INTRODUCTION

Fuzzy systems embraces the whole field of imprecisely described systems. Since 1965, the year in which L.A. Zadeh introduced the concept, fuzzy systems are being studied from various angles.

The concept of fuzzy subgroups was introduced by A. Rosenfeld in 1971. However, not much attention has been paid to study fuzzy structures on rings.

In this work we have introduced the concept of a fuzzy ideal which generalizes the concept of an ideal of a ring. The word ' fuzzy ideal' was also used by A. Rosenfeld. But it stands for different concept.

This dissertation consist of two Chapters. In Chapert-I, the elementary results from fuzzy set theory and fuzzy subgroups are stated which are used in the subsequent part of the thesis. We have supplied the proofs of some important results. At times these proofs are different from those which are available in the literature.

The second Chapter constitute the main body of the thesis. In the section 2.1 we have introduced the concept of fuzzy ideal of a ring and proved many interesting properties. Notably, we establish that if $f:R \longrightarrow R'$ is an epimorphism of rings then there is one-to-one correspondance between fuzzy ideals of R' and fuzzy ideals of R which are constant on kernel of f .

We have defined a fuzzy coset of a fuzzy ideal of a ring R and proved that the set of all fuzzy cosets of a fuzzy ideal is a ring. There is natural epimorphism from the ring R onto this ring.

The section 2.2 discusses prime fuzzy ideals and their properties.

The section 2.3 deals with fuzzy radical of a fuzzy ideal and primary fuzzy ideals.

The characteristic function of an ideal is a fuzzy ideal and it has only two values. In the last section we have tried to answer the question ' Under what condition a fuzzy ideal is finite value ? ' In other words, how much fuzzy the fuzzy ideals are ?

We have proved that any prime fuzzy ideal P of a ring R is two valued if $P(x) = P(0)$ for some $0 \neq x \in R$; where the ring R is one of the following ;
(i)   Principal ideal domain
(ii)   Boolean ring
(iii)   Artinian ring

It is proved that if R is both Noetherian and Artinian ring then every fuzzy ideal of R is finite valued.

We have defined the concept of fuzzy ideal generated by a fuzzy set. It is proved that fuzzy ideal of a ring R is finite valued if and only if it is generated by a finite valued fuzzy set.

Lastly, it is observed that most of the results are true for any lattice with 0 and 1 and for any ring R. But for simplicity we have proved these results for commutative ring with unity and the closed interval $[0, 1]$ of the real line as a lattice.