

PREFACE

The concept of Fuzzy Sets was introduced by L.A.Zadeh in 1965. Since its inception, the theory of fuzzy sets has developed in many directions and it is finding applications in a wide variety of fields. As an outgrowth of these attempts, the concept of Multiple Boolean Algebra is introduced in 1985 by Silvano Di Zenzo [1]. We have tried to study these structures on fuzzy sets.

Though this study is ^a combine ^d one, the concept of multiple Boolean algebra and that of fuzzy sets are independent. A multiple Boolean algebra is an attempt to generalise the Boolean algebra, where as Fuzzy set is generalisation of ordinary set. The set of all subsets of a set becomes a Boolean algebra. On the other hand Zenzo shows that the set of all fuzzy subsets of a set becomes a multiple Boolean algebra if the binary operations on it are defined in a suitable manner.

Zenzo has considered the case of finite multiple Boolean algebras only. We have defined multiple Boolean algebra which also includes the infinite case. Our definition comes as an innate generalisation of his. Example of infinite multiple Boolean algebra can be easily given using his considerations. Moreover, his

considerations also offer us some new fuzzy connectives defined along purely algebraic lines.

We have introduced isomorphism between two multiple Boolean algebras. Through this isomorphism we have proved that all the examples of multiple Boolean algebra given by Zenzo are nothing but the powers of the basic multiple Boolean algebra defined by him. Our main contribution seems to be the ^Eexample 3 in ^Cchapter 1 which probably becomes an obstacle in our way of structure determination. In this view, we suggest an improvement in the definition of multiple Boolean algebra. Further, as one naturally tempts to do, we have defined ^{ja} sub-multiple Boolean algebra.

The dissertation consists of four chapters. In ^Cchapter 0, basic definitions, from fuzzy set theory and Boolean algebra theory are stated along with some known results which are used throughout in the subsequent part of the dissertation.

Chapter 1 deals with the notion of multiple Boolean algebra as introduced by Zenzo. The proofs of the existence theorems are given in more details. Isomorphism of multiple Boolean algebras is defined which is followed by some structure determining theorems. A 'different' multiple Boolean algebra is given.

^C In ²chapter 2, new fuzzy connectives are introduced

and it is shown that the algebraic structure so determined over the fuzzy sets is that of a pair of multiple Boolean algebras.

Chapter 3 contains insertion of some more fuzzy relations which seem to be appropriate through ^{which} simple theorems ^{are} proved. A definition of sub multiple Boolean algebra is given.

Finally, more appropriate title of the dissertation would have been "A Study of Multiple Boolean algebra and Fuzzy Sets." However for some technical reasons, this change could not be incorporated.

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" Science is what we know and Philosophy
is what we don't know "

- Bertrand Russel.

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