CHAPTER - ONE

INTRODUCTION

1.1 The concept of generalised functions as a functional on a certain function space was formulated by S.L.Sobolev in 1936. He introduced generalised functions as a continuous linear functional in the space of functions. He applied generalised functions to the uniqueness problem of the solution of the Cauchy problem for a hyperbolic equation. In 1950-51 L. Schwartz gave the theory of generalised functions systematically in his book "Theorie des distributions".

The advantage of the generalised functions and distributions is that by widening the class of functions, many theorems and applications are free from tedious restrictions. The extension of integral transformations from classical functions to generalised functions is possible. Lions was the first, who extended Hankel transformation to a generalised functions in such a way that an inversion formula could be stated for it. Recently Zemanian extended various types of transforms to a certain class of generalised functions. Zemanian gave an alternative theory designed specifically for the Hankel transformation. He has defined H_{μ} as a testing function space. A generalised function on the open interval I is any continuous linear functional on any testing function space.

At present, the theory of generalised functions has numerous applications in Physics, Mathematics and Engineering.

L. Schwariz first used the spaces "S" as fundamental spaces in his "Theorye des distributions". The authors proposed this kind of spaces with conditions of a strong decrease in fundamental functions and their derivatives at infinity. Later, L. Schwartz, A.IA. Lepin, A.D.Myshkis, G.E.Shilov constructed the theory of countably normed spaces. Spaces of the type 'S', later introduced and studied by G.E.Shilov, are a broader and more natural class of spaces. The authors used them to construct an operator method in the problem of uniqueness of the solution of Cauchy's problem. The idea of considering the still broader class of generalised spaces of type 'S' is due to I.M.Gel'fand. The construction and utilization of these spaces is connected with the results of the theory of quasi-analytic functions and Phragmen-Lindelof theorem.

1.2 DEFINITIONS

Definition 1.2.1 -

If a function g(x) satisfies the inequality $g(x) < C. \exp(-a|x|^{p})$, a > 0, then it is known that $g(x)$ decreased exponentially with an order $\geqslant p$ and a type $\geqslant a$.

Definition 1.2.2 -

The space $K\{M_p\}$ consists of all infinitely differentiable functions $\phi(x) = \phi(x_1 \dots x_n)$ for which the products $M_p(x)\ D^q\ \phi(x)$, $(\ q \leqslant p\ )$ are everywhere continuous and bounded in the whole space , Where $M_p(x)$ is a sequence of functions satisfying the inequalities. $1 < M_o(x) < M_1(x) < \dots$.

Definition 1.2.3 -

A sequence of elements $\phi_n (n = 1, 2 \dots)$ of a normed space is said to be fundamental, if for any $\epsilon > 0$, there is an integer $n_o = n_o(\epsilon)$ such that $\gamma, \mu > n_o$ implies $\|\phi_\nu - \phi_\mu\| < \epsilon$.

Definition 1.2.4 -

A space in which every fundamental sequence converges to some element of the space is called complete.

Definition 1.2.5 -

Two norms $\|\phi\|_1$ and $\|\phi\|_2$ defined in a linear space Φ are said to be compatible if every sequence $\phi_\nu \in \Phi$, $\nu = 1, 2 \dots$. which is fundamental with respect to both norms and converges to the zero element with respect to one of the norms also converges to zero element with respect to the other norm.

Definition 1.2.6 -

A linear space ϕ in which a topology is defined by a countable family of compatible norms is called a countably normed space.

Definition 1.2.7 -

A complete countably normed space in which all the bounded sets are compact is called a perfect space.

Definition 1.2.8 -

A sequence $\phi_{\nu}(x)$ converges correctly to zero if for any q, the functions $\phi_{\nu}^{q}(x)$ converges uniformly to zero in any segments $|x| \leq x_0 \leq \infty$.

Note - A sequence $\phi_{\nu} \in H_{\mu,\lambda,\alpha}$ converges to zero if and only if the sequence $\phi_{\nu}(x)$ converges correctly to zero and norms $\|\phi_{\nu}\|_p$ are bounded for any p.

Definition 1.2.9 -

A conventional function is said to be infinitely smooth if all its derivatives of all orders are continuous at all points of its domain.

Definition 1.2.10 -

The support of a continuous function $f(t)$ defined on some open set Ω is the closure with respect to Ω of the set of points t where $f(t) \neq 0$. Notation - suppf.

Definition 1.2. 11 -

The space D is a space of all complex valued smooth functions whose support is a compact set.

Definition 1.2.12 -

Let V be a countably multinormed space. A rule that assigns a unique complex number to each $\phi \in V$ is called a functional on V.

Definition 1.2.13 -

A functional is called linear if for any ϕ and $\psi \in V$ and for any complex numbers α, β we have

$$< f, \alpha \phi + \beta \psi > = \alpha < f, \phi > + \beta < f, \psi >$$

definition 1.2.14 -

A functional f is called continuous at the point $\phi \in V$ if for each $\epsilon \geq 0$, there is a neighbourhood Ω of ϕ in V such that $|< f, \psi > - < f, \phi >| < \epsilon$ whenever $\psi \in \Omega$.

Defihition 1.2.15 -

A space is said to be non-trivial if it contains at least one function which is not identically zero.

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