and for the case of uniform suction v_o = constant

$$C_Q = \frac{-v_o}{U_\infty} \qquad \ldots \text{ iii}$$

Assuming incompressible two dimensional flow we have the following differential inequalities.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{-1}{e} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \qquad \ldots \text{ iv}$$

with the boundary conditions

$$\left.\begin{array}{ll} y = 0: & u = 0, \quad v = U_o(x) \\ y = \infty: & u = U(x) \end{array}\right) \qquad \ldots \text{ v}$$

Evidently, the integration of the above system of equations for the general case of arbitrary body shape, employing an arbitrary velocity function U(x), presents no fewer difficulties than does the case with no suction.

10. Forced convection in a laminar boundary layer on a flat plate :

Consider the steady flow of a viscous incompressible fluid past a thin semi finite flat plate at a constant temperature T_w (or insulated) placed along the direction of a uniform stream of velocity U_∞ and a temperature T_∞. Let the origin of co-ordinates be at the leading edge of the plate, the x-axis along the plate and y-axis normal to it.

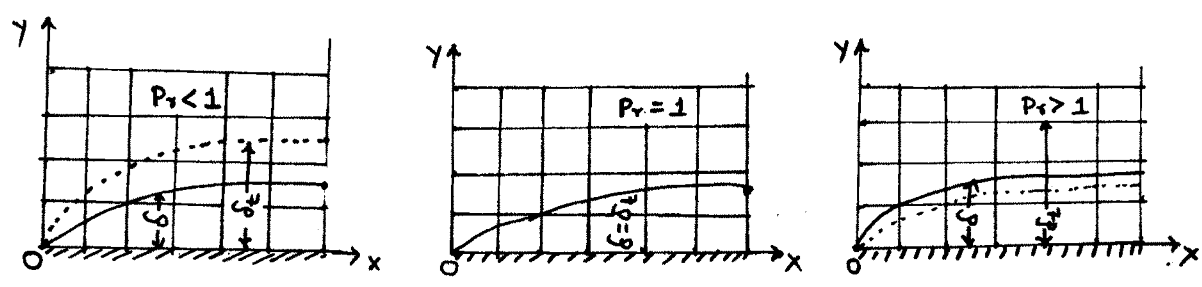

Fig.3. Comparison between velocity and thermal boundary layer on an isothermal plane wall $Pr \lesseqgtr 1$.

11. Free convection from a heated vertical plate:

A flat plate is heated to a temperature T_w is placed vertically under gravity in a large body of fluid (air) which is otherwise at rest and has a temperature T_{∞} and density ϱ_{∞}. It is assumed that the temperature difference between the plate and the fluid is small, so that fluid properties may be taken as constant but the small motion of the fluid in the neighbourhood of a vertical plate is caused by a buoyancy force due to density variations. Hence the dissipation term in the thermal boundary layer equation may be neglected in this problem, but a body force due to gravity must be added in the velocity boundary layer equation.

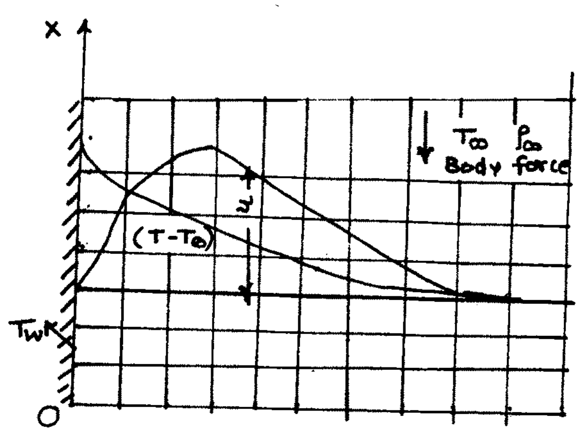

Fig.4. Temperature and velocity distributions in free convection from a heated vertical plate.

REFERENCES

1. Howarth, L : A solution of the laminar boundary layer equations for retarded flow. ARC R and M, No. 2426 (1949)

2. Falkner, V.M. and Skan, S.W. Some approximate solutions of boundary layer equations Phil. Mag. 12/865-896 (1931)

3. Hartree, D.R. : On an equation occurring in Falkner and Skan's approximate treatment of equations of boundary layer Proc. Camb. phil. Soc. 23, pt.II: 223-39 (1937)

4. Goldstein, S : A note on boundary layer equatiohs Proc. Camb. Phil. Soc. 35, (1937)

5. Mangler, W : Die 'ahnliche' Lo sungonder Prandtlschen Grenzschichtgleichangen ZAMM, 23 : 241-47 (1973)

6. Pohlhausen, K : Zur nalierungsweise Integration der differential gleichung der Grenzschicht ZAMM, 1, 252-63 (1921)

7. Gortler, H : A new series for calculation of steady laminar boundary layer flows. Journ. of Maths. & Mech. 6 : 1-66 (1957).

8. Prandtl, L : Veber Flussigkeitschewegung beischer Kleiner Reibung Proc. third Intern. Maths. Congress. Bidelberg (1904).

9. Moore, L.L.: A solution of laminar boundary layer equation for a compressible fluid with variable properties including dissociation.
Jour. Sci. 19 No.8, 1952: 505-18.

10. Blasius, H : Grenzschichten in Flussigkeiten mit Kleiner Reibung Zeit. Math. Phys. 56, 1908, P.L. Also NACA, TM No. 1256.

11. Blasius, H : (1908)
Z. Math. Phys. 66 : 1-37.

12. Pohlhausen, E : Der warneaustatusch zwischen festen Korpern and Flussigkeiten mit kleiner Reibung and kleiner.
Warmeleitung ZAMM 1, (1921), p.121.

13. Boltz, E. (1908) :
Gottingen dissertation.

14. Blasius, H : Grenzschichten in Flussigkeiten mit Kleiner Reibung
Z. Math. Phys. 56: 1-36 (1908).

15. Himenz, L : Die Grenzschient an eniem inden gleich formigen Flussigkeiten eingetauchten geraden Kreiszylinder.
Thesis Gottingen 1911, Dingl. Polytechn.J. 326, 32 (1911).

16. Tani, I : (1954) : J. Aeronant Sci. 21 : 487-95

17. Cebeci, T : Proc. Roy. Soc. London ser. A 355, no. 1081 : 225-38.

18. Goldstein, S. (1948). G.J.Mech. Appli. Math.1:43-49.

19. Stewartson, K. (1954): Proc. Cambridge Philos Soc. 50: 454-65.

20. Landu, L.D. and Lifeschitz E.M. (1959): Fluid Mech. Oxford. Pergaman Press.

21. Hartree, D.R. (1937): Proc. Cambridge Philos.Soc. 33 : 223-39.

22. Prandtl, L. (1905): Verb. Int. Math. Konger 3rd Heidebberg (1904): 484-91, Transl. 1928, NACA Memo No-452.

23. Blasius, H. (1900) : Z.Maths. Phys. 66: 1-37.

24. Falkeno, V.M. and Skan S.W. (1930) : Aeronant Res. Coun, Rep. Mem. No. 1314.

25. Goldstein, S. (1948): G.J. Mech. Appl.Maths.1:43-49.

26. Schlichting, H: (1933) : Z.Agew. Math. Mech.13: 260-63.

27. Valiullin, A.N. and Jganter V.N. A certain scheme of increased accuracy for the calculation of a two dimensional laminar boundary layers.
Cist. Metody Mech. Splosnon Seredy 1,3 70-74 (1970).

28. Schlichting, H: Boundary layer theory P-223. MacGraw Hill, New York (1968).

29. Hsu, Chen, Chi,: A special Galerkin method for a class of steady two dimensional incompressible boundary layer flows.
ARL 73-0167 (1973).

30. Busemann, A. (1931): In Hand buckelar experimental physik, ed. W. Wien. F. Harms 4(f): 341-360 Laipzing Akademigche.

31. Busemann, A. (1935) : Z.A. New Math. Mech.15: 23-25.

32. Wada, K. (1944) : Rep. Aeromant Res Inst. Tokyo, Imp Liniv No. 302.

33. Howarth, L. (1948): Proc. Cambridge Philos.Soc. 33: 223-39.

34. Illingworth, C.R. (1949): Proc. R.Soc. London Ser. A-199, 533-57.

35. Dinemann, W.: Berchnungdes wantemperatur, ZAMM 33, 89-109 (1953)

36. Wuest, W: Grenzschichten an zulindrischen korpern mit ni Chtstationarer Querbewegung ZAMM 32, 172-78 (1952)

37. Howarth, L.: The boundary layer in three-dimensional flow. I Derivation of the equations of flow along a general curved surface. Phil. Mag. 42, (7th Series), No. 326, March, 1951, pp. 239-43.

38. Nigam, S.D.; Quarterly Amer. Math. 9, 89-91 (1951)

39. Thiriot, K.H.: Uber die laminare Anlaufstromung einer Flussi gkeit uber einens rotierenden Bodenbel plotzlicher Anderungdes Preh ungszustandes ZAMM 20, 1, (1940).

40. Botz, E: Grenzschichten an Rotution korpern in Flussigkeiten mit kleiner Reibung Thesis Gottingen (1908).

41. Schlichting, H: Laminar Stralausbreitung ZAMM 13: 260 (1933).

42. Bodewadt, Die Drestromung uber festem Grande ZAMM 20: 241 (1940).

43. Eiffel, G. (1912). C.R.Acad. Sci. Paris Ser A. 155-1597-99.

44. Tofer, C. (1912): Z. Math. Phys. 60 : 397-98.

45. Prandtl, L. (1914): Nachr. Geg-Wiss. Tottingen Math. Phys. Kl. pp. 177-90.

46. Hiemenz, K. (1911) : Dingler S J. 326, 321-24, 344-88, 357-62, 372-76, 391-93, 407-10.

47. Prandtl, L. (1910): Phys. Z.11: 1072-78.

48. Van Byke, M. (1964): Pertubation method in fluid Mechanicis New York and London Academic 1916.

oOo