CHAPTER - 0

PRELIMINARIES

This section is devoted to a summary of known concepts and results which will be used in subsequent sections.

An algebra $(L, \wedge, V)$ is called a lattice if L is nonvoid set, $\wedge$ and V are binary operations on L satisfying following properties - (i) idempotency (ii) commutativity (iii) associativity and (iv) absorption identities. A lattice $(L, \wedge, V)$ is said to be distributive if $\wedge$ is distributive over V or dually. The least and the greatest elements, whenever they exist, will be denoted by 0 and 1 respectively. If both 0, 1 are in L then L is said to be bounded. A bounded lattice L is called complemented, if for every x in L there exists y in L such that $x \wedge y = 0$ and $x V y = 1$. A Boolean algebra is a system $(L, \wedge, V, ', 0, 1)$ where $(L, \wedge, V)$ is a distributive lattice, the complementation ' ' ' is unary operation, and 0, 1 are nullary operations. If for every element x in a lattice L with 0 there exists an element x^* in L such that $x \wedge x^* = 0$ and $x \wedge y = 0 \implies y \leq x^*$, L is said to be pseudocomplemented. A S-lattice is nothing but a bounded pseudocomplemented lattice L in which $a^* V a^{**} = 1$ holds for every a in L.

By a semilattice S we mean a meet semilattice $(S, \wedge)$, where S is a nonempty set with a commutative, associative, idempotent binary operation $\wedge$ on S. We define a partial order on S by $a \leq b$ iff $a \wedge b = a$. If both 0 and 1 are in S then S is called bounded. A semilattice S is distributive if $w \geq a \wedge b$ implies that there exist x and y in S such that $x \geq a$, $y \geq b$ and $x \wedge y = w$. A semilattice S is said to be very weakly distributive if $x_1 \vee x_2$ exists in S then for all x in S, $(x \wedge x_1) \vee (x \wedge x_2)$ exists and is equal to $x \wedge (x_1 \vee x_2)$. A bounded semilattice S is said to be complemented if for any x in S there exist an element y (complement of x) in S such that the join $x \vee y$ exists in S and is equal to 1 and $x \wedge y = 0$. A semilattice S with 0 is said to be pseudocomplemented if for any a in S there exists a^* in S such that $a \wedge x = 0$ and $a \wedge a^* = 0$ implies $x \leq a^*$. An element a in a pseudocomplemented semilattice S is said to be normal if $a = a^{**}$.

An ideal of a semilattice S is a nonempty subset I of S satisfying

(i) $a \in I$, $b \leq a$ $(b \in S) \implies b \in I$ and

(ii) the join of any finite number of elements of I whenever it exists, belongs to I.

The filter i.e. dual ideal of a semilattice is a nonempty subset F of S such that $x \wedge y \in S$ if and only if $x \in F$ and

$y \in F$. An ideal (filter) generated by a nonempty subset A of S is denoted by (A] ([A)). The principal ideal (filter) i.e. the ideal (filter) generated by $\{a\}$, $a \in S$ is denoted by (a] ([a)) i.e. $(a] = \{x \in S : x \leq a\}$ ($[a) = \{x : x \geq a\}$). A proper ideal i.e. $I \neq S$ in S is called prime if $x \wedge y \in I$ then $x \in I$ or $y \in I$. The proper filter F in S is said to be prime if $x \vee y$ exists and is in F imply $x \in F$ or $y \in F$. Also, a filter F of S is prime if $\emptyset \neq F_1 \cap F_2 \subseteq F \Longrightarrow F_1 \subseteq F$ or $F_2 \subseteq F$ for any two filters F_1 and F_2 in S. A proper ideal (filter) I is called maximal if I is not contained in any other proper ideal (filter) of S. The minimal element in the set of all prime ideals of S is called a minimal prime ideal. The concepts of minimal prime filter is defined in a dual fashion.

An equivalence relation θ (i.e. reflexive, symmetric and transistive binary relation) on a semilattice S is called a congruence relation of S if $a_0 \equiv b_0$ (θ) and $a_1 \equiv b_1$ (θ) implies $a_0 \wedge a_1 \equiv b_0 \wedge b_1$ (θ). For $a \in S$, we write $[a]^{\theta}$ for the congruence class containing a i.e. $[a]^{\theta} = \{x : x \equiv a(\theta)\}$. For congruence θ on S the kernel and cokernel are denoted and defined by $\ker \theta = \{x \in S : x \equiv 0\ (\theta)\}$ and $\mathrm{Coker}\ \theta = \{x \in S : x \equiv 1(\theta)\}$ respectively.

A topological space is a non-empty set A and a collection T of subsets of A closed under finite intersections and arbitary unions; a member of T is called an open set. Call a set closed in (A, T) if its complement is open. A family of nonvoid sets $B \subseteq T$ is a base for open sets if every open set is a union of members of B. A family of nonvoid sets $C \subseteq P(A)$; the power set of A, is a subbase for open sets if the finite intersection of members of C form a base for open sets. Let A be topological space and let $X \subseteq A$. Then smallest closed set $\bar{X}$ containing X is called closure of X. Let A and B be topological spaces then a map $f : A \rightarrow B$ is called continuous if for every open set U of B, $f^{-1}(U)$ is open in A. f is a homeomorphism if it is one-to-one and onto and if both f and f^{-1} are continuous. A map $f : A \rightarrow B$ is open if $f(U)$ is open in B for every open $U \subseteq A$. A subset X of a topological space A is compact if $X \subseteq \cup (U_i : U_i$ is open, $i \in I)$ implies that $X \subseteq \cup (U_i : i \in I_1)$ for some finite $I_1 \subseteq I$. A space A is a Hausdorff space (T_2-space) if for $x, y \in A$ with $x \neq y$, there exist disjoint open sets U, V such that $x \in U$, $y \in V$. A space A is totally disconnected if for $x, y \in A$ $x \neq y$, there exists a closed-open set U with $x \in U$, $y \notin U$.

We list here important results that are needed in the sequel.

Result 1 [11] : Any proper filter of a semi-lattice S with O is contained in a maximal filter.

Result 2 [4] : In any pseudocomplemented semilattice S, the following results hold :

(i) $a \leq a^{**}$ (ii) $a^{***} = a^{*}$ (iii) $a \leq b \Rightarrow a^{*} \geq b^{*}$

(iv) $(a \wedge b)^{*} = (a^{**} \wedge b^{**})^{*}$ (v) $(a \wedge b)^{**} = a^{**} \wedge b^{**}$

(vi) S has the greatest element 1 and $0^{*} = 1$.

Result 3 [9] : Let S be a semilattice with O. A proper filter M in S is maximal if and only if for any element $a \notin M$ $(a \in S)$ there exists an element $b \in M$ with $a \wedge b = 0$.

Result 4 [11]: Any maximal filter of a pseudocomplemented semilattice is prime.

Result 5 [12] : A filter F of a pseudocomplemented semilattice S is maximal if and only if F contains precisely one of x, x^{*} for every x in S.

Result 6 [12] : If M is maximal filter of S, $x^{**} \in M \Rightarrow x \in M$ where S is a pseudocomplemented semilattice.

Result 7 [1] : The set N of all normal elements of a pseudocomplemented semilattice $(S, \wedge)$ forms a Boolean algebra $(N, \wedge, \underline{\vee}, *, 0, 1)$ the join ($\underline{\vee}$) of any two elements a,b of N is $(a^* \wedge b^*)^*$ and their meet is same as that in S.

<u>Result 8 [12]</u> : Let N be a Boolean algebra of normal elements of a pseudocomplemented semilattice S. Then for any ideal I of N $a \in I_e$ if and only if there exists an element b in I such that $a \leq b$.

<u>Result 9 [12]</u> : Let N be a Boolean algebra of normal elements of a pseudocomplemented semilattice S. Then the extension I_e of an ideal I of N is the least ideal of S meeting N in I i.e. $I_e \cap N = I$.