CHAPTER - I

" General Relativity is the prime example of a physical theory built on a mathematical 'leap in the dark'. It might have remained undiscovered for a century if a man with Einstein's peculiar imagination had not lived."

... DYSON, F. J.

CHAPTER - I

1) INTRODUCTION

In recent years there has been a revival of interest in the Einstein-Cartan theory of space-time. In this theory the intrinsic spin of matter is incorporated as the source of the torsion of the space-time manifold. According to relativistic quantum mechanics mass and spin are the two fundamental characters of an elementary particle system.

In Einstein theory of general relativity mass plays a dynamical role but not the spin. The density of energy-momentum is the source of curvature. By introducing torsion and relating it to spin one can obtain an interesting link between the theory of gravitation and the theory of special relativity.

By introducing torsion and relating it to the density of intrinsic angular momentum the Einstein-Cartan theory restores the analogy between mass and spin. The similarity between mass and spin extends to the principle of equivalence at least in its weak form. According to this principle the world line of a spinless test particle moving under the influence of gravitational fields only depends on its initial position and velocity but not on its mass. Similarly the motion of a spin depends on the initial data but not on the magnitude of the spin of the particle.

The notion of an affine connection was casually mentioned by Eddington [12] in discussing possible extensions of general relativity. He pointed out that applications in macro-physics are conceivable but did not develop his idea. Torsion as the anti-symmetric part of an asymmetric affine connection was introduced by Elie Cartan [7] in 1922. He suggested a simple generalization of Einstein's theory of gravitation.

In 1922 Elie Cartan proposed to consider, as a model of space-time, a four-dimensional differentiable manifold with a metric tensor and a linear connection compatible with the metric but not symmetric, in general. According to Cartan, the torsion tensor of the connection should be related to the density of intrinsic angular momentum of matter and it should vanish in matter free regions. Independently of Cartan, similar ideas were
4 put forward by Wagoner [74], Nordtvedt [40] and Bergmann [6]. The generalization due to Cartan constitutes only a slight departure from the Einstein's theory: the field equations in empty space remain inchanged.

The desirability of such an analysis may be related to recent discoveries in astronomy. It is conceivable that torsion may produce observable effects inside those objects which, as the neutron stars, have built-in strong magnetic fields, possibly accompanied by a substantial average value of the density of spin. One is tempted to speculate that intrinsic angular momentum may influence-or even prevent- the occurrence of singularities in gravitational collapse and cosmology. A recent result of Kopczyński [25] on the geometry of a Universe filled with a spherically-symmetric distribution of mass and spin, supports this idea.

2) EINSTEIN-CARTAN THEORY

In recent years, there has been a growing interest in the foundation of Einstein's theory of general relativity. A number of new relativistic theories of gravitation were put forward by Brans and Dicke [4], Bergamann [6], Wagoner [74], Nordtvedt [40] and Sen and Dunn [55]. Their predictions with the observational data and the available experimental results are compared with those of the older theories.

Thorne and his co-workers [64] have undertaken a systematic study of what they call "metric theories of gravitation". These are the theories which may be formulated in terms of Riemannian geometry in space-time, possibly with supplementary structures added to it. The total stress-energy tensor of matter is assumed to satisfy a differential conservation law determined by the Riemannian linear connection of space-time.

The question of singularity in general relativity is a much discussed problem. Penrose [44], Hawking [15] and Geroch[14] have shown that the occurrence of space-time singularities is a general prediction of the theory and not just the consequence of the symmetry of the models. Modifying Einstein's equations of general relativity has been one one of the techniques followed to avoid space-time singularities. Recently Trautman [69] has proposed that spin and torsion may avert gravitational singularities, by considering a Friedmann type of universe in the framework of Einstein-Cartan theory and obtaining a minimum radius R_c at $t = 0$.

According to Hehl Einstein-Cartan theory is an even more beautiful theory than Einstein's general relativity because of its relation to the Poincare group.

In Trautman's opinion the Einstein-Cartan theory is the simplest and the most natural modification of the original, Einstein's theory of gravitation. This modification deserves to be analysed in detail, in precedence over the theories requiring an additional scalar field to describe gravitational phenomena.

3) HISTORICAL SURVEY OF EINSTEIN-CARTAN THEORY

Einstein-Cartan theory begins with Scima [56] and Kibble[24]. It was further developed by Trautman [67-70], Adamovicz [1], Kerlick [22,23], Kuchowicz [30-34], Hehl [16,17], Hehl et al.[18]. Tafel [62,63], Stewart and Hajicek [58], Kopczyński [25-27], Raychaudhuri [54] and Prasanna [45-49].

Since the predictions of the Einstein-Cartan theory differ from those of general theory of relativity only for the regions filled with matter. An important field of application for the theory is relativistic astrophysics which deals with the theories of stellar objects like neutron stars with some alignment of spins of the constituent particles and under the conditions when torsion may produce some observable effects. Hence it is desirable to understand the implications of the Einstein-Cartan theory in full for finite distributions like fluid spheres with non-zero pressure. With this view the problem of static fluid spheres in Einstein-Cartan theory were considered by Prasanna[46], Kerlick [23], Kuchowicz [33], Skinner and Webb [60].

In 1964, István Ozsvàth [42] had solved the Einstein's field equations with incoherent matter for the case of homogeneous space-time, i.e. for metrics allowing a four parametric simply transitive group of motions. He has obtained two sets of new solutions by using a spinor-technique. Misra, Pandey and Srivastava [37,38] have investigated the Einstein-Maxwell equations for a stationary gravitational field in 1972 and for an axially symmetric stationary gravitational field in 1973. Assuming the space is filled with charged incoherent matter, they have shown that if the Lorentz force vanishes everywhere, the charge density bears a constant ratio to the mass density and this constant ratio may assume arbitrary values.

Hehl, Heyde and Kerlick [18] have considered the field equations of general relativity with spin and torsion (U_4 theory) to describe correctly the gravitational properties of matter on a macrophysical level. By an averaging procedure one can arrive at a macroscopic field equation, which under normal matter densities coincides with Einstein's equation of conventional general relativity. They have shown how the singularity theorems of Penrose and Hawking must be modified to apply in U_4 theory and all known cosmological models in U_4 theory which prevent singularities have also shown to violate an energy condition of a singularity theorem.

Following the work of Trautman [70], Prasanna [46] has described briefly the Einstein-Cartan equations with special reference to a perfect fluid distribution and then obtained three solutions adopting Hehl's [16,17] approach and Tolman's[65] technique. He has found that a space-time metric similar to the Schwarzschild interior solution will no longer represent a homogeneous fluid sphere in the presence of spin density.

Arkuszewski et al [3] discussed the junction conditions in Einstein-Cartan theory. Raychaudhuri and Banerji [54] constructed a specific solution corresponding to a collapsing sphere and showed that it bounces at a radius greater than the Schwarzschild radius. Banerji [5] has pointed out that Einstein-Cartan sphere must bounce outside the Schwarzschild radius if it bounces at all. Nduka [39] generalized the Prasanna's [46] work by considering a static charged fluid sphere in Einstein-Cartan theory. Singh and Yadav [59] studied the static fluid sphere in Einstein-Cartan theory and obtained the solutions in an analytic form by the method of quadrature.

Kopczyński [25-27] studied and developed the Einstein-Cartan theory of gravitation. The specialization of the technique gives the solutions of Adler [2], Kuchowicz [33], Whitman [75], Krori et al. [28,29] and Mehra [35]. Spatially homogeneous cosmological models of Bianchi types VI and VII based on Einstein-Cartan theory were considered by Tsoubelis [71]. Som and Bedran [61] got a class of solutions that represents a static incoherent spherical dust distribution in equilibrium under the influence of torsion and spin. Krori et al [29] gave a singularity free solution for a static charged fluid sphere in Einstein-Cartan theory. Pandey et al [43] solved the Einstein-Cartan field equations for a static spherically symmetric fluid sphere by a suitable assumption on the metric potential g_{11}.

In 1982, Rao and Reddy [50] have shown that there are no spherically symmetric static conformally flat solutions of Nordtvedt-Barker field equations respectively perfect-fluid distribution with disordered radiation obeying the equation of state $\rho = 3p$, except for the trivial empty flat space-time of Einstein's theory. Ray and Smalley [51,52] have presented an improved perfect-fluid energy momentum tensor including spin and torsion with use of a Lagrangian variational principle based upon the tetrad formalism of Halbwach and the method of constraints of Ray. Nurgaliev and Ponomarev [41] have shown that for a specific value of the constant of interaction between spins, which must be introduced from gauge considerations, the Einstein-Cartan equation without the Λ-term admits a cosmological solution in the form of a steady-state deSitter metric.

In 1983, Srivastava and Prasad [57] have discussed the boundary conditions at the interface of spherically symmetric perfect-fluid distribution and the exterior vacuum and as a consequence they have established the following theorem : " Uniform expansion or contraction of a perfect fluid sphere obeying an equation of state with nonuniform density is not admitted by the field equations". It is further shown that the Wyman metric is not suitable on physical ground to represent a cosmological solution. Zhu Shi-Chang [76] has obtained some conformal flat interior solution of the Einstein-Maxwell equations for a charged stable static sphere which satisfy physical conditions inside the sphere. Hirohisa Ishikawa [19] has presented the exact non-static solutions for the coupled repulsive sourceless massless scalar field and the gravitational field and shown that these solutions have the same form as the spatially conformally flat static metric

$$ds^2 = - e^{2\lambda}dt^2 + e^{-2\lambda} (dx^2 + dy^2 + dz^2).$$

deRitis et al [11] have studied spin fluid in Einstein-Cartan theory : A variational principle and an extension of the velocity potential representation. Ray and Smalley [53] have considered an Eulerian variational principle for a spinning fluid in Einstein-Cartan metric-torsion theory and they have shown that the symmetric energy-momentum tensor is a sum of a perfect-fluid term and a spin term.

Recently Kallyanshetti and Waghmode [20] have considered a static conformally flat spherically symmetric perfect fluid distribution in Einstein-Cartan theory. They have solved the field equations by adopting Hehl's [16,17] approach with the assumption that spin of the particles composing the fluid are all aligned in the radial direction alone. They have observed that the density ϱ will not be constant as observed by Narlikar in 1950 for conformally flat spherically symmetric perfect fluid distribution.

Faulkes [13] has shown that shear-free solutions of the Einstein-Maxwell field equations can be found by solving a single second order non-linear differential equation containing two arbitrary functions of the radial co-ordinate. But in this year, Chattarjee [10], in his work, a general method has proposed to solve this nonlinear equation which, in effect, extends an earlier work of Wyman to its electromagnetic.

4) THE STRUCTURE EQUATIONS OF EINSTEIN-CARTAN THEORY

Let M be a four-dimensional differentiable manifold of class C^{∞}, oriented connected Hausdorff and let g be a Lorentz metric defined on it. All geometric objects on M, other than forms, will be described by their components with respect to a field Θ^i of coframes in the cotangent spaces of M which are linearly independent at each point of M. Since we are interested in spinor fields we take the Θ^i to be in general anholonomic and the associated tetrad to be orthonormal. The metric g and the connection w are described with respect to the co-frame Θ^i chosen by the metric components g_{ij} and by a set of one-forms $w^i{}_j$.

Therefore we have

$$g = ds^2 = g_{ij}\theta^i \otimes \theta^j \qquad \ldots (4.1)$$

and $w^i{}_j$ are completely determined by the functions Γ^i_{kj} such that

$$w^i{}_j = \Gamma^i_{kj}\,\theta^k \qquad \ldots (4.2)$$

If ϕ_A (A,B, ... = 1, ..., N) is a tensor valued p-form, the (p + 1)-form

$$D\phi_A = d\phi_A + \sigma^{Bj}_{Ai}\, w^i{}_j \wedge \phi_B$$

is called the covariant exterior derivative of ϕ_A , the constants σ 's are related to the tensorial type of ϕ_A. In particular, if ϕ_A is a 0-form, its covariant exterior derivative becomes covariant derivative

$$D\phi_A = \nabla_i \phi_A \theta^i \;,$$

and if ϕ is a scalar p-form

$$D\phi = d\phi \;.$$

The covariant exterior derivative of the vector-valued 1-form Θ^i is the torsion 2-form of w

$$\Theta^i = D\theta^i = d\theta^i + w^i{}_j \wedge \theta^j \qquad \ldots (4.3)$$

The curvature 2-form of w is

$$\Omega^i{}_j = dw^i{}_j + w^i{}_k \wedge w^k{}_j \qquad \ldots (4.4)$$

These two form satisfy the Bianchi identities

$$D\Omega^i{}_j = 0, \; D\Theta^i = \Omega^i{}_j \wedge \theta^j \qquad \ldots (4.5)$$

We shall use the tensors $Q^i{}_{jk}$ and $R^i{}_{jkl}$ and the one-forms $Q^i{}_j$ and $R^i{}_{jk}$ of the torsion and the curvature respectively

$$\begin{aligned} \Theta^i &= \tfrac{1}{2}\,\theta^j \wedge Q^i{}_j \\ &= \tfrac{1}{2}\,Q^i{}_{jk}\theta^j \wedge \theta^k \end{aligned} \qquad \ldots (4.6)$$

$$\begin{aligned} \Omega^i{}_j &= \tfrac{1}{2}\,\theta^k \wedge R^i{}_{jk} \\ &= \tfrac{1}{2}\,R^i_{jkl}\;\theta^k \wedge \theta^l \end{aligned} \qquad \ldots (4.7)$$

From (4.3), (4.4), (4.6) and (4.7) we have

$$\begin{aligned} \Theta^i &= D\theta^i \\ &= d\theta^i + w^i{}_j \wedge \theta^j \\ &= \tfrac{1}{2}\,Q^i{}_{jk}\theta^j \wedge \theta^k \end{aligned} \qquad \ldots (4.8)$$

$$\begin{aligned} \Omega^i{}_j &= dw^i_j + w^i{}_k \wedge \theta^k \\ &= \tfrac{1}{2}\,R^i{}_{jkl}\;\theta^k \wedge \theta^l \end{aligned} \qquad \ldots (4.9)$$

Here (4.8) and (4.9) are called as first and second Cartan's structural equations respectively.

5) THE FIELD EQUATIONS

If the manifold M is four-dimensional and has a metric tensor, it is possible to introduce the completely antisymmetric pseudo-tensor η_{ijkl} , where

$$\eta_{1234} = \left|\det g_{ij}\right|^{1/2}$$

Together with η_{ijkl}, the forms

$$\eta_{ijk} = \Theta^l \eta_{ijkl} \quad , \quad \eta_{ij} = \frac{1}{2}\, \Theta^k \wedge \eta_{ijk} \, ,$$
$$\eta_i = \frac{1}{3}\, \Theta^j \wedge \eta_{ij} \quad , \quad \eta = \frac{1}{4}\, \Theta^i \wedge \eta_i \qquad \ldots (5.1)$$

Span the Grassmann algebra of M and

$$\Theta^m \eta_{ijkl} = \delta^m_l\, \eta_{ijk} - \delta^m_k\, \eta_{lij} + \delta^m_j\, \eta_{kli} - \delta^m_i\, \eta_{jkl} \, ,$$
$$\Theta^l \wedge \eta_{ijk} = \delta^l_k \eta_{ij} + \delta^l_j \eta_{ki} + \delta^l_i\, \eta_{jk} \, ,$$
$$\Theta^k \wedge \eta_{ij} = \delta^k_j \eta_i - \delta^k_i \eta_j \, ,$$
$$\Theta^j \wedge \eta_i = \delta^j_i\, \eta \qquad \ldots (5.2)$$

The Einstein-Cartan field equations are obtained from the variational principle.

$$\delta \int (S + KL) = 0 \qquad \ldots (5.3)$$

where L is the material Lagrangian four-form and is given by

$$L = L\,(\psi_A, D\psi_A, \Theta^i, g_{ij}) \, .$$

It is depending locally on the spinor or tensor fields ψ_A, their covariant derivatives $D\psi_A$ and the metric; K is the gravitational constant and S is the Ricci four-form defined globally as

$$S = \frac{1}{2}\, \eta^l_k \wedge \Omega^k{}_l = \frac{1}{2}\, R\eta, \qquad \ldots (5.4)$$

where $R = g^{ln}\, \delta^m{}_k\, R^k{}_{lmn}$; and η is the volume four-form.

Varying the total action with respect to the metric, i.e. Θ^i since g_{ij} are fixed, the connection $w^i{}_j$ and the fields ψ_A independently, we get the following equations

$$e_i = kt_i, \quad e^j{}_i = k\delta^j{}_i \;, \quad \frac{\delta L}{\delta \psi_A} = 0 \qquad \ldots (5.5)$$

where in

$$e_i = \tfrac{1}{2}\eta_{ijk} \wedge \Omega^{jk} \;, \quad c^j{}_i = -Dn^j{}_i \;,$$
$$t_i = \frac{\delta L}{\delta \Theta^i} \;, \qquad S^j{}_i = \frac{1}{2}\frac{\delta L}{\delta w^i{}_j} \qquad \ldots (5.6)$$

The orthonormality of the frames together with the fact the connection as a metric connection ($Dg_{ij} = 0$) tells us that an infinitesimal variation in connection induces tetrad rotation. Hence $S^j{}_i$ is the spin density of the system and t_i is the energy-momentum vector-valued three form. In the general case we have the energy-momentum symmetric four-form,

$$T^{ij} = \frac{1}{2}\frac{\delta L}{\delta g_{ij}} \;,$$

which, along with t_i and $S^j{}_i$ satisfies the identity

$$T^j_i = \Theta^j \wedge t_i - \frac{1}{2} DS_i{}^j \qquad \ldots (5.7)$$

From the equations (5.5) and (5.6) by using (4.8) and (4.9) we obtain Einstein-Cartan equations as

$$R^j_i - \frac{1}{2} R\delta^j_i = -kt^j{}_i \qquad \ldots (5.8)$$

$$Q^i_{jk} - \delta^i_j Q^l_{lk} - \delta^i_k Q^l_{jl} = -kS^i_{jk} \qquad \ldots (5.9)$$

where $t^j{}_i$ and $S^l{}_{jk}$ are defined through the relations

$$t_i = \eta_j t^j{}_i \;, \quad S_{ij} = \eta_k S^k{}_{ij} \qquad \ldots (5.10)$$

In finding the solutions of Einstein-Cartan equations, we use a classical description of spin as follows

$$S^{i}{}_{jk} = u^{i}S_{jk} \;, \quad \text{with} \quad u^{k}S_{jk} = 0 \qquad \ldots (5.11)$$

where u^i is the velocity four-vector and S_{jk} is the intrinsic angular momentum tensor.

In case of a perfect fluid distribution with isotropic pressure the cononical tensor for such a distribution is given by

$$t_{kj} = (p + \varrho)V_i V_j - pg_{ij} \qquad \ldots (5.12)$$

together with $g^{ij}V_i V_j = 1$,

where p is the pressure, ϱ is the density and V_i is the flow vector which describes the radial motion of the fluid.

6) COMPARISON WITH EINSTEIN'S THEORY

The comparison between the Einsteinian theory of gravitation and the Einstein-Cartan theory is summarized in the following Table[27

	Einstein theory	Einstein-Cartan theory
Sources	T	t + s
gravitational field description	g	g + w or g + Q

In the Einstein theory the energy-momentum of matter T is the only one source of gravitation. This situation may seem unjustified from the point of view of elementary particle physics, where it is difficult to answer the question, which of two invariants of the Poincare group is more "important" : mass or spin ? Taking this into account, in the Einstein-Cartan theory we have two quantities which serve as sources of gravitational field: except of the energy-momentum tensor t, we have the spin tensor S. The energy-momentum tensor t couples to the curvature of the metric but assymetric connection w in a similar way as in the Einstein theory. Since the connection w is metric, one can express it by the metric field g and the torsion field Q. Therefore, we can consider this theory as the theory of two tensor fields - g and Q - however the role of these tensor fields is different : the torsion is algebraically connected with sources.

Torsion does not propagate

Torsion is only found inside spinning matter. In the vacuum we have the usual Riemann space-time geometry where the Einstein tensor vanishes. Torsion cannot propogate in vacuum as it is tied to matter. The propagation of gravity is the same as in Einstein's theory in the vacuum. The difference springs for the metric dependent part of gravity from redefined sources. The metric energy-momentum tensor of General Relativity in this context is replaced by the combined energy-momentum tensor of Einstein-Cartan theory. The sources look different but the field is the same.

Spin contact interaction

The long distance behaviour of Einstein-Cartan theory is the same as in General Relativity but the short distance behaviour is distinctly different. Adamovicz [1] shown that in the case of the linear approximation Einstein-Cartan theory and the general relativity gives the same metrics of space-time.

Scalar matter, Photons and Neutrinos

If spin $S \neq 0$, mass $m \neq 0$, it is valid for spinning massive matter. Matter without spin ($S = 0$) i.e. a scalar field produces no torsion. Spinning massless matter deserves special attention. Maxwell's field ($s = 1$, $m = 0$) cannot produce a gauge invariant torsion and cannot be coupled to the U_4. Neutrinos ($s=1/2$, $m=0$) play an interesting role and their special relativistic Lagrangian is not invariant in a U_4. At ordinary matter densities we can get the results of General Relativity by safely neglecting the U_4 correction.

Equation of Motion

In General Relativity, the test particles which are point like and neutral fall along geodesics of the Riemannian space-time V_4 of General Relativity. This behaviour can be derived from the energy-momentum law of General Relativity or from the field equations. In Einstein-Cartan theory a typical massive test mass carries spin and therefore falls neither along a straight line nor along a shortest line (geodesic).

7) COMPARISON OF EINSTEIN MODEL WITH ACTUAL UNIVERSE

The most satisfactory feature of the Einstein model is its correspondence with a universe which could actually contain a finite concentration of uniformly distributed matter. In this respect it gives us a cosmology which is superior to that provided by the dé-Sitter model. This advantage is gained only at the expense of introducing the extra cosmological term Λg_{ij} into Einstein's original field equations which is a device similar to the modification in Poisson's equation proposed in order to permit a uniform static distribution of matter in flat space of the Newtonian theory.

The most unsatisfactory feature of the Einstein model as a basis for the cosmology of the actual universe is, that it provides no reason to expect any systematic shift in the wave-length of light from distant objects. In the actual universe, however, the work of Hubble and Humason shows a definite red-shift in the light from the nebulae which increases with the distance. This is of course the main consideration which will lead us to prefer non-static to static models of the universe as a basis for actual cosmology.

8) REASONS FOR CHANGING TO NON-STATIC MODELS

The original static universes of Einstein and of de Sitter are certainly very important in furnishing examples of the kind of cosmological models those can be constructed within the theoretical frame-work of General Relativity. It is evident that neither of these models give a satisfactory description of the present state of the actual universe, the one because it permits no shift in the wave-length of light from the nebulae, and the other because it permits no matter or radiation to be present in space.

We must hence turn to some less restricted class of models in our attempts to describe the behaviour of the actual universe.

There are several reasons which make it natural to abandon this assumption that our cosmological models should necessarily be static in character. They are :

(1) The non-static models which we shall study are, to be sure, mathematically more complicated than static ones.

(2) It is of course evident that any increase in generality which can be brought about by the removal of previous restrictions will be of advantage in increasing the range of possible applicability.

(3) Although there was some observational evidence for ascribing a reasonably stationary character to our surroundings at a time when our knowledge of the universe was practically limited to the stars in our own galaxy, this evidence now be replaced by the observed red-shift in the light from the extra-galactic nebulae which at least leads to the presumption that these objects are not static but are moving away from each other.

(4) Even if some successful alternative hypothesis should be proposed for explaining this red-shift which certainly lead to changes in gravitational field with the time and hence necessarily to a non-static universe.

(5) We shall find that an originally static Einstein universe would in any case not be stable but would start to expand or contract as a result of disturbances.

By dropping the restriction to static models, we study a considerable group of non-static homogeneous models, which were first theoretically investigated by Friedmann and first considered in connection with the phenomena of the actual universe by Lemaître.

9) SURVEY OF OUR INVESTIGATION

In Chapter I, following the work of Kopczyński [25] and Prasanna [46] we have described briefly the structure equations of Einstein-Cartan theory and the field equations.

In Chapter II, a non-static conformally flat spherically symmetric perfect-fluid distribution in Einstein-Cartan theory is considered. With the assumption that the spins of the particles composing the fluid are all aligned in the radial direction alone, we obtain the connection forms, curvature forms and Riemann tensors and hence Ricci tensors and scalar of curvature are obtained in Section-2. In Section-3, the field equations are obtained. Adopting the Hehl's approach [16,17], these field equations are solved in Section-4. Finally, in Section-5, we discussed the particular cases of this solution.

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