CHAPTER - I

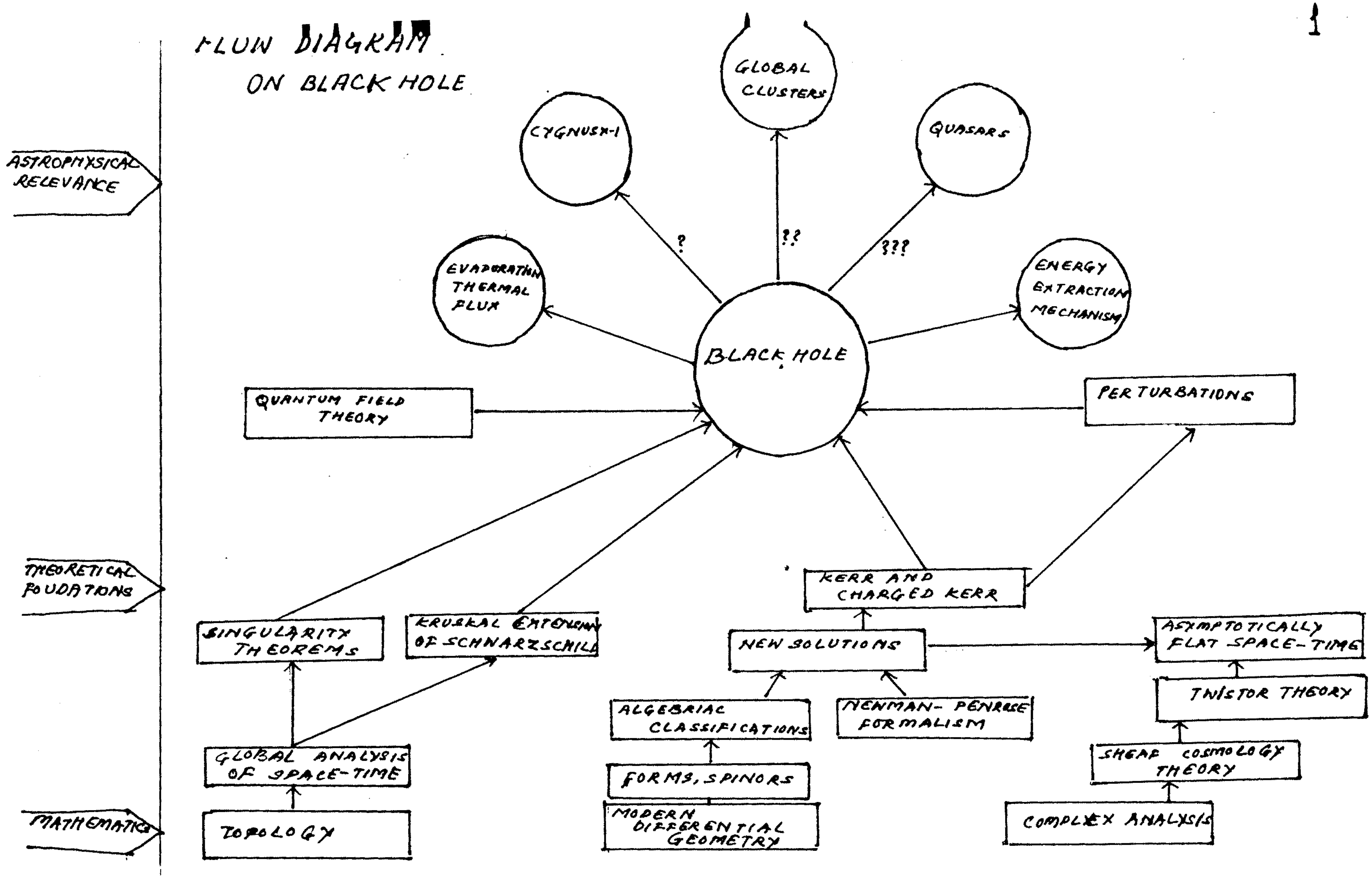
BLACK HOLES AND NEWMAN-PENROSE FORMALISM

"The mathematical theory of black
holes is a subject of immense
complexity. But its study has
convinced me of the basic truth
of the ancient mottoes:

'The simple is the seal of the true
and
Beauty is the splendour of truth."

... Prof. S.Chandrasekhar
Stockholm (Nobel Prize)address 1983.

FLOW DIAGRAM ON BLACK HOLE



CHAPTER-1

1. Historical aspects of Black Hole

What is a black hole ?

Black Holes are at present enjoying a certain prevailing fashion in astrophysics. In standard textbooks black holes are described as exotic. The theoretical astronomers have come up with interesting concepts of the black hole with the help of careful observation. The concept of black hole is not a difficult one, although it does lead into various conceptual differences. According to Prof.S.Chandrasekhar "they are very simple objects." Both their mathematical and physical ideas are very simple. The basic definition of a black hole is that it is a region of space towards which the gravitational attraction is so great that not even light can escape. This idea was first thought of by pierre Laplace in 1796, when he stated that "there exists, in the immensity of space, opaque bodies as considerable in magnitude, and perhaps equally as numerous as the stars." This prediction is remarkably close to what we believe today. We all know that if a rocket is boosted from the Earth it has to have a certain minimum speed before it can escape the gravity of the Earth (Escape velocity of the Earth: 7 miles/sec). Similarly if one projects a rocket from a planet where the surface gravity is much larger than we have to boost it with a much larger speed e.g. on Jupiter it will be 840 miles/sec. Consequently it is not difficult to consider an object with a force of gravity so large that even light cannot escape from it, since the velocity of light is the limiting velocity of any body, it follows that black hole is the strongest object in the universe. The general theory of relativity predicts that light rays will be deflected by gravity in exactly the same way as the particles. This was verified in the eclipse expedition of 1919; The light rays grazing the sun are deflected by 1.75 seconds of arc which is very small. On the other hand, if for the same mass like the sun, it had been compressed to a radius 10 times smaller, then the deflection would have been 10 times greater. Hence in order that light should be deflected by say about 90^{o}, so that instead of being deflected and going into infinity it should go round and round the sun. It has been calculated that if the sun from its radius of 700,000 km were compressed to a radius of 3.75 km then light would go round that object in the same way that planets circulate round the sun. And if the radius be contracted still further to 2.5 km then no light can escape from it. That is the reason for the name "Black Hole". The mathematical theory of black holes is based upon finding solutions to Einstein's vacuum equation which satisfy the two conditions of having an event horizon and a space time which is asymptotically flat.

Evolution of neutron stars and black holes

Neutron stars are the endpoints in stellar evolution in which a star that has a mass more than the Chandrasekhar limit, that is, 1.44 solar masses, but less than 3 solar masses, will contract until the neutrons in the star resist being pushed together any further. It is supported by compressed "degenerate" neutrons. But now the question is would objects like the sun and stars similar to the sun, evolve in the natural course of events so that a state like black holes could be formed ? Suppose take a star of given mass, at a state in which it is large and suppose that it obeys perfect gas law. Then if it loses its source of energy, it contacts. Now one question arises; Is there some way in which this process of contraction could be arrested? If the star has mass less than this limiting mass, then clearly it would settle down to a finite state of equilibrium. The next question then arises: What happens if the stars had a larger mass? Naturally when it contracts, its mean density becomes, say 100 million or 50 million, then at that stage the degeneracy pressure cannot arrest the contraction and hence the contraction will proceed still further. Clearly one could say that a limit must be reached when all the nuclei are placed closed together. Then the density of such matter will be more like 10^{13}, 10^{14} or 10^{15} grams per cubic centimeter. Thus a star of mass greater than the limiting mass could find a stable equilibrium at a state in which all the nuclei are compressed together. This is similar to what has been discovered as pulsars. Pulsars are the magnetised neutron stars with a mass of about one solar mass and a radius of 10 km. These are spinning rapidly with about 1-30 revolutions per second and give off pulses of radiation in radio and other bands with this period. Their observation some 16 years ago confirmed the earlier theoretical predictions of the existence of neutron stars. It is suggested that electrons moving rapidly in their magnetic field produce narrow beams of radiation which sweep around the sky as the pulsar spins. As of early 1980s, about 300 pulsars are known, all located within the miky way Galaxy. The primary constituents of such compressed nuclei will be neutrons. Therefore in the first instance, it would appear that stars of large mass would end as neutron stars. But on the other hand in general relativity we cannot have stable equilibrium when mass exceeds limiting values, that is we cannot have a neutron star with a mass greater than a certain limit (This limit is established to about two or three solar masses). Now if a star is more massive, let us say five solar masses and goes on contracting, under those conditions it will become a black hole. Thus the possibilities are if a star is of sufficient mass and it does not eject a sufficient amount of matter during the process of collapse, it will end as a black hole.

On the detection of black holes

There are some observational signals of black holes predicted by astronomers from 1965. Black Holes do not emit any light of their own but one could hope to detect them from their gravitational effect on nearby stars and matter. In 1972 the American Satellite UHURU detected a strong and rapidly fluctuating source of x-rays called Cygnus X-1. This turned out to consist of a large normal star in orbit around a small massive object that could not be seen. Material from the outer layers of the large star seemed to be blown off and to fall onto the compact object. As it approached the compact object it developed a spiral motion like water running out of a bath and it got very hot, emitting the observed x rays (see the figure; page 28). So one assumes that if you have a massive compact object that emits x-rays, then it is the secondary effect of the black hole that one is observing. The black hole simply represents the possibility of matter acquiring velocities close to that of light. And as a specific signal of black hole one expects intensity variations which are of the order of milliseconds. This variation represents the size of the black hole.

Artists impression of a double star system containing a black hole by Prof. J.V.Narlikar shows that the matter falling into the black hole forms a disc around it, called the 'accertion disc'. Many believe such a system exists at the location of the X-ray source cygnus X-1 which is suspected as a black hole.

Characteristics features of Black Hole

Last few years ago it was thought that the black holes were complete dead ends, that matter and energy which fell into a black hole were lost forever and they could not be recovered in any form. However it was found that when quantum mechanical effects were taken into account, so called black holes were not completely black after all : radiation would leak out of them at a steady state. This is due to Heisenberg uncertainty principle which states that one cannot simultaneously measure accurately both the position and the velocity of a particle. It also suggests that the energy of a system cannot be accurately defined over a short interval of time. The rate at which a black hole leaks radiation depends on its size. For a black hole of the mass of the sun the rate is so low that the effect would be quite undetectable. But it is possible that there may also be very much smaller black holes which might have been formed in the very hot and dense conditions that are thought to have existed in the early stages of the universe soon after the Big Bang, the beginning of the universe such primordial black holes might have masses around 100 million tons (the mass of a mountain) and sizes of about one ten million millionth of an inch (the size of the elementary particle). They would be emitting radiation in the form of gamma rays and high energy particles at the rate of about 6000 megawatts. As they radiate energy, their mass would decrease and the rate of emission go up. In course of time they would disappear completely in a huge explosion which will be equivalent to many million of H bombs. The scarcity of observational evidence has not prevented theoreticians from coming up with beautiful results on how black holes should behave as physical objects. Whether they will be successful will depend on whether the conditions in the early universe were such as to produce significant number of primordial black holes. But the quantum mechanical emission from black holes will be very important conceptually as it completely changes our notions of black holes. The commentary on black holes delivered by Prof. S.Chandrasekhar in Dec.1983 on the eve of the Nobel Prize presentation is quite valuable: "Black holes are macroscopic objects with masses varying from a few solar masses to millions of solar masses. To the extent they may be considered as stationary and isolated, to that extent, they are all, every single one of them, described exactly by the Kerr solution. This is the only instance we have of an exact description of a macroscopic object. Macroscopic objects, as we see them all around us, are governed by a variety of forces, derived from a variety of approximations to a variety of physical theories. In contrast, the only elements in the construction of black holes are our basic concepts of space and time. They are thus, almost by definition the most perfect macroscopic objects there are in the universe. And since the general theory of relativity provides a single unique two-parameter family of solutions for their descriptions, they are the simplest objects as well".

2. TYPES OF BLACK HOLES :

There are four different types of Black Holes. All these types of black holes are named after scientists who obtained their mathematical description.

(i) Schwarzschild Black Hole :

Schwarzschild discovered the first rigerous solution in spherically symmetric space time to the Einstein field equations for empty space. This famous but simple solution is described how space time is warped by the gravitational field of massive collapsed star or black hole. If the black hole has mass and no electric charge and angular momentum it is the simplest type of black hole and is called the Schwarzschild black hole.

(ii) Reissner-Nordström Black Hole :

A black hole with mass and electric charge but no angular momentum is called the Reissner-Nordström Black Hole. It is spherically symmetric and it is also believed to represent the ultimate state of an irrotational collapsing massive body with electric charges and current.

(iii) Kerr Black Hole :

A black hole with mass and angular momentum but no charge is called the Kerr black hole. The Kerr black hole is exactly symmetric but not spherically symmetric.

(iv) Kerr-Newman Black Hole (KNBH) :

The most general type of black hole within the framework of Maxwells electromagnetic theory and Einsteins General theory of relativity is the socalled Kerr Newman black hole. The

Kerr-Newman black hole is an exact solution of the Einstein's field equations possessing mass angular momentum and charge. The dynamical and geometrical features of KNBH are delineated, exploiting the null formalism in Section 4.

" Black Hole Has No Hair "

"A black hole has no hair !" This celebrated statement by J.A.Wheeler implies that when a body undergoes gravitational collapse to form a black hole, very few items of information survive to tell outside observes what the physical characteristic of the black hole are (Anything or any one falling into a black hole could never get out again or even signal for help). The basis for this remark is a theorem known as "No-hair theorem". When matter falls down a black hole left by a dead star, the matter loses all identity. The only quantities left after material falls down a black hole are mass charge and angular momentum the object carried with it. Such things as the chemical composition of the material, the colour of the object and its size are all lost. This is due to the fact that space is asymptotically flat far from the singularity. But if the entire universe itself falls down into its own black hole, then even the fundamental quantities are lost. This is because there is no flat space on to which we could connect our solutions to the field equations.

3. NEWMAN-PENROSE FORMALISM (NP-FORMALISM) :

Introduction :

This is the most efficient artifact in finding the exact solutions of Einsteins field equations in different fields. A beautiful review on the exact solutions is available in the book of Kramer, Stephane, Hearlt and MacCollum (1980). Conspicuous usage of this technique can be found in the study of Black Holes (Hawking and Ellis,1973); Chandrasekhar (1978,1979,1980) and in the study of asymptotically flat spaces. All these applications justify that this is an "AMAZINGLY USEFUL TOOL". (Flaherty 1974)

Advantages :

NP-formalism is exquisitely useful because,

(i) it is suitable for computational work,

(ii) it is adaptable to other formalisms,

(iii) it utilizes all the 24 Bianchi identities,

(iv) it makes Einstein field equations transparent.

In the 4-dimensional space-time continnum of the general theory of relativity different types of tetrad formalisms have been employed. The most prominent among the formalism is proposed by Newman and Penrose in 1962, which uses four 'invisible vectors' in the sense that their magnitudes vanish. It has several exquisite advantages over the standard tensorial presentation of Einsteins field equations of gravitation in the presence of matter.

At first we observe that Einstein had utilized only four (contracted) Bianchè identities, viz. $(R^{ab} - \frac{1}{2}Rg^{ab})_{;b} = 0$. Leading to the energy-balance equations in Continnum Mechanics:

$$T^{ab}_{\ ;b} = 0 \ .$$

Unfortunately these four equations do not indicate the interaction of free gravitational field Weyl tensor (C_{abcd}) and the matter field, T_{ab}. One has to look out for the twenty four Bianchi identities in a 4-dimensional Riemannian-space. The credit of utilizing all the twenty four identities for studying the interaction of C_{abcd} and T_{ab} lies in the Newman and Penrose formalism (NP-formalism in short).

Next advantage in this spin coefficient formalism is its easy adaptability to other formalism. From null tetrad formalism we can switch over very easily to a tetrad comprising of one-time like and three space like vectors. Eisenhart (1964), Lichnerewicz (1955), Shaha (1974) have used such formalism consisting of one null vector field and three space like vector fields, while studying the Serret-Frenet formulae of a curve representing the history of a massless particle. Computational ease is an asset for the NP-formalism. The covariant derivative of a vector field is again in terms of the (outer product of) four null vectors. In particular the derivatives of null vector fields is again in terms of null vector. As a matter of fact even the computer time is economized if the NP method is adopted. Campbell and Wainright (1977) have claimed that the null formalism affords a saving of 60 % of the computer time as compared to the classical methods of Einstein.

The efficiency of the Newman Penrose formalism lies in making the tensor equations transparent. This means that the necessary and sufficient conditions for the validity of a TENSOR equation will be expressed in terms of independent SCALAR equations. This is the reason for calling the Newman Penrose formalism as an amazingly useful formalism.

EXPOSITION :

The four null vector fields :

Newman and Penrose (1962) invented a set of four null vectors

$$X_i^a = \left\{ \underline{l}^a, m^a, \bar{m}^a, n^a \right\}$$

where $\underline{l}^a$, n^a are two real vectors and m^a, $\bar{m}^a$ (an over head bar denotes complex conjugation) are complex vectors which satisfy the following conditions :

$$\underline{l}^a \underline{l}_a = m^a m_a = n^a n_a = \bar{m}^a \bar{m}_a = 0 \qquad \ldots (3.1)$$

(Null relations)

$$\underline{l}^a m_a = \underline{l}^a \bar{m}_a = n^a m_a = n^a \bar{m}_a = 0 \qquad \ldots (3.2)$$

(orthogonal relations)

$$\underline{l}^a n_a = - m^a \bar{m}_a = 1 \qquad \ldots (3.3)$$

(Normal relations)

The relation between this tetrad and the geometry of space-time is

$$g_{ab} = \underline{l}_a n_b + n_a \underline{l}_b - m_a \bar{m}_b - \bar{m}_a m_b \qquad \ldots (3.4)$$

(completeness relation).

Differential relations :

The simplification of differential relations become possible because of the fact that the covariant derivative of a null vector field is expressible as an algebraic (linear) combination of the four null vector fields.

To illustrate this we note that,

$$\underline{l}_{a;b} = (\gamma+\bar{\gamma})\underline{l}_a\underline{l}_b - (\alpha+\bar{\beta})\underline{l}_a m_b - (\bar{\alpha}+\beta)\underline{l}_a\bar{m}_b + (\epsilon+\bar{\epsilon})\underline{l}_a n_b - - \bar{\tau}\, m_a\underline{l}_b + \sigma m_a m_b + \bar{\rho}\, m_a\bar{m}_b - \bar{k}\, m_a n_b - \tau\, \bar{m}_a\underline{l}_b + + \rho\, \bar{m}_a m_b + \sigma \bar{m}_a\bar{m}_b - k\, \bar{m}_a n_b \,. \qquad \ldots (3.5)$$

The Twelve Spin Coefficients :

This formalism combines 24 Ricci rotation coefficients into 12 complex spin coefficients and ten components of Weyl tensor into five complex spinor components. The Ricci rotation coefficients are defined by

$$\gamma_{ijk} = Z_{ib;a}\, Z^b_j Z^a_k \,, \qquad \ldots (3.6)$$

which are antisymmetric in first two indices. In NP formalism these are known as spin coefficients which are defined as follows: (The following 12 "Greek" letters have been famous in all the works on gravitational radiation)

$$K = l_{a;b} m^a \underline{l}^b \,, \qquad \ldots (SC\ 1)$$

$$\rho = l_{a;b} m^a \bar{m}^b \,, \qquad \ldots (SC\ 2)$$

$$\sigma = \underline{l}_{a;b} m^a m^b \;, \qquad \ldots(\text{SC } 3)$$

$$\tau = \underline{l}_{a;b} m^a n^b \;, \qquad \ldots(\text{SC } 4)$$

$$\nu = -n_{a;b} \bar{m}^a n^b \;, \qquad \ldots(\text{SC } 5)$$

$$\mu = -n_{a;b} \bar{m}^a m^b \;, \qquad \ldots(\text{SC } 6)$$

$$\lambda = -n_{a;b} \bar{m}^a \bar{m}^b \;, \qquad \ldots(\text{SC } 7)$$

$$\pi = -n_{a;b} \bar{m}^a \underline{l}^b, \qquad \ldots(\text{SC } 8)$$

$$\alpha = \frac{1}{2} (\underline{l}_{a;b} n^a \bar{m}^b - m_{a;b} \bar{m}^a \bar{m}^b), \qquad \ldots(\text{SC } 9)$$

$$\beta = \frac{1}{2} (\underline{l}_{a;b} n^a m^b - m_{a;b} \bar{m}^a m^b), \qquad \ldots(\text{SC } 10)$$

$$\gamma = \frac{1}{2} (\underline{l}_{a;b} n^a n^b - m_{a;b} \bar{m}^a n^b), \qquad \ldots(\text{SC } 11)$$

$$C = \frac{1}{2} (\underline{l}_{a;b} n^a \underline{l}^b - m_{a;b} \bar{m}^a \underline{l}^b) \;. \qquad \ldots(\text{SC } 12)$$

<u>Ricci Scalars</u> :

The enumeration of the eleven scalars which are just the tetrad components of the Ricci tensor R_{ab} and R (Ricci scalar) is given by :

$$\phi_{00} = -\frac{1}{2} R_{ab} \underline{l}^a \underline{l}^b \;, \qquad \ldots(\text{R1})$$

$$\phi_{01} = -\frac{1}{2} R_{ab} \underline{l}^a m^b \;, \qquad \ldots(\text{R2})$$

$$\phi_{02} = -\frac{1}{2} R_{ab} m^a m^b \;, \qquad \ldots(\text{R3})$$

$$\phi_{10} = -\frac{1}{2} R_{ab} \underline{l}^a \bar{m}^b \;, \qquad \ldots(\text{R4})$$

$$\phi_{11} = -\frac{1}{4} R_{ab} (\underline{l}^a n^b + m^a \bar{m}^b), \qquad \ldots(\text{R5})$$

$$\phi_{12} = -\frac{1}{2} R_{ab} n^a m^b \;, \qquad \ldots(\text{R6})$$

$$\phi_{20} = -\frac{1}{2} R_{ab}\, m^a \bar{m}^b , \qquad \text{...(R7)}$$

$$\phi_{21} = -\frac{1}{2} R_{ab} n^a \bar{m}^b, \qquad \text{...(R8)}$$

$$\phi_{22} = -\frac{1}{2} R_{ab} n^a n^b, \qquad \text{...(R9)}$$

The scalar curvature R is identified by

$$\Lambda = \frac{1}{24} R \qquad \text{...(C1)}$$

<u>The Five complex Weyl scalars</u> :

The free gravitational part of the curvature tensor R_{abcd} (which is locally not defined by matter tensor T_{ab}) is the Weyl tensor C_{abcd}. The Weyl tensor C_{abcd} in NP formalism is expressed by (Campbell and Weinright, 1977)

$$\begin{aligned} C_{abcd} = R_e \, [& -2\psi_0 U_{ab}U_{cd} + 4\psi_1 (U_{ab}M_{cd} + M_{ab}U_{cd}) - \\ & - 2\psi_2 (U_{ab}V_{cd} + 4 M_{ab} M_{cd} + V_{ab} U_{cd}) + \\ & + 4\psi_3 (V_{ab}M_{cd} + M_{ab}V_{cd}) - 2\psi_4 V_{ab}V_{cd}] , \end{aligned} \qquad \text{... (3.7)}$$

where,

$$U_{ab} = 2\, \bar{m}_{[a} n_{b]}$$

$$V_{ab} = 2\, l_{[a} m_{b]}$$

and

$$M_{ab} = l_{[a} n_{b]} - m_{[a} \bar{m}_{b]} .$$

Then the tetrad components of C_{abcd} are as follows :

$$\Psi_0 = - C_{abcd}\, l^a m^b l^c m^d \,, \qquad \ldots \text{(W1)}$$

$$\Psi_1 = - C_{abcd}\, l^a n^b l^c m^d \,, \qquad \ldots \text{(W2)}$$

$$\Psi_2 = - C_{abcd}\, \bar{m}^a n^b l^c m^d \,, \qquad \ldots \text{(W3)}$$

$$\Psi_3 = - C_{abcd}\, \bar{m}^a n^b l^c n^d \,, \qquad \ldots \text{(W4)}$$

$$\Psi_4 = - C_{abcd}\, \bar{m}^a n^b \bar{m}^c n^d \,, \qquad \ldots \text{(W5)}$$

Types of C_{abcd} :

Petrov type	Propogation vector	Form of C_{abcd}
I	n_a	$C_{abcd} = - \Psi_0\, U_{ab}U_{cd} - \bar{\Psi}_0\, \bar{U}_{ab}\bar{U}_{cd}$.
II	n_a	$C_{abcd} = 2\, \Psi_1 (U_{ab}M_{cd} + M_{ab}V_{cd}) + \text{C.C.}$
III	l_a	$C_{abcd} = 2\, \Psi_3 (U_{ab}M_{cd} + M_{ab}V_{cd}) + \text{C.C.}$
D	n_a and l_a	$C_{abcd} = - \Psi_2 (U_{ab}V_{cd} + 4\, M_{ab}M_{cd} + V_{ab}U_{cd}) + \text{C.C.}$
N	l_a	$C_{abcd} = - \Psi_4\, V_{ab}\, V_{cd} + \text{C.C.}$

...(3.8)

It is clear that the Weyl tensor (generally) is specified by the fi~~v~~e complex scalars Ψ_0 , Ψ_1, ... Ψ_4 . Hence it is convenient to have a general formula which expresses the different components of the Weyl tensor in terms of the five scalars, viz.,

$$
\begin{aligned}
C_{abcd} = & -(\Psi_2 + \bar{\Psi}_2)\Big[\{l_a n_b l_c n_d\} + \{m_a \bar{m}_b m_c \bar{m}_d\} + \\
& + (\Psi_2 - \bar{\Psi}_2)\{l_a n_b m_c \bar{m}_d\}\Big] + \Big[\Big[-\Psi_0 \{n_a \bar{m}_b n_c \bar{m}_d\} - \Psi_4 \{l_a m_b l_c m_d\} \\
& + \Psi_2 \{l_a m_b n_c \bar{m}_d\} - \Psi_1 \big[\{l_a n_b n_c \bar{m}_d\} + \{n_a \bar{m}_b \bar{m}_c m_d\}\big] + \\
& + \Psi_3 [\{l_a n_b l_c m_d\} - \{l_a m_b m_c \bar{m}_d\}] + \text{complex conjugate}\Big]\Big] .
\end{aligned}
\qquad \ldots (3.9)
$$

<u>Electromagnetic field as source of gravitational field</u> :

In NP version the electromagnetic bivector (Debney and Zund, 1971) is

$$
F_{ab} = -2 \operatorname{Re} \phi_1 l_{[a} n_{b]} + 2 i \operatorname{I_m} \phi_1 m_{[a} \bar{m}_{b]} + \phi_2 l_{[a} m_{b]} + \bar{\phi}_2 l_{[a} \bar{m}_{b]} -
$$

$$
- \phi_0 n_{[a} m_{b]} - \phi_0 n_{[a} \bar{m}_{b]} . \qquad \ldots (3.10)
$$

where Re(...) and Im(...) denote the real and imaginary parts of (...) respectively.

The tetrad components of the electromagnetic field F_{ab} are given by the three complex scalars (NP formalism).

$$\phi_0 = F_{ab} l^a m^b \qquad \ldots (3.11a)$$

$$\phi_1 = \frac{1}{2} F_{ab} (l^a n^b + \bar{m}^a m^b) \qquad \ldots (3.11b)$$

$$\phi_2 = F_{ab} \bar{m}^a n^b . \qquad \ldots (3.11c)$$

4. KERR NEWMAN BLACK HOLE : (KNBH)

The most general type of Black Hole within the framework of Maxwells electromagnetic theory and Einsteins General theory of relativity is the so called Kerr Newman Black Hole. The Kerr Newman black hole is an Exact solution of the Einsteins field equations possessing mass, angular momentum and charge. The metric describing this solution is (in "Boyer-Lindquist coordinates") :

$$ds^2 = -(1 - 2\,mrH^{-1})dt^2 - (4\,mar\,\sin^2\theta\,H^{-1})\,dt\,d\phi + \Delta^{-1}H^{-1}dr^2 + $$
$$+ H^{-1}d\theta^2 + (r^2 + a^2 + 2m^2 ar\,\sin^2\theta\,H^{-1})\sin^2\theta\,d\phi^2 \quad \ldots (4.1)$$

where,

$$a^2 + e^2 = m^2$$
$$\Delta \equiv r^2 - 2\,mr + a^2 + e^2$$
$$H \equiv (r^2 + a^2\cos^2\theta)^{-1}\,.$$

where m, e, a are respectively mass, charge and angular momentum. The metric coefficients are independent of t and ϕ, So $\xi_{(t)} = \partial/\partial t$ and $\xi_{(\phi)} = \partial/\partial\phi$ are killing vectors. Among the properties of this solution which follows from the metric are the orbital equations for the test particles :

$$\dot{r} = \pm\,(V_r)^{1/2}\,H\,. \quad \ldots (4.2)$$
$$\dot{\theta} = \pm\,(V_\theta)^{1/2}\,H\,. \quad \ldots (4.3)$$
$$\dot{\phi} = -(aE - L_z/\sin^2\theta) + a/\Delta P)\,H \quad \ldots (4.4)$$

$$\dot{t} = \left[- a(aE \sin^2\theta - L_z) + r^2 + a^2 p \Delta^{-1}\right] H \quad \ldots (4.5)$$

Here "dot" indicates the derivative with respect to proper time or affine parameter and

$$P \equiv E(r^2 + a^2) - L_z a - e^* er \quad \ldots (4.6)$$

$$V_r = P^2 - \Delta [u^2 r^2 + (L_z - aE)^2 + \Omega] \quad \ldots (4.7)$$

$$V_\theta = \Omega - \cos^2\theta \, [a^2(u^2 - E^2) + L_z^2 / \sin^2\theta] \quad \ldots (4.8)$$

where,

E ≡ conserved total energy.

L_z = conserved z component of angular momentum

u ≡ rest mass of particle

e^* ≡ charge of particle

Ω ≡ conserved quantity related to total angular momentum

The dynamical features and geometrical features of KNBH are : delineated below with the help of Kerr Newman metric (Carmeli,1977).

<u>Gravitational potentials</u> :

<u>Contravariant components of gravitational potentials</u> :

$$g^{00} = - a^2\sin^2\theta / (r^2 + a^2 \cos^2\theta)$$

$$g^{01} = (r^2 + a^2) / (r^2 + a^2 \cos^2\theta)$$

$$g^{03} = -a / (r^2 + a^2 \cos^2\theta)$$

$$g^{11} = (2 mr - r^2 - a^2 - e^2) / (r^2+a^2 \cos^2 \theta).$$

$$g^{13} = a / (r^2 + a^2 \cos^2\theta)$$

$$g^{22} = -1 / (r^2 + a^2 \cos^2\theta)$$

$$g^{33} = -1/(r^2 + a^2 \cos^2\theta) \sin^2 \theta . \qquad \ldots (4.9)$$

Covariant components of gravitational potentials :

$$g_{00} = \frac{r^2+e^2-2mr+a^2\cos^2\theta}{r^2+a^2\cos^2\theta} = 1 + \frac{e^2-2\,mr}{r^2+a^2\cos^2\theta}$$

$$g_{01} = 1$$

$$g_{02} = 0$$

$$g_{03} = \frac{a(2mr-e^2)\sin^2\theta}{r^2+a^2\cos^2\theta}$$

$$g_{11} = 0$$

$$g_{12} = 0$$

$$g_{13} = -a\sin^2\theta$$

$$g_{22} = -(r^2 + a^2 \cos^2\theta)$$

$$g_{23} = 0$$

$$g_{33} = -(r^2+a^2)\sin^2\theta + \frac{(e^2-2mr)\,a^2\sin^4\theta}{r^2+a^2\cos^2\theta} \qquad \ldots \qquad (4.10)$$

Orthogonal tetrads :

$$l^u = (0, 1, 0, 0)$$

$$m^u = -2^{-1/2}\bar{\rho}\,(ia\sin\theta, 0, 1, i\operatorname{cosec}\theta).$$

$$n^u = \rho\bar{\rho}\,(r^2 + a^2, -\Delta/2, 0, a) \qquad \ldots \qquad (4.11)$$

where $\Delta = r^2 + a^2 + e^2 - 2\,mr$.

$l_u = (1, 0, 0, -a\sin^2\theta)$

$m_u = -(\bar{\rho}/\sqrt{2})\ [\ ia\sin\theta,\ 0,\ -1/\rho\bar{\rho}\ ,\ -i(a^2+r^2)\sin\theta\ ]$

$n_u = [\Delta\rho\bar{\rho}\ /\ 2,\ 1,\ 0,\ -(\Delta\rho\bar{\rho}\ a/2)\ \sin^2\theta\ ]$ ... (4.12)

where $\Delta = r^2 + a^2 + e^2 - 2\,mr$

5. PHYSICAL COMPONENTS FOR KNBH :

Spin coefficients :

The spin coefficients for the Kerr-Newman metric are (Carmeli,1977).

$$\kappa = \nu = o = \lambda = \epsilon = 0$$

$$\rho = -(r - ia\cos\theta)^{-1}$$

$$\tau = -2^{-1/2}\ ia\ \sin\theta\rho\bar{\rho}\ ,$$

$$\pi = 2^{-1/2}\ ia\ \sin\theta\rho^2\ ,$$

$$\alpha = \pi - \bar{\beta}\ ,$$

$$\beta = -2^{-3/2}\ \cot\theta\bar{\rho}\ ,$$

$$\mu = 2^{-1}\Delta(\rho^2\bar{\rho})\ ,$$

$$\gamma = \mu + (r-m)\rho\bar{\rho}\ /2\ . \qquad \text{... (5.1)}$$

where $\Delta = r^2 + a^2 + e^2 - 2\,mr$

Physical components of the Weyl tensor and Maxwell scalars

(i) Weyl scalars :

$$\Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 = 0 \qquad \Psi_2 = \rho^3(m+e^2\bar{\rho}) \qquad \text{...(5.2)}$$

i.e., Black hole is of Petrov type D since

$\Psi_2 = \rho^3 (m + e^2 \bar{\rho})$ and other four Weyl scalars

$\Psi_0, \Psi_1, \Psi_3, \Psi_4$ vanish.

(ii) Maxwell scalars :

$$\phi_0 = \phi_2 = 0 , \quad \phi_1 = e\, \rho^2/2 . \qquad \ldots (5.3)$$

i.e., out of the three maxwells sclars only, $\phi_1 = e\rho^2/2$ is non vanishing.

Then the electromagnetic bivector for KNBH in NP version is

$$F_{ab} = \mathrm{Re}\phi_1 l_a n_b + \mathrm{Re}\phi_1 n_a l_b + i\mathrm{Im}\phi_1 m_a \bar{m}_b - i\mathrm{Im}\phi_1 \bar{m}_a m_b \qquad \ldots (5.4)$$

where Re(...) and Im(...) represents Real part of (...) and Imaginary part of (...) respectively.

APPENDIX (i)

NP concomitants :

$$l_{a;b} = (\gamma+\bar{\gamma}) l_a l_b - (\alpha+\bar{\beta}) l_a m_b - (\bar{\alpha}+\beta) l_a \bar{m}_b + (\epsilon + \bar{\epsilon}) l_a n_b -$$
$$- \bar{\tau}\, m_a l_b - \tau\, \bar{m}_a l_b + \bar{\sigma}\, m_a m_b + \sigma\, \bar{m}_a \bar{m}_b + \bar{\rho}\, m_a \bar{m}_b +$$
$$+ \rho\, \bar{m}_a m_b - \bar{k}\, m_a n_b - k\, \bar{m}_a n_b\ .$$

$$n_{a;b} = \nu\, m_a l_b + \bar{\nu}\, \bar{m}_a l_b - \lambda\, m_a m_b - \bar{\lambda}\, \bar{m}_a \bar{m}_b - \mu\, m_a \bar{m}_b -$$
$$- \bar{\mu}\, \bar{m}_a m_b + \pi\, m_a n_b + \bar{\pi}\, \bar{m}_a n_b - (\gamma + \bar{\gamma}) n_a l_b +$$
$$+ (\alpha + \bar{\beta}) n_a m_b + (\bar{\alpha} + \beta) n_a \bar{m}_b - (\epsilon + \bar{\epsilon}) n_a n_b .$$

$$m_{a;b} = \bar{\nu}\, l_a l_b - \bar{\mu}\, l_a m_b - \bar{\lambda}\, l_a \bar{m}_b + \bar{\pi}\, l_a n_b + (\gamma-\bar{\gamma}) m_a l_b +$$
$$+ (\bar{\beta}-\alpha) m_a m_b + (\bar{\alpha}-\beta) m_a \bar{m}_b + (\epsilon - \bar{\epsilon}) m_a n_b - \tau\, n_a l_b +$$
$$+ \rho\, n_a m_b + \sigma\, n_a \bar{m}_b - k\, n_a n_b \ .$$

$$\bar{m}_{a;b} = \nu\, l_a l_b - \lambda\, l_a m_b - \mu\, l_a \bar{m}_b + \pi\, l_a n_b + (\bar{\gamma}-\gamma) \bar{m}_a l_b +$$
$$+ (\alpha-\bar{\beta}) \bar{m}_a m_b + (\beta-\bar{\alpha}) \bar{m}_a \bar{m}_b + (\bar{\epsilon} - \epsilon) \bar{m}_a n_b - \bar{\tau}\, n_a l_b +$$
$$\bar{\sigma}\, n_a m_b + \bar{\rho}\, n_a \bar{m}_b - \bar{k}\, n_a n_b\ .$$

For Kerr Newman black hole :

$$k = \nu = o = \lambda = \epsilon = 0\ .$$

$$l_{a;b} = (\gamma+\bar{\gamma}) l_a l_b - (\alpha+\bar{\beta}) l_a m_b - (\bar{\alpha}+\beta) l_a \bar{m}_b - \bar{\tau}\, m_a l_b -$$
$$\bar{\tau}\, \bar{m}_a l_b + \bar{\rho}\, m_a \bar{m}_b + \rho\, \bar{m}_a m_b \ .$$

$$n_{a;b} = -\mu\, m_a\bar{m}_b - \bar{\mu}\, \bar{m}_a m_b + \pi\, m_a n_b + \bar{\pi}\, \bar{m}_a n_b -$$

$$- (\gamma + \bar{\gamma}) n_a l_b + (\alpha + \bar{\beta}) n_a m_b + (\bar{\alpha} + \beta) n_a \bar{m}_b \,.$$

$$m_{a;b} = - \bar{\mu}\, l_a m_b + \bar{\pi}\, l_a n_b + (\gamma - \bar{\gamma})\bar{m}_a l_b + (\bar{\beta} - \alpha)\bar{m}_a\bar{m}_b +$$

$$+ (\bar{\alpha} - \bar{\beta}) m_a \bar{m}_b - \bar{\tau}\, n_a l_b + \rho\, n_a \bar{m}_b \,.$$

$$\bar{m}_{a;b} = -\kappa\, l_a \bar{m}_b + \pi\, l_a n_b + (\bar{\gamma} - \gamma)\bar{m}_a l_b + (\alpha - \bar{\beta})\bar{m}_a m_b +$$

$$+ (\beta - \bar{\alpha})\bar{m}_a \bar{m}_b - \bar{\tau}\, n_a l_b + \bar{\rho}\, n_a m_b \,.$$

APPENDIX (ii)

Intrinsic Derivatives of Tetrad Vectors :

$$l_{a;b} l^b = (\epsilon + \bar{\epsilon}) l_a - \bar{k}\, m_a - k\, \bar{m}_a \,,$$

$$l_{a;b} m^b = (\bar{\alpha} + \beta) l_a - \bar{\rho}\, m_a - \sigma \bar{m}_a \,,$$

$$l_{a;b} \bar{m}^b = (\alpha + \bar{\beta}) l_a - \rho\, \bar{m}_a - \bar{\sigma} m_a \,,$$

$$l_{a;b} n^b = (\gamma + \bar{\gamma}) l_a - \bar{\tau}\, m_a - \tau\, \bar{m}_a \,,$$

$$n_{a;b} l^b = \pi m_a + \bar{\pi}\bar{m}_a - (\epsilon + \bar{\epsilon}) n_a \,,$$

$$n_{a;b} m^b = \mu m_a + \bar{\lambda}\bar{m}_a - (\bar{\alpha} + \beta) n_a \,,$$

$$n_{a;b} \bar{m}^b = \lambda m_a + \bar{\mu}\bar{m}_a + (\alpha + \bar{\beta}) m_a \,,$$

$$n_{a;b} n^b = \nu m_a + \bar{\nu}\bar{m}_a - (\gamma + \bar{\gamma}) n_a \,,$$

$$
\begin{aligned}
m_{a;b}\underline{l}^b &= \bar{\pi} l_a + (\epsilon - \bar{\epsilon}) m_a - k n_a \ , \\
m_{a;b} m^b &= \bar{\lambda} \underline{l}_a - (\bar{\alpha} - \beta) m_a - \sigma n_a \ , \\
m_{a;b} \bar{m}^b &= \bar{\mu} \underline{l}_a - (\bar{\beta} - \alpha) m_a - \rho n_a \ , \\
m_{a;b} n^b &= \bar{\nu} \underline{l}_a + (\gamma - \bar{\gamma}) m_a - \tau n_a \ ,
\end{aligned}
$$

$$
\begin{aligned}
\bar{m}_{a;b}\underline{l}^b &= \pi \underline{l}_a + (\bar{\epsilon} - \epsilon) \bar{m}_a - \bar{k} n_a \ , \\
\bar{m}_{a;b} m^b &= \mu \underline{l}_a + (\bar{\alpha} - \beta) \bar{m}_a - \bar{\rho} n_a \ , \\
\bar{m}_{a;b} \bar{m}^b &= \lambda \underline{l}_a - (\alpha - \beta) \bar{m}_a - \bar{\sigma} n_a \ , \\
\bar{m}_{a;b} n^b &= \nu \underline{l}_a + (\bar{\gamma} - \gamma) \bar{m}_a - \bar{\tau} n_a \ ,
\end{aligned}
$$

The projections of the tetrad vectors can be obtained in the following forms :

$$
\begin{aligned}
\underline{l}^a \underline{l}_{a;b} &= 0 \\
m^a \underline{l}_{a;b} &= \tau \underline{l}_b - \rho m_b - \sigma \bar{m}_b + k n_b \ , \\
\bar{m}^a \underline{l}_{a;b} &= \bar{\tau} \underline{l}_b - \bar{\sigma} m_b - \bar{\rho} \bar{m}_b + \bar{k} n_b \ , \\
n^a \underline{l}_{a;b} &= (\gamma+\bar{\gamma}) \underline{l}_b - (\alpha+\bar{\beta}) m_b - (\bar{\alpha}+\beta) \bar{m}_b + (\epsilon+\bar{\epsilon}) n_b \ .
\end{aligned}
$$

$$
\begin{aligned}
\underline{l}^a n_{a;b} &= -(\gamma+\bar{\gamma}) \underline{l}_b + (\alpha+\bar{\beta}) m_b + (\bar{\alpha}+\beta) \bar{m}_b - (\epsilon+\bar{\epsilon}) n_b \ , \\
m^a n_{a;b} &= -\bar{\nu} \underline{l}_b + \bar{\mu} m_b + \bar{\lambda} \bar{m}_b - \bar{\pi} n_b \ , \\
\bar{m}^a n_{a;b} &= -\nu \underline{l}_b + \lambda m_b + \mu \bar{m}_b - \pi n_b \ , \\
n^a n_{a;b} &= 0 \ ,
\end{aligned}
$$

$$\underline{l}^a m_{a;b} = -\tau \underline{l}_b + \rho m_b + \sigma \bar{m}_b - k n_b \,,$$

$$m^a m_{a;b} = 0 \,,$$

$$\bar{m}^a m_{a;b} = -(\gamma - \bar{\gamma})\underline{l}_b - (\bar{\beta} - \alpha)m_b - (\bar{\alpha} - \beta)\bar{m}_b - (\epsilon - \bar{\epsilon})n_b,$$

$$n^a m_{a;b} = \bar{\nu}\underline{l}_b - \bar{\mu} m_b - \bar{\lambda}\bar{m}_b + \bar{\pi} n_b \,,$$

$$\underline{l}^a \bar{m}_{a;b} = -\bar{\tau}\underline{l}_b + \bar{\rho}\bar{m}_b + \bar{\sigma} m_b - \bar{k} n_b \,,$$

$$m^a \bar{m}_{a;b} = -(\bar{\gamma} - \gamma)\underline{l}_b - (\beta - \bar{\alpha})\bar{m}_b - (\alpha - \bar{\beta})m_b - (\bar{\epsilon} - \epsilon)n_b \,,$$

$$\bar{m}^a \bar{m}_{a;b} = 0 \,.$$

$$n^a \bar{m}_{a;b} = \nu \underline{l}_b - \mu \bar{m}_b - \lambda m_b + \pi n_b \,.$$