

I N T R O D U C T I O N

The purpose of this chapter is to give the historical background for the investigations of this dissertation.

1) History

Fluid mechanics is a part of applied mathematics, physics and of many branches of engineering particularly civil, mechanical, chemical and aeronautical engineering and of naval architecture and geophysics, with astrophysics and biological and physiological fluid dynamics to be added. Significant contributions to the theory of airfoils came early in the century. During the whole of the first half of the century applied aerodynamics was to be probably the major incentive, dealing with questions which was important also in mechanical and civil engineering; but geophysical questions, certainly not without charm and fascination, received much attention.

Curiosity about at least two of the branches of fluid mechanics and their applications have a long and distinguished history, for in the proverb of Solomon, the son of David, who was the king of Israel, which was stated in the words of Agur, the son of Jaken that "There be three things which are too wonderful for me, " Yea four which I know not", of which two were, "The way of an eagle in the air" and

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"The way of a ship in the midst of the sea", which I take to be questions of aerodynamics and naval architecture, questions that concern as still.

Until the turn of this century the study of fluids was undertaken essentially by two groups hydraulicians and mathematicians. Hydraulicians worked along empirical lines while mathematicians concentrated on analytical lines. The vast and often ingenious experimentation of the former group yielded much information of indispensable value to the practicing engineer of the day. However lacking the generalising benefits of workable theory, these results were of restricted and limited value in novel situations, mathematicians, meanwhile, by not availing themselves of simplified as to render their results very often completely at odds with reality.

It becomes clear to such eminent investigators as Reynolds, Froude, Prandtl and Van Karman that the study of fluids must be a blend of theory and experimentation. Such was the beginning of the science of fluid mechanics as it is known today. Our Modern research and test facilities employ mathematician, physicists, engineers and skilled technicians, who working in terms, bring both view points in varying degrees to their work.

Boundary layer theory is the oldest branch of modern

fluid dynamics. Essentially three branches of fluid dynamics have become particularly well developed during the last ninety years, they include boundary layer theory, gas dynamics and aerofoil theory. Since about the beginning of the current century modern research in the field of fluid dynamics has achieved great successes and has been able to provide a theoretical clarification of observed phenomena which the science of classical hydrodynamics of the preceding century failed to do.

Boundary Layer theory was found by L. Prandtl [32] in his 1904 lecture to the International Congress of Mathematics, on the boundary layer condition of no slip and circumstances in a thin layer of fluid near a solid wall. This concept made it possible to clarify many phenomena which occur in flows and which had previously been incomprehensible. Most important of all it has become possible to subject problems connected with the occurrence of drag to a theoretical analysis. In 1908 Blasius [5] published in a more accessible, more conventional, medium of communication a fuller account of the derivation of the boundary layer equation and a detailed investigation of the flow along a flat plate parallel to a stream, but even after that there was not exactly a rush of acceptance, exposition or further investigation of boundary layer theory. Later on Toffer [41] and Howorth [19]

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tackled the same problem by using numerical methods to obtain the solution.

With the publication in 1921 of Karman's [24] momentum equation and the Karman-Pohlhausen [35] approximate method of integration, and published in 1924 of the experiments of J.M. Burgers [8], boundary layer theory at last became the subject of more attention and acceptance.

The science of aeronautical engineering was making rapid progress and was soon able to utilize these theoretical results in practical applications. It did on the other hand, post many problems which could be solved with the aid of the new boundary layer theory. In the other fields of machine design in which problems of flow occur, in particular layer made much slower progress, but in modern times three new concepts have come to the fore in such applications as well.

Flows through porous media occur in filtration of fluids and seepage of water in river beds. Movements of underground water and oil are some other important examples of flow through porous media. An oil reservoir mostly consists of porous sedimentary formation such as limestone and sandstone in which oil is entrapped. Oil can be obtained from such reservoirs by drilling wells in oil bearing area down to the oil reservoir and then either allowing or causing the oil to flow through porous oil bearing rocks into the

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well. To study the underground water resources also one needs to investigate the flow of fluid through porous media is the seepage under dam. There are numerous other practical uses of fluid flow through porous media.

The theory of energy (heat) and mass transfer is one of the domains of contemporary science, it is of great practical importance for increasing the process rate in heat-power engineering, power engineering, and chemical engineering in various branches of industry and agriculture. Heat and mass transfer have become especially important in Modern technology, particularly atomic power engineering and space research, whose rapid development gave rise to the theory of energy and mass transfer.

For the past two decades this branch of science has been considerably extended and advanced. Numerous applications of the theory may be found in power plant and industrial power engineering, technological processes, chemical engineering the construction industry and agriculture. It should be noted that the level of scientific research and development varies in different branches of engineering whereas in the newest fields of scientific research the development of heat and mass transfer theory is at a rather high level, applications in other fields lag far behind.

In heat transfer problems involving porous media such as high temperature reactor technology, heat technology, chemical technology and geothermal energy extraction, the working medium is subjected a high temperature and will have high optical density. When the medium has high optical density, then the optical mean free path is commensurate with the molecular mean free path. In such cases thermal radiation also serves as a model of heat transfer. Hence the transfer of thermal energy is governed by convective and radiative components which represents the most general case of heat transfer with heat being transferred not only by radiation but also by conduction and convection.

Convective heat transfer in a porous medium has attracted considerable interest in recent years due to its numerous applications in industrial and geo-physical problems.

Sparrow and Chya [37] and Sunden [38] concluded that although the conventional fin theory model based on the prescribed uniform heat transfer coefficient gives a good estimate of the overall heat transfer rate from the fin substantial errors could arise in the prediction of the local heat transfer rate. The conjugate mixed convection conduction heat transfer problem for a plate fin embedded vertically in a saturated porous medium has recently been analysed by Liu [25].

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B.C.Chandrasekhara and P.Nagaraju [9] studied the composite heat transfer in case of steady laminar flow of a gray fluid with large optical density past a horizontal plate embedded in a saturated porous medium and obtained velocity and temperature distribution by using series method and observed that the ratio of heat fluxes increases with increase in B (the porous parameter).

A.K.Kolar and V.M.K.Sastri [21] studied the numerical and experimental investigation into the upstream transpiration problem in free convection, and obtained that the downstream heat transfer depends not only on the upstream transpiration velocity, but also on the length over which transpiration is applied. A. Nakayama and H.Koyama [50] investigated the free convective heat transfer over anisothermal body.

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BASIC CONCEPTS

The basic concepts, required for the discussions of our problems have been explained. x

1) Fluid

All materials exhibits deformation under the action of forces. The matter is usually divided into two classes namely, the fluids and the solids. When some external force is applied, it is the deformation which is important, i.e. there is a relationship between the external forces and the deformation caused in the matter is called a solid. / 9. If the deformation in the material increases continuously without limit under the action of shearing forces, however small the material is called a fluid.

Another criterion for the classification of the matter between solids and fluids is, in solids the different particle of the matter (volume elements) have definite relative position and these elements change their relative position only when some external forces is applied. In fluids the different elements can change their positions even without any external forces and therefore when a fluid is put into a container the elements rearrange themselves so as to take the shape of the container. ?

A fluid may be either a liquid or a gas. A liquid can have a free surface, that is a surface from which all pressure is removed except that of its own vapour. A liquid has intermolecular forces which hold it together so that it possesses volume but no definite shape. A liquid is relatively incompressible. A gas consists of molecular in motion which collide with each other tending to disperse it so that a gas has no set volume or shape. A gas is very compressible, and when all external pressure is removed, it tends to expand indefinitely. A gas is therefore, in equilibrium only when it is completely enclosed.

ii) Viscosity

The viscosity of a fluid is a measure of its resistance to shear or angular deformation. Consider two parallel plates as in Fig.(1) sufficiently large so that edge conditions may be neglected, placed a small distance y apart, the space between being filled with the fluid. Assume that the upper one is moved relative to the lower one with a velocity U by the application of a force F corresponding to some area A of the upper plate. Such a condition is approximated in the clearance space of a flooded journal bearing. Particles of fluid in contact with each plate will adhere to the surfaces, and if the distance y is not too great or the velocity U too high, the velocity gradient will be a

straight line. The action is much as if the fluid were made up of a series of thin sheets, each of which would slip of little relative to the next. Experiment has shown that for a large class of fluids $F \sim AU/y$.

It may be seen from similar triangles that U/y can be replaced by the velocity gradient du/dy . If a constant of proportionality μ is introduced, the shearing stress between any thin sheets of fluid is $T = F/A = \mu U/y = \mu \frac{\partial u}{\partial y}$ which is called Newton's equation of viscosity.

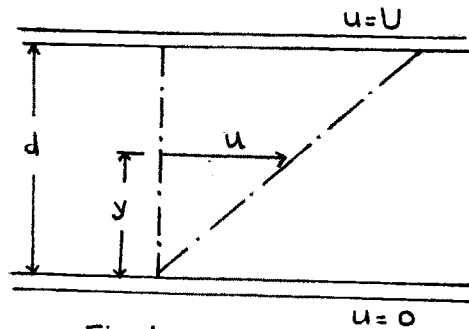


Fig. 1

By transformation $\mu = T / \frac{du}{dy}$, which is called coefficient of viscosity, the absolute viscosity, or the dynamic viscosity. | ?

$$\begin{aligned} \mu &= \frac{\text{Shearing stress}}{\text{Velocity gradient}} \\ &= \frac{\text{Force/area}}{\text{Velocity/length}} = ML^{-1}T^{-1} \end{aligned}$$

In the metric absolute, or physicists', system the dimensions of absolute viscosity are dyne-second per square

centimeter. This unit is called the poise (the metric gravitational system is rarely used).

At most fluids have low viscosities, the centipoise (= 0.01 poise) is frequently a more convenient unit. It has the further advantage that since the viscosity of water at 68.4°F is 1 centipoise, the value of viscosity in centipoises is an indication of the viscosity of any fluid relative to that of water. Therefore the value in centipoises is numerically equal to the specific viscosity (a dimensionless ratio) of the fluid relative to that of water at 68.4°F.

iii) Kinematic viscosity :

In many problems involving viscosity, there frequently appears the value of viscosity divided by density. This quotient is called kinematic viscosity because no forces is involved, the only dimensions being length and time.

$$\nu = \mu/\rho = \frac{ML^{-1}T^{-1}}{ML^{-3}} = L^2T^{-1}$$

In the metric system viscosity in poises divided by density in gram per cubic centimeter gives kinematic viscosity in square centimeters per second and its unit is called stoke. The centistoke (= 0.01 stoke) is often a more convenient unit and is obtained by dividing centipoises by gram per

cubic centimeter.

iv) Steady and Unsteady Fluids :

A fluid flow is called a steady flow if the velocity field which does not depend upon time, and if it depends on time then the flow is called an unsteady flow. If the flow variables like velocity, pressure etc. oscillate then, the flow is called quasi-steady. The steadiness and unsteadiness of a flow depend upon the observer. To some observer a flow may appear to be steady, while to another the same flow may appear to be unsteady. In practice, we always try to choose, if possible, the frame of reference in such a manner that the flow should appear to be steady, because this simplifies the calculations to a great extent.

v) Non-dimensional Numbers

a) Reynolds Number :

The dimensionless quantity Re defined as

$$Re = \frac{UL\rho}{\mu} = \frac{UL}{\nu}$$

where U, L, ρ and μ are some characteristic values of the velocity, Length, density and viscosity of the fluids respectively, is known as the Reynolds number in honour of the British scientist Osborne Reynolds who in 1883 demonstrated the importance of Re in the dynamics of viscous fluids.

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It can be easily seen from the equation of motion that the inertia forces [terms like $\rho u \frac{\partial u}{\partial x}$] are of the order of $\rho U^2/L$ and the viscous forces [terms like $\mu \frac{\partial^2 u}{\partial x^2}$] are of the order of $\mu U/L^2$

Therefore

$$\frac{\text{Inertia forces}}{\text{Viscous forces}} \sim \frac{\rho U^2/L}{\mu U/L^2} = \frac{UL}{\nu} = Re$$

For this reason Reynolds number is sometimes spoken of as the ratio of inertial to viscosity forces. It is in fact a parameter for viscosity, for if Re is small the viscous forces will be predominant and the effect of viscosity will be felt in the whole flow field. On the other hand, if Re is large the inertial forces will be predominant and in such a case the effect of viscosity can be considered to be confined in a thin layer, known as boundary layer, adjacent to a solid boundary.

b) Prandtl Number

The ratio of the kinematic viscosity to the thermal diffusivity of the fluid, i.e.

$$\frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{K/\rho C_p} = \frac{\mu C_p}{K} = Pr.$$

is designated as the Prandtl number named after the German

scientist Ludwig Prandtl. It is a measure of the relative importance of heat conduction and viscosity of the fluid. The Prandtl number like the viscosity and thermal conductivity, is a material property and is thus varies from fluid to fluid. For air $Pr = 0.7$ (approx.) and for water at $60^{\circ}F$, $Pr = 7.0$ (approx.). For liquid metals the Prandtl number is very small e.g. for mercury $Pr = 0.044$, but for highly viscous fluids it may be very large, e.g. for glycerine $Pr = 7250$.

c) Dimensionless Coefficient of Heat Transfer
(Nusselt number)

In the dynamics of viscous fluids one is not much interested to know all the details of the velocity and temperature fields but would certainly like to know quantity of heat exchanged between the body and the fluid. This quantity of heat transfer can be calculated with the help of coefficient of heat transfer $\alpha(x)$, which is defined by Newton's law of cooling as follows :

If $q(x)$ is the quantity of heat exchanged between the wall and the fluid, per unit time, at a point x , then $q(x) = \alpha(x)(T_w - T_o)$ (Newton's Law of cooling) where $(T_w - T_o)$ is the difference between the temperature of the wall and heat of the fluid. Since at the boundary the heat

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exchanged between the fluid and the body is only due to conduction, according to Fourier's Law, we have

$$q(x) = -k \left[\frac{\partial T}{\partial \eta} \right]_{\eta=0}$$

where η is the direction of the normal to the surface of the body. From these two laws we can define a dimensionless coefficient of heat transfer which is generally known as the Nusselt number as follows :

$$Nu = \frac{\alpha(x)L}{k} = \frac{L}{(T_w - T_o)} \left[\frac{\partial T}{\partial \eta} \right]_{\eta=0}$$

where L is some characteristic length in the problem.

vi) Flow Through Porous Media

A porous medium is literally a solid which contains a number of small holes distributed throughout the solid. These holes may be effective or ineffective. By effective holes we mean those holes through which the fluid can actually pass. It is these holes which contribute towards the porosity of the material. By ineffective holes we mean those holes through which the fluid cannot pass. These holes may either be so fine that fluid cannot move through them due to surface tension or the holes may not be interconnected. If the holes are not interconnected then the fluid cannot pass through them and thus these become ineffective. In

future by holes we shall mean only the effective holes. The holes may be very small or moderately large. Some of the examples of porous media are spuncles, a pack of sand, cotton and woolen padings, wood dust, soil, wood leather, sandstone and foamed plastics.

vii) Permeability

The permeability of a porous medium is its most useful fluid, flow property. The permeability is a measure of the ease with which a fluid will flow through a medium; the higher the permeability, the higher the flow rate for a given pressure gradient. The permeability is a statistical average of the fluid conductivities of all the flow channels in the variations in size, shape, direction, and interconnections of all the flow channels.

The most commonly used unit of permeability in the darcy. The American petroleum Institute defines a darcy as follows :

"A porous medium has a permeability of one darcy when a single phase fluid of one centipoise viscosity that completely fills the voids of the medium will flow through it under condition of viscous flow at the rate of one cubic centimeter per second equivalent to hydraulic gradient of one atmosphere per centimeter."

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For linear, horizontal, isothermal flow fluid, the equation is $q = \frac{KA}{\mu} \frac{dp}{dL}$ in which the common laboratory units are used :

q = flow rate cm^3/sec

K = permeability, darcys

A = Cross-section area cm^2

μ = Viscosity, centipoises,

$\frac{dp}{dL}$ = Pressure gradient.

viii) Porosity :

The porosity of a porous medium is defined as the void volume, or volume of pore space divided by the total volume of the medium. Void, or pore, volumes are usually determined by measuring either gravimetrically or volumetrically the amount of liquid needed to saturate the dry medium. Pore volume is determined from measurements of the external dimensions of the medium or from the volume of liquid displaced by immersion of the saturated medium. Porosities are expressed as either in fractions or in percentage.

The average porosity of a very large porous medium such as a oil-bearing sand may be determined from the porosity of a number of small core samples of the reservoir rock. A simple-arithmetic average will suffice when sufficient

samples are available to get a statistical distribution of porosities in the core samples.

The porosity of material is defined as the fraction of the total volume of the material which is actually occupied by the holes. To obtain the apparent density of a porous material we first calculate the density of the pore-free material ρ_s and then the density ρ_a of the dry porous material. From this the porosity is defined as $\epsilon = 1 - \rho_a / \rho_s$.

ix) Two-dimensional Boundary Layer Equations :

The Prandtl boundary layer equations for a two dimensional unsteady incompressible flow are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$$

and $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

The boundary conditions, under which these equations are usually integrated are :

$$y = 0; u = v = 0; y \rightarrow \infty; u = U(x, t)$$

in which the first is the no slip condition and the condition of non-porous wall and the second is obtained from the consideration that the velocity u , in the boundary layer must join smoothly on to the main stream velocity.

x) Characteristic Boundary Layer Parameters

a) Boundary layer thickness (δ)

Although the transition from the velocity in the boundary layer to that of potential flow takes place asymptotically, a value which is very close to the potential flow velocity is practically attained in a small distance from the boundary. Hence the boundary layer thickness has tentatively been regarded as that distance from the wall where the velocity in the boundary layer (i.e. 'u') differs from the potential flow velocity $U(x)$ by 1%.

$$\text{Thus } \delta = (y)_{u=99\%U}$$

In the case of flat plate

$$\delta \approx \left[\eta \sqrt{\frac{\nu x}{U_0}} \right]_{u=99\% U_0}$$

It can be seen that $u = 99\% U_0$ at $\eta = 5$ (approx.). Hence the boundary layer thickness on a flat plate is given by

$$\delta = 5 \sqrt{\frac{\nu x}{U_0}} = \frac{5x}{\sqrt{Re x}}$$

It may be noted that the growth of the boundary layer thickness with x is parabolic since the definition of the boundary layer thickness is somewhat arbitrary, a more physically meaningful thickness via., 'displacement thickness' is introduced.

b) Displacement thickness (δ_1)

The defect is the volume rate of flow, at a section $x = \text{constant}$ caused by the action of friction is given by

$$\int_0^{\infty} (U-u) dy$$

If we denote this defect in the volume rate of flow by $\delta_1 U$ then δ_1 is known as the displacement thickness. Thus

$$U\delta_1 = \int_0^{\infty} (U-u) dy$$

A physical significance to the displacement thickness may be given in the following manner.

In the adjoining figure, for the velocity distribution, drawn a line DE such that area OCD = area CEB, then OD = δ_1 since area OAED = area OAECB

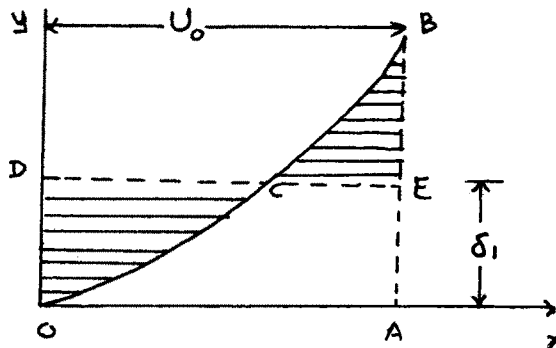


Fig. 2.

$$\text{or } U\delta_1 = \int_0^{\infty} (U-u) dy$$

$$\text{or } \delta_1 = \int_0^{\infty} (1-u/U) dy$$

Thus δ_1 signifies the distance by which the stream lines of

the potential flow are shifted owing to the formation of the boundary layer (the velocity) causes the displacement of the potential flow stream lines).

In analogy to the displacement thickness another thickness, known as momentum thickness, is also introduced in boundary layer calculations which will be used later.

c) Momentum thickness (δ_2)

The loss of momentum in the boundary layer, as compared with the potential flow, is given by

$$\int_0^{\infty} \rho u U dy - \int_0^{\infty} \rho u \cdot u dy$$

or $\rho \int_0^{\infty} u(U-u) dy.$

If $\rho U^2 \delta_2$ denotes the loss of momentum, then δ_2 is known as momentum thickness.

Thus

$$\rho U^2 \delta_2 = \rho \int_0^{\infty} u(U-u) dy$$

or $\delta_2 = \int_0^{\infty} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy .$

d) Skin-Friction

The shearing stress on the plane boundary is given by

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$$\tau_w = \mu \left[\frac{\partial u}{\partial y} \right]_{y=0}$$

In the case of flow past a flat plate

$$\tau_w = \mu U_0 \sqrt{\frac{U_0}{\nu} x} f''(0) = \frac{0.332}{\sqrt{Re x}} \rho U_0^2$$

Hence the dimensionless shearing stress, which is also known as local skin-friction is given by

$$C_f = \frac{\tau_w}{\rho U_0^2 / 2} = \frac{0.664}{\sqrt{Re x}}$$

The drag per unit width, for one side of the plate of length l is calculated as

$$D = \int_0^l \tau_w dx = 0.664 \cdot \rho U_0^2 \sqrt{\frac{\nu l}{U_0}}$$

This shows that the drag-coefficient in this case is

$$C_D = 2D / \rho U_0^2 l = \frac{1.328}{\sqrt{Re l}}$$

where $Re l = U_0 l / \nu$

xi) Thermal Boundary Layers.

The fluid at a large distance from the surface is not materially effected by the heated body. This narrow region (thin layer) near the surface of the body is known

as thermal boundary layer analogous to the concept of velocity boundary layer. The problem of thermal boundary layers may be classified into two categories : viz.,

i) Forced convection and ii) Free convection. By forced convection we mean the flow in which the velocities arising from the variable density (i.e. due to the force of bouyancy) are negligible in comparison with the velocity of the main or forced flow, whereas in free convection, also known as natural convection, the motion is essentially caused by the effect of gravity on the heated fluid of variable density.

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