CHAPTER - III

Steady Laminar Flow Past a Heated Horizontal Plate Embedded In a Saturated Porous Medium

1) Introduction

Blasius [1] was the first person who solved the Prandtl boundary layer equation for an incompressible viscous flow over a flat plate by assuming a series solution. Later Toffer [2] and Hawarth [3] tackled the same problem by using the numerical methods to obtain the solution. In all these analysis it is clear that a boundary layer of thickness 'δ' is formed and in this layer the streamwise component of the flow velocity 'u' varies from zero on the plate to U_o on the free stream. But no mention is being made about transverse component V which has considerable influence on motion of viscous fluid. Kankov [4] was the first mathematician who investigated in 1960 the formation of two boundary layers associated with two components of velocity u and v such that the thickness δ component of velocity u varies from zero to U_o and in thickness Δ the component v varies from zero to the plate to zero at the free stream.

Howa?

Recently B.C.Chandrasekhara [5] has obtained on interesting solution of the existence of transverse and axial velocity components for thermal boundary layers in the case of flow past a horizontal plate embedded in saturated porous medium. A.K.Kolar and V.M.K.Sastri [6] by using the implicit Crank-Nicolson-Predictor-corrector method investigated the influence of step discontinuity in porous plate temperature in free convection with transpiration.

By using implicit C.N.P.C.Method of finite difference scheme [6] and [7] we obtained a solution of the boundary layer equation for steady laminar flow past a heated horizontal plate embedded in saturated porous medium, following the formalism of B.C.Chandrashekara which is one of the interesting problems in fluid Mechanics. B.P.Jadhav and B.B.Waghmode [9] worked on Laminar Boundary Layer of a Power Law Fluid past a continuously Moving Porous flat plate by using zeroth approximate method.

2) Physical Model and Mathematical Analysis

The governing equations of motion for two dimensional boundary layer are given by

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K} u \qquad \dots (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \dots (2)$$

where K : permeability of the porous medium.

ν : Kinematic viscosity of the fluid with boundary conditions

$$\left.\begin{array}{l} u = U_o, \quad v = V_o \quad \text{as} \quad y = 0 \\ u \to 0 \text{ as } y \to \infty \end{array}\right\} \qquad \ldots (3)$$

3) <u>Analysis of the Problem</u>

To solve the above equation we define a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \qquad \ldots (4)$$

Substituting equation (4) in (1) we get

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3} - \frac{\nu}{K}\frac{\partial \psi}{\partial y} \qquad \ldots (5)$$

$$\text{i.e. } \frac{\partial \psi}{\partial y}\frac{o^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 u}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3} - B\frac{\partial \psi}{\partial y}$$

where $B = \nu / K$

Introducing a similarity transformation as

$$\eta = \sqrt{\frac{U_o}{\nu x}} \cdot y, \quad \psi(\eta) = \sqrt{\nu U_o x}\; f(\eta) \qquad \ldots (7)$$

then

$$
\left.
\begin{aligned}
&\frac{\partial \psi}{\partial y} = U_o\, f'(\eta) \\
&\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) = U_o \frac{\partial f'}{\partial x} \\
&\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} = U_o^2\, f' \frac{\partial f'}{\partial x} \\
&\frac{\partial^2 \psi}{\partial y^2} = U_o\, f''(\eta) \sqrt{\frac{U_o}{\nu\, x}} \\
&\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = U_o^2\, f''(\eta) \frac{\partial f}{\partial x} + \\
&\qquad\qquad + \frac{U_o^2\, f(\eta) f''(\eta)\, x^{-1}}{2} \\
&\nu\, \frac{\partial^3 \psi}{\partial y^3} = \frac{U_o^2\, f'''(\eta)}{x}
\end{aligned}
\right\} \quad \ldots (8)
$$

Putting the values of equation (8) in equation (5) we get

$$
U_o^2\, f' \frac{\partial f'}{\partial x} - U_o^2\, f'(\eta) \frac{\partial f}{\partial x} - \frac{U_o^2\, f(\eta)\ f''(\eta) x^{-1}}{2} = U_o^2\, f'''(\eta)\ x^{-1} - B\ U_o\ f'(\eta) \quad \ldots (9)
$$

On simplifying we get

$$
f'''(\eta) + \frac{1}{2}\, f(\eta)\ f''(\eta) - S f'(\eta) = 0 \quad \ldots (10)
$$

where $S = \left(\frac{B}{U_o}\right) x$

with boundary conditions

$$\left.\begin{array}{lll} \eta = 0 & f = \alpha & f' = 1 \\ \eta = \infty & & f' = 0 \end{array}\right\} \quad \ldots (11)$$

where α is suction/injection parameter ($\alpha > 0$ for suction and $\alpha < 0$ for injection equation (10) can be solved by using the method of successive approximation.

Making zeroth approximation for f and it derivatives as

$$\left.\begin{array}{l} f_o(\eta) = \alpha - \frac{1}{\lambda} e^{-\lambda\eta} + \frac{1}{\lambda} \\ f_o'(\eta) = e^{-\lambda\eta} \\ f_o''(\eta) = -\lambda e^{-\lambda\eta} \end{array}\right\} \quad \ldots (12)$$

where λ is a constant to be determined. The solution of equation (10) subject to boundary conditions (11) can be obtained for different approximates from the equation

$$f_i''' = -\frac{1}{2} f_{i-1} f_{i-1}'' + S f_{i-1}' \quad \ldots (13)$$

$$i = 1, 2, 3 \ldots$$

for the first approximation we have

$$f_1''' = -\frac{1}{2} f_o f_o'' + S f_o' \quad \ldots (14)$$

Using the boundary condition (11) along with equation (14) gives

$$f_1''' = -\frac{1}{2}\left[\alpha - \frac{1}{\lambda}e^{-\lambda\eta} + \frac{1}{\lambda}\right]\left[-\lambda e^{-\lambda\eta}\right] + Se^{-\lambda\eta}$$

$$f_1''' = \left[\frac{\alpha\lambda}{2} + \frac{1}{2} + S\right]e^{-\lambda\eta} - \frac{1}{2}e^{-2\lambda\eta} \qquad \ldots (15)$$

On integrating equation (15) we get

$$f_1''(\eta) = \left[\frac{\alpha\lambda + 1 + 2S}{-2\lambda}\right]e^{-\lambda\eta} + \frac{e^{-2\lambda\eta}}{4\lambda} \qquad \ldots (16)$$

Integrating again we get

$$f_1'(\eta) = \left[\frac{\alpha\lambda + 1 + 2S}{2\lambda^2}\right]e^{-\lambda\eta} - \frac{e^{-2\lambda\eta}}{8\lambda^2} \qquad \ldots (17)$$

Integrating again we get

$$f_1(\eta) = \left[\frac{\alpha\lambda + 1 + 2S}{-2\lambda^3}\right]e^{-\lambda\eta} + \frac{e^{-2\lambda\eta}}{16\lambda^3} + C_1 \qquad \ldots (18)$$

where C_1 is constant of integration to be determined by using the boundary conditions

$$C_1 = \alpha + \frac{\alpha\lambda + 1 + 2S}{2\lambda^3} - \frac{1}{16\lambda^3} \qquad \ldots (19)$$

$$f_1(\eta) = \left[\frac{\alpha\lambda + 1 + 2S}{-2\lambda^3}\right]e^{-\lambda\eta} + \frac{e^{-2\lambda\eta}}{16\lambda^3} + \alpha + \frac{\alpha\lambda + 1 + 2S}{2\lambda^3} - \frac{1}{16\lambda^3} \qquad \ldots (20)$$

$$f'(\eta) = \left[\frac{\alpha\lambda + 1 + 2S}{2\lambda^2} \right] e^{-\lambda\eta} - \frac{e^{-2\lambda\eta}}{8\lambda^2}$$

Putting $\eta = 0$ $f' = 1$, then

$$\lambda = \frac{\frac{\alpha}{2} \pm \frac{1}{2}\sqrt{\alpha^2 + 2(3 + 8S)}}{2}$$

$$\lambda = \frac{\alpha}{2} \pm \sqrt{\frac{\alpha^2}{4} + 4(S + \frac{3}{8})}\Big/2 \qquad \ldots (21)$$

$$f_1''(0) = - \left[\frac{\alpha\lambda + 1 + 2S}{2\lambda} \right] + \frac{1}{4\lambda} \qquad \ldots (22)$$

The values of λ, $f_1(\eta)$, $f_1'(\eta)$, $f_1''(\eta)$ have been obtained for different values of S and different values of suction (α). The values have been shown analytically and graphically.

RESULTS AND DISCUSSION

a) Boundary Layer Thickness (δ)

The boundary Layer thickness (δ) is defined as that distance from the moving surface at which

$$\frac{u}{U_o} = f'(\eta) = 0.01$$

from the table 1 to 3 it has been observed that as the porous parameter (S) increases so the skin friction and the boundary layer thickness (δ) decrease.

b) Also from the table (4) as suction (α) is zero, porous (S) = 0 and λ = 0.62 we see that as η increases the velocity distribution decreases.

Also for α = 0, S = 0.5 and λ = 0.62 we see that as η increases the velocity distribution decreases.

Again we check for α = 0, S = 0 and λ = 0.76. We see here again that as η increases the velocity distribution decreases.

CONCLUSIONS

The effect of porous parameter makes the skin friction and boundary layer decrease [Refer table 1 to 3].

As the suction (α) increases the boundary layer thickness (δ) decreases [Refer table 1 to 3]. As the parameter S increases, the velocity distribution decreases as η varies from 1 to 10 [Refer table 4].

The above results are compared with the results of B.P.Jadhav and B.B.Waghmode [10] and found that they are approximately same.

see page 55

Table 1

for Section (α) = 0

S	λ	$-f''(0)$	δ
0	0.62	0.40	13
0.5	0.95	0.79	10
1	1.17	1.07	9
1.5	1.37	1.28	8
2	1.56	1.46	7

Table 2

for Section (α) = 0.5

S	λ	$-f''(0)$	δ
0	0.76	0.58	10
1.5	1.07	0.96	9
1	1.31	1.21	8
1.5	1.5	1.41	7
2	1.68	1.59	6

Table 3

for Section (α) = 1

S	λ	$-f''(0)$	δ
0	0.92	0.77	11
0.5	1.22	1.12	9
1	1.45	1.36	8
1.5	1.65	1.56	7
2	1.82	1.76	6

Table 4

For Section (α) = 0 Porosity (S) = 0, λ = 0.67

and S = 0.5

η	$A = f'(\eta)$	$B = f'(\eta)$	$C = f'(\eta)$
0	0.9755	1.3788	0.9782
1	0.6056	0.8225	0.5113
2	0.3492	0.4658	0.2509
3	0.1946	0.2574	0.1199
4	0.1066	0.1404	0.0566
5	0.0579	0.0761	0.0266
6	0.0313	0.0411	0.0125
7	0.0169	0.0221	0.0058
8	0.0091	0.0119	0.0027
9	0.0049	0.0064	0.0013
10	0.0026	0.0034	0.0006

$A = f'(\eta)$ for $\alpha = 0$ $S = 0$ $\lambda = 0.67$
$B = f'(\eta)$ for $\alpha = 0$ $S = 0.5$ $\lambda = 0.92$
$C = f'(\eta)$ for $\alpha = 0.5$ $S = 0$ $\lambda = 0.76$

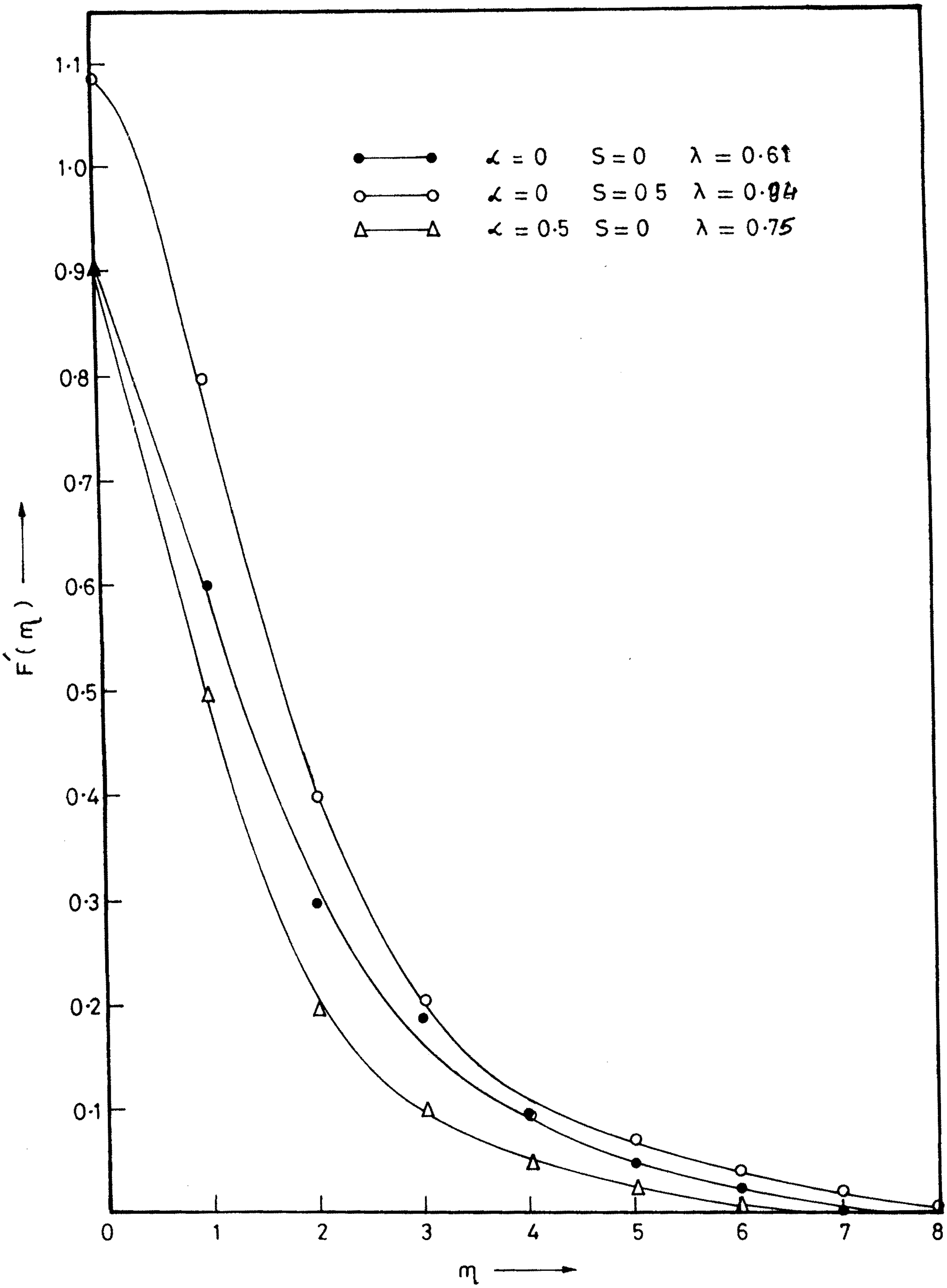

Fig. 1 — VELOCITY DISTRIBUTION.

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