<u>CHAPIER I</u>

SOME DEFINITIONS

ABSTRACT

In the present Chapter We give in detail the definitions and statements of known results which we are making use in the course of our research.

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Notation Definition :- Let $E = \{Z : Z \text{ is a complex number and} |z| < 1 \}$ tion $f(\mathbf{z})$ is said

to be holomorphic in a domain D in the complex plane if it is differentiable at every point of the domain D.

Definition :- A single valued function f is said to be univalent (or schlicht) in a domain D \subset \bigcirc \bigcirc denoting the complex plane, if it never takes the same value twice, that is, if $f(Z_1) \neq f(Z_2)$ for all points z_1 and z_2 in D, with $z_1 \neq z_2$

Definition :- Let S be the class of all functions f(z)holomorphic and univalent in E and normalised by f(o)=0, f'(o)=1.

mivalent

This family of functions is designated as the normalised functions. We will in general confine our attention to this above family of functions.

Definition :- A domain containing the origin is starlike with respect to the origin if it is intersected by any straight line through the origin in a linear segment. Starlike with respect to the origin will be referred to as simply starlike.

Definition :- Let S* be the subclass of S whose members map every disc $|z| < \beta$, $(o < \beta < i)$ onto a starlike domain . <u>Definition</u> :- Let f(z) be holomorphic at z=0 and satisfy f(o)=0, $f'(o) \neq 0$ there. Then the radius of univalence, denoted by Uo is defined to be the largest value of 'r' such that f(z) is holomorphic and univalent for |z| < r.

Definition :- let f(z) be holomorphic at z = 0 and satisfy f(o)=0 and f'(o)=0, there. Let \prec be a real number satisfying $0 \leq \prec < 1$. The radius of starlikeness of order denoted by S_{\prec} , is defined to be the largest value of 'r' such that f(z) is holomorphic and $\operatorname{Re}\left\{\frac{Z - f'(z)}{f(z)}\right\} > \ll$ for |z| < r.

<u>Definition</u> :- Let k be the subclass of S, whose members map every disc $|z| \leq \beta$, $0 \leq \beta < |$ onto a convex domain.

Definition := Let f(z) be holomorphic at z=0 and satisfy f(o)=0 and $f'(o)\neq0$ there, let \prec be a real number satisfying $0 \leq \prec < 1$. The radius of convexity of order \prec , C_{\prec} defined to be the largest value of 'r' such that f(z) is holomorphic and $\operatorname{Re}\left\{1 + \frac{z}{f'(z)}\right\} > \ll f_{0r} |z| < r$ Theorem - $\langle 6, \text{ page 221} \rangle$ Suppose f(z) is holomorphic in E and f(o)=0, f'(o)=1, then $f(z) \in S^*$ if and only if $\operatorname{Re}\left\{2f'(z)/f(z)\right\} > 0$ $f_{0r} |z| \in E$ Theorem - $\langle 6| \text{ page 223} \rangle$ Suppose f(z) is holomorphic in E and f(o)=0 and f'(o)=1, then $f(z) \in K$, e the class of convex functions if and only if $\operatorname{Re}\left\{1+\frac{z}{f'(z)}\right\} > 0$ $Z\in E$ f(z) Theorem - (5) Suppose $f(z) \in K$, then for $Z \in E$ Re $\{Z f'(Z)/f(Z)\} > \frac{1}{2}$ Definition :- (Kaplan (24). A function $f(Z) \in S$ is

close-to-convex with respect to the convex function $e^{ix}g(z)$ # where $g(z) \in K$ and $0 \leq \alpha < 2\pi$ if

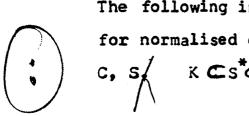
 $\operatorname{Re}\left\{f'(z)/\operatorname{eig}(z)\right\} > 0 \right\} / ($

For $z \in E$ let C be the subclass of S of close-toconvex functions ie $f(z) \in C$ if f(z) is close-toconvex with respect to some $e^{i \swarrow} g(z), g(z)$ in K.

<u>Definition</u> :- Let f(z) be holomorphic at z=0 and satisfy f(o)=0, f'(o)=0 there. Then the radius of close-to-convexity is defined to be the largest value of r such that f(z)is holomorphic and close-to-convex for |z| < r.

<u>Remark</u> :- A function satisfying either the close-to-convex condition, the starlike condition, or the convex condition is univalent.

Theorem - (4) page 173 Suppose f(z) is holomorphic in E, $f'(z) \neq 0$, $(Z \in E)$, and f(o)=0, f'(o)=1, Then $f(z) \in C$ if and only if $\begin{cases} \theta_2 \\ \theta_1 \end{cases}$ Re $\left(1 + Z f''(z)/f'(z)\right) d\theta > -\pi$, $Z = re^{i\theta}$ for any r, θ_1 and θ_2 , 0 < r < 1, $0 \le \theta_1 < \theta_2 \le 2\pi$ REMARKS :



The following inclusion relationships hold for normalised classes of functions, K, S^* , C, S/ K C S^* C C S.

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